



EFFICIENCY EVALUATION OF INSURANCE INDUSTRY BY DYNAMIC AND NETWORK DEA: CASE STUDY OF NON-LIFE INSURERS IN THAILAND

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Keywords

Dynamic Network Data Envelopment Analysis (DNDEA), Dynamic Black-Box Data Envelopment Analysis (DBDEA), Static Network Data Envelopment Analysis (SNDEA), Insurance Companies, Multi-Objective Programming (MOP)

Abstract

The operational performance of insurance companies is a matter of keen interest to both investors and policyholders. Building on previous research that has focused on the performance evaluation and process improvement of insurance firms, this study aims to evaluate the efficiencies of non-life insurance companies in Thailand over the period from 2017 to 2022 via dynamic and network data envelopment analysis. In this work, three models are designed: dynamic black-box data envelopment analysis (DBDEA), static network data envelopment analysis (SNDEA), and dynamic network data envelopment analysis (DNDEA), all of which are formulated and solved by multi-objective programming (MOP). The results show that the developed models are useful and possess discriminating power in evaluating the non-life insurers. This study contributes in designing the evaluation modes for Thailand based on an innovative formulation of MOP that optimizes departmental and yearly efficiencies of insurance companies simultaneously.

1. Introduction

The insurance industry is a type of service business that provides safety and security to individuals or businesses. There are various types of insurance, such as life insurance, travel insurance, health insurance, motor vehicle insurance and property insurance. Another type of insurance is re-insurance, which provides insurance to other insurance companies for the financial risks they assume.

Insurance helps to mitigate the effects of risks that are encountered in daily life. The underlying operating principle of any insurance company is to spread risk among a large number of people. A large population takes out insurance policies and pays premiums to the insurer.

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Whenever a loss occurs, the policy holder is compensated out of the corpus of funds collected from the policyholders collectively. On the other hand, the insurance company hedge against the risk they assume by using investments. In other words, the insurance company generates capital by way of premiums from policyholders, and, in turn, directs this accumulated capital into productive investments.

The insurance industry is an important element of any country's economy, and it becomes an increasingly crucial sector as a country develops. In order to gauge the success of its acumen in business and its contribution to the overall growth of the economy in general, the operating performance of an insurance company can be assessed by several techniques. There are two primary methodological approaches for evaluating the efficiency of insurance firms. The statistics-based econometric approach takes an assumed production function and measures efficiency based on both random and firm-specific efficiency components. The other method for estimating insurer firms' efficiency involves a mathematical programming approach called data envelopment analysis (DEA) (Eckles & Saardhom, 2007).

DEA has been applied in various domains, such as insurance (Hwang & Kao, 2008), insurance asset management (Liu, 2021), manufacturing (Gupta et al., 2016; Zhu et al., 2019), and many others (Kaffash et al., 2019). Evidently, DEA has also proven to be a popular approach in insurance studies (Eling & Luhnen 2019).

Insurance structures, by nature, lend themselves well to network data envelopment analysis (Network-DEA), which is handled with the "black-box" problem. The original network-DEA structure comprises two connected sub-processes, whereby the outputs of the first production process are then taken as inputs of the second production process, and this approach has been adopted in several fields including the insurance industry (Kaffash et al., 2019).

In recent years, the number of companies competing in Thailand's insurance market has decreased, mainly due to firms encountering financial difficulties or as a result of mergers and acquisitions. In terms of the latter, such mergers and acquisitions can function to increase the companies' size as well. The insurance sector is primarily divided into three categories: life insurance, non-life insurance (general insurance), which includes all other aspects of insurance such as motor insurance, property insurance, travel insurance, marine insurance, fire insurance, accident and health insurance, and re-insurance. In 2018, Thailand's non-life insurance industry was ranked second in the ASEAN insurance market. In fact, the total income derived from premiums of the Thai life insurance and non-life insurance industry accounted for 23.34% of the ASEAN market's total premium income, lagging only behind that of Singapore (27.97%). In addition, the Thai insurance industry was highly ranked in the non-life insurance industry in Asia 2018.

Previous research on the Thai insurance industry by Karin et al. (2005) examined the cost efficiency of life insurance firms in Thailand from 1997 to 2002. That study suggests that increasing efficiency should be the main priority for the Thai life insurance industry. Efficiency and productivity are not only crucial for the survival of the non-life insurance industry, but also for the economic growth of Thailand (Yaisawarn et al., 2014).

This study's main objective is to empirically investigate and measure efficiencies of the Thai non-life insurance industry using the dynamic network data envelopment analysis model. The specific objectives of this research include: (1) to measure and analyze the efficiency of service and finance departments of the insurance industry in Thailand; (2) to evaluate the aforementioned efficiency via network data envelopment analysis using multi-objective programming (MOP); (3) to compare the efficiency evaluations of the DEA models (DBDEA, SNDEA, and DNDEA); and (4) to compare and rank the insurance companies' performance.

The rest of this article is organized as follow. The related work and literature are reviewed in Section 2, while Section 3 addresses the methods and research process. The results are shown in section 4. Finally, the discussion and conclusions are given in Section 5.

2. Related Works

This section reviews and discusses related background materials and research work, including data envelopment analysis, dynamic and DEA, multi-objective programming, and related studies in insurance industry evaluation.

2.1 Data Envelopment Analysis (DEA)

The methodology for measurement of productive efficiency was originally researched by Farrell (1957), who investigated the efficiency of a simple case with a production frontier. Mathematical programming was used to evaluate the efficiency of single output and single input, and the efficiency ratio was determined as: $\text{efficiency} = \text{output}/\text{input}$. Even though the Farrell method is a simple concept for measuring efficiency, it is inapplicable for the multivariate nature of efficiency indicators in the real-world industry. The pioneer Data Envelopment Analysis (DEA) model is proposed by Charnes, Cooper and Rhodes and called the CCR model (1978). This model can deal with multiple inputs and multiple outputs to measure efficiency. The efficiency calculation formula in the CCR model is: $\text{weighted sum of multiple outputs}/\text{weighted sum of multiple input}$.

The CCR model is used to calculate the efficiency value by constant return to scale (CRS), which means for every increase in input, a proportional increase in output follows. However, the inefficiency of production may be caused by improper operation scale, which is technically an ineffective rate (Technical Efficiency, TE). To deal with this issue, the BCC model was developed by Banker, Charnes and Cooper (1984). The BCC model assumes that the production process conforms to variable returns to scale (VRS), to calculate the efficiency score.

In addition, the CCR model and BCC model can be divided into two types: input-based efficiency and output-based efficiency. For input-based efficiency, the focus is on examining current output to decide how much input needs to be used to achieve efficiency. Output orientation refers to comparing the efficiency of output with the same input. The data envelopment analysis model evaluates the weighted total ratio of maximum output and input of decision-making units (DMU) as its efficiency. In this way, the relative efficiency of the DMU is obtained. The following section describes the CCR and BCC models.

Consider n DMUs ($j = 1, 2, \dots, n$) to be evaluated, and each DMU has m inputs ($i = 1, 2, \dots, m$) and s outputs ($r = 1, 2, \dots, s$), and the calculation method of the efficiency score of the k th decision-making unit is expressed by formula (1). By calculating the maximum efficiency score, the weighted weight of input and output corresponding to the sum is the ratio of the output weighted total output multiplied by the output weight to the input weighted total input multiplied by the input weight. To be informed, all variables are non-negative and all weighted values are positive, and between 0 and 1.

CCR Ratio Model

$$\begin{aligned}
 \text{Max } E_k &= \frac{\sum_{r=1}^s u_r Y_{rk}}{\sum_{i=1}^m v_i X_{ik}} \\
 \text{s.t. } &\frac{\sum_{r=1}^s u_r Y_{rj}}{\sum_{i=1}^m v_i X_{ij}} \leq 1 \\
 &u_r, v_i \geq \varepsilon
 \end{aligned} \tag{1}$$

For computation of global optimality, one extra constraint $\sum_{i=1}^m v_i X_{ik} = 1$ can be added to (1) to convert the ratio model, as given above, to a linear programming model.

The BCC model is extended from CCR, where the BCC model has one extra parameter u_0 , called the scale factor. When u_0 is negative, it represents increasing returns to scale (IRS). In such cases, when input increases by a certain proportion, the proportion of the output increase is greater than the proportion of input increase. When u_0 is zero, it represents the constant return to scale (CRS) when the input increases by a certain proportion, and the proportion of output increase will be equal to the proportion of input increase. When u_0 is positive, it represents Decreasing Return to Scale (DRS), which means that when the input increases by a certain proportion, the proportion of output increase will be less than the proportion of the input increase.

The BCC model assumes that the production process has variable returns to scale, and is a model that can measure technical efficiency (TE) and scale efficiency (SE).

BCC Ratio Model

$$\begin{aligned}
 \text{Max } E_k &= \frac{\sum_{r=1}^s u_r Y_{rk} - u_0}{\sum_{i=1}^m v_i X_{ik}} \\
 \text{s.t. } &\frac{\sum_{r=1}^s u_r Y_{rj} - u_0}{\sum_{i=1}^m v_i X_{ij}} \leq 1 \\
 &u_r, v_i \geq \varepsilon \\
 &u_0 \text{ unrestricted in sign}
 \end{aligned} \tag{2}$$

To linearize the BCC model, we convert the BCC model by restricting the denominator in (2) with the constraint $\sum_{i=1}^m v_i X_{ik} = 1$.

2.2 Network and Dynamic Data Envelopment Analysis

The conventional DEA models deal with measurement of the relative efficiency of DMUs regarding multiple inputs or multiple outputs, excluding the internal or linking activities. This blind internal activity is referred to as a “black box”; that is to say, there is nothing revealed about what happens inside them (Tone & Tsutsui, 2009).

To deal with this problem, the slacks-based network DEA model was proposed. This model can evaluate departmental efficiencies, along with the overall efficiency of decision-making units (DMUs) (Tone & Tsutsui, 2009).

Network data envelopment analysis evaluates the efficiency of multi-departmental organizations. This model solves for the comparative overall efficiency of the organization along with the departmental efficiencies in a unified framework. In addition, the network DEA model can be applied to various industries; for instance, electric power companies, hospitals, broadcasting companies, and financial holding companies, as shown by Tone and Tsutsui (2009).

The network DEA deals with multiple departments connected by links of network structures within each period. However, for real-world problems, special concern needs to be given to intertemporal efficiency changes in DEA, which combine with the network structure by means of carry-over activities between two succeeding periods. To comprehensively solve this problem, Fare and Grosskopf (1997) proposed the first dynamic DEA model that dealt with the interconnecting activities. Tone and Tsutsui (2014) subsequently developed a dynamic DEA model with network structure, accounting for the fact that a carry-over comes from a department to the same department in the next period. They proved that the model can evaluate: (i) the overall efficiency over the entire observed period; (ii) dynamic changes in the period efficiency; and (iii) dynamic changes in departmental efficiency. The aforementioned model can be implemented in input-oriented, output-oriented and non-oriented (both input-oriented and output-oriented) forms under a constant-returns-to scale (CRS) or variable-returns-to-scale (VRS) assumption about the production possibility set.

2.3 Evaluation of the Non-life Insurance Industry

The traditional performance assessment framework for insurance companies is based on the production process of manufacturing systems. The flow of money in the insurance system requires the regeneration of money through investment. The financial portfolio theory for an insurance company views the activity of an insurer as a “leveled investment operation” that borrows funds, by issuing risky obligations (premium accumulation) and investing part of these funds in securities. An insurance company is widely considered to be a financial intermediary that generates capital by selling a diversified portfolio of insurance claims (capital generating opportunities), and invests the funds in a balanced portfolio of financial instruments (investment opportunities). Indeed, for a firm-level analysis of cash flow activities, the insurance company is viewed as financial intermediary that aims to maximize profits.

Prior research has used the DEA technique to study the efficiency of insurance companies. Yang (2005) used a two-stage data envelopment analysis model for estimating the performance of the Canadian life and health (L&H) insurance industry. In the first stage, a production and investment model for the computation of efficiency was used. The inputs of the first-stage model were labor expenses, general operating expenses, capital equity and claims incurred. The outputs were net premiums written and net income. The next stage utilized a dummy with value 1 as the input and the production efficiency score and inverse of the investment efficiency score as the outputs. This model allows integration of the production performance and investment performance for the insurance companies. The results showed that the Canadian L&H insurance industry operated fairly efficiently during the period that was examined.

Kao and Hwang (2008) used data from 2001-2002 to analyze the performance of 24 Taiwanese non-life insurance businesses using a two-stage DEA model. The system's first inputs included operational expenses, such as staff wages and various sorts of costs incurred in everyday operations, and insurance expenses, such as fees paid to agencies, brokers, and solicitors, as well as other charges involved with marketing the insurance business. Direct writing premiums (premiums received from insured clients) and reinsurance premiums (premiums received from ceding firms) are the outputs of the marketing sub-process, and these were used as inputs to the investment sub-process. Underwriting profit (profit earned from the insurance business) and investment profit (profit earned from the investment portfolio) were the system's outputs. The study divided insurance firm activities into two sub-processes. In comparison to the black box model, the results of that investigation indicated several unique discoveries. In particular, none of the insurers studied were found to be completely efficient in both stages.

Ram (2019) uses the network DEA models to assess non-life insurance businesses' performance in India. In the model, the first stage's inputs were reinsurance and operating expenses, while the second stage's output was net premium income, and the final outputs were incremental assets and benefits paid. Reinsurance expenses were the only input used in the first step of the Tone-Tsutsui model, which produced net premium income. This is the variable that connects the first and second stages. The two outputs were created in the second stage by combining net premium income with operating expenses. For the financial years 2011-2012 and 2013-2014, data was gathered from the IRDA Annual Reports and the Handbooks of Indian Insurance Statistics. The impact of return on equity and solvency ratio on the efficiency performance of the in-sample insurance companies was investigated in the study.

Zhao et al. (2021) examined the factors influencing the profitability of fifty-three Chinese property insurers from 2013 to 2017, and their evolution. Profitability was evaluated by using the DEA methodology's profit ratio efficiency, and a profit ratio change index was used to compare the performance of those insurers over time. Previous studies on insurer performance were used to select the indicators used in the study. Several factors influencing profitability were investigated, using Tobit regression models. The empirical findings highlighted the importance of an insurer's expenses and revenues being properly aligned, as well as the impacts of company size, age, and product specification on profitability.

Kaffash et al. (2019) reviewed and analyzed 132 DEA-application studies in the insurance industry, published between 1993 and July 2018, which included both applications and methodological discussions. They discovered the impact of current improvements such as InsurTech and market transparency in the application. In terms of methodologies, they discovered a gap in the literature for newly-developed DEA approaches such as shared resources dynamic network DEA, modified directional distance function, satisficing DEA, and fuzzy DEA.

There have also been more recent attempts to measure the efficiency of insurance companies. Cummins et al. (2010) estimated cost, revenue, and profit efficiency utilizing DEA. Eling and Jia (2019) developed an understanding of the efficiency and profitability relationship in business management and in the life and nonlife insurance industries. Barros et al. (2010) analyzed the efficiency, estimated by DEA, of the Greek insurance market using the innovative two-stage procedure. Tone et al. (2019) proposed a dynamic two-stage network DEA model for evaluating the corporate performance of 30 insurance companies in Malaysia, from 2008 to 2016. Grmanova and Strunz (2017) applied the DEA to measure the technical efficiency score of insurance companies and then to discover the relationships between efficiency and other indicators. Abdin et al. (2022) analyzed the efficiency of insurance companies in Indonesia by using two stages of DEA. DEA has been shown to be a very useful performance analytics tool that supports benchmarking systems in measuring, comparing, and enhancing various notations of efficiency, and the above observations lead us to consider a dynamic network DEA model to measure the efficiency of insurance companies, based on a review of the relevant research. Liu et al. (2025) design an evaluation indicator system and proposes the dynamic two-stage improved base point slacks-based measure (IBP-SBM) model for handling negative data. Smetek et al. (2024) evaluates the applicability of the Dynamic Financial Analysis for assessing the financial performance of insurers by incorporating into the analysis various areas of risk affecting the financial position, along with Asset Liability Management and Data Envelopment Analysis.

3. Methods

This section presents the data gathered and methods used in this research, including variables selection, dynamic black-box data envelopment analysis (DBDEA), static network data envelopment analysis (SNDEA), dynamic network data envelopment analysis (DNDEA) and multi-objective programming (MOP). Compared to slacks-based measure approach (Tone & Tsutsui, 2009; 2014; Tone et al., 2019), this study is based on BCC models and treats the departmental and yearly efficiencies as objective functions instead of constraints. Different DEA forms are equivalent and are feasible for evaluation.

The variable notation is shown in Table 1. Consider n DMUs ($j=1, \dots, n$) consisting of K departments ($k=1, \dots, K$) over T time periods ($t=1, \dots, T$). Let m_k and r_k be the numbers of inputs and outputs to department k , respectively. We denote the link leading from department k to department h by $(k, h)_l$ and the set of links by L_{kh} . The inputs, intermediate links and outputs for this study derived from previous studies on insures' performance are defined in Table 2.

The data for our study was obtained from annual financial statements of the non-life insurance industry in Thailand for the period from 2017 and 2022. The data comprises entries for 49 companies, as listed in Appendix 1. Because the dataset has imbalances and negative numbers and zero values, it was normalized. The mean normalization technique was used to resolve data imbalance. In addition, the translation variance technique was used to deal with negative numbers and zero numbers by adding a sufficiently large positive constant to the values of the inputs or outputs that have the non-positive numbers.

Table 1 *Parameters and Variables*

General	
n	Number of DMUs
T	Number of periods
K	Number of departments (functions)
M	The number of inputs to a DMU
R	The number of outputs from a DMU
r_k	Number of outputs from department k
m_k	Number of inputs to department k
(k, h)	The intermediate link from department k to department h
$(t, t+1)$	The carry-over link from period t to $t+1$
L	The set of links
DBDEA	
X_{ij}^t	The i -th input to department k at DMU $_j$ in period t
Y_{rj}^t	The r -th output from department k at DMU $_j$ in period t
$Z_{jp}^{(t-1,t)}$	The p -th link of DMU j from period t to $t+1$
$\phi_{(t,t+1)}$	The number of links from period t to $t+1$
SNDEA	
X_{ij}^k	The i -th input to department k at DMU $_j$
Y_{rj}^k	The r -th output from department k at DMU $_j$
$Z_{jp}^{(k,h)}$	The p -th link from department k to department h
$\phi_{(k,h)}$	The number of links from department k to department h
DNDEA	
X_{ij}^{kt}	The i -th input to department k at DMU $_j$ in period t
Y_{rj}^{kt}	The r -th output from department k at DMU $_j$ in period t
$Z_{jp}^{(k,h)t}$	The p -th link from department k to h at DMU j in period t
$Z_{jq}^{k(t-1,t)}$	The q -th link at department k , DMU j from period $t-1$ to t .
$\phi_{k(t-1,t)}$	The number of links at department k from period $t-1$ to t

Table 2 *Descriptions of the Variables*

Variables	Definitions	Type of variables
Department 1: Business and service department		
Operating expenses	Operating expenses are operating expenses not related to underwriting and claims, which are recognized as expenses on an accrual basis.	Input
Other underwriting expenses	Other underwriting expenses are other insurance expenses for both direct and indirect expenses, including contributions, which are recognized as expenses on an accrual basis.	Input
Commission and brokerages	Commission and brokerage expenses are recorded as deferred expenses and will be gradually amortized as expenses proportionately to the earned premium.	Input
Cash and Cash equivalents	Cash and cash equivalents report the value of a company's assets that are cash or can be converted into cash immediately.	Input carry-over
Gross Claim	The total amount of gross benefits and claims paid, minus the claims ceded to reinsurers for the year.	Output
Earned premium	The total amount of gross earned premiums minus premiums ceded to reinsurers for the year.	Output (link)
Department 2: Finance department		
Investment in securities	Investment securities are a category of securities-tradable financial assets such as equities or fixed income instruments-that are purchased with the intention of holding them for investment.	Input
Investment revenue	The total amount of generated income from all investment activities for the year.	Output
Net profit	The total amount of income after deducting all operating and tax expenses for the year.	Output

3.1 Dynamic Black-Box Data Envelopment Analysis (DBDEA)

The dynamic black-box DEA acts as a basic structure, which means that it ignores the internal activities of each DMU. During evaluation, only inputs, outputs and inter-period carry-over are considered. The structure of DBDEA is shown in Fig 1 and formulated as (3)-(5), where the objective function is to maximize the efficiency of every DMUo in each period.

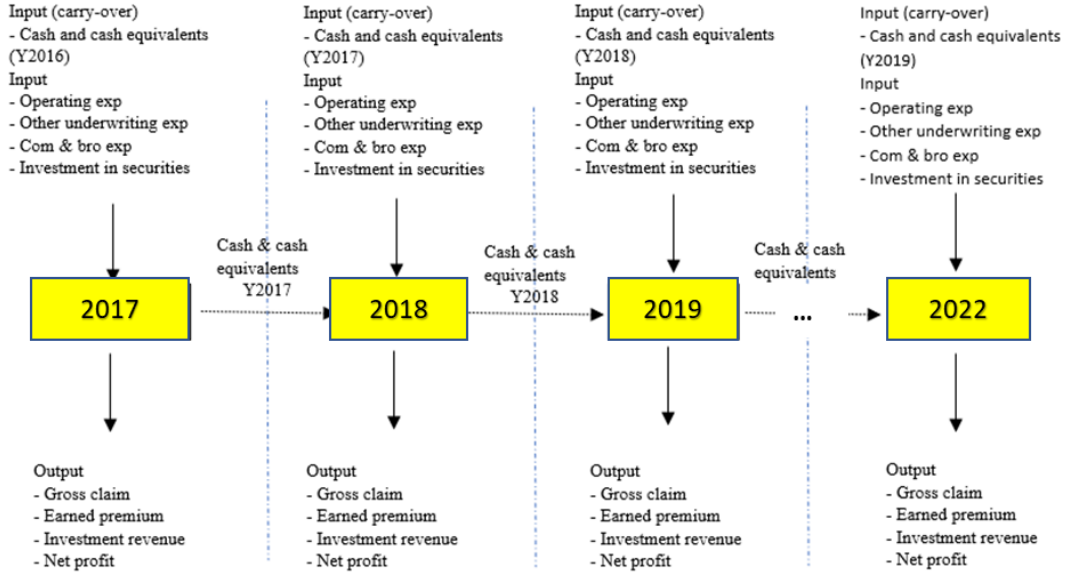


Fig 1 The Dynamic Black-box Data Envelopment Analysis (DBDEA) Model

$$\text{Max } E_0^t \quad t = 1, 2, \dots, T \quad (3)$$

s. t.

$$\frac{\sum_{r=1}^R u_r^t Y_{rj}^t + \sum_{\forall k} \sum_{p=0}^{\phi_{(t,t+1)}} \mu_{t+1}^t Z_j^{k(t,t+1)} - \alpha_0}{\sum_{i=1}^M v_i^t X_{ij}^t + \sum_{\forall k} \sum_{q=0}^{\phi_{(t-1,t)}} \omega_{t-1}^k Z_j^{k(t-1,t)}} \leq 1, \quad j = 1, 2, \dots, n, t = 1, 2, \dots, T \quad (4)$$

$$\sum_{i=1}^M v_i^t X_{ij}^t + \sum_{\forall k} \sum_{q=0}^{\phi_{(t-1,t)}} \omega_{t-1}^k Z_j^{k(t-1,t)} = 1 \quad (5)$$

$$j = 1, 2, \dots, n$$

$$u_r^k, v_i^k \geq \varepsilon > 0, \quad \alpha_k \text{ unrestricted in sign}, \quad r = 1, 2, \dots, r_k; \quad t = 1, 2, \dots, T; \quad i = 1, 2, \dots, m_k$$

3.2 Static Network Data Envelopment Analysis (SNDEA)

The static network data envelopment analysis model aims to measure the efficiency score for each department (sub-process) including the internal activities (linkage) of the DMU in the same period. However, it neglects the transition between periods (carry-over). The SNDEA structure is displayed in Fig 2 and is formulated as (6)-(9)

$$\text{Max } E_0^k, k = 1, 2, \dots, K \quad (6)$$

s. t.

$$E_o^k = \frac{\sum_{r=1}^{r_k} u_r^k Y_{ro}^k + \sum_{\forall(k,h)} \sum_{p=1}^{\phi_{k,h}} \mu_h^k Z_{op}^{(k,h)} - \alpha_k}{\sum_{i=1}^{m_k} v_i^k X_{io}^k + \sum_{\forall(g,k)} \sum_{q=1}^{\phi_{g,k}} \omega_g^k Z_{oq}^{(g,k)}} \quad , \quad k = 1, 2, \dots, K \quad (7)$$

$$\sum_{i=1}^{m_k} v_i^k X_{io}^k + \sum_{\forall(g,k)} \sum_{q=1}^{\phi_{g,k}} \omega_g^k Z_{oq}^{(g,k)} = 1 \quad , \quad k = 1, 2, \dots, K \quad (8)$$

$$\frac{\sum_{r=1}^{r_k} u_r^k Y_{rj}^k + \sum_{\forall(k,h)} \sum_{p=1}^{\phi_{k,h}} \mu_h^k Z_{jp}^{(k,h)} - \alpha_k}{\sum_{i=1}^{m_k} v_i^k X_{ij}^k + \sum_{\forall(g,k)} \sum_{q=1}^{\phi_{g,k}} \omega_g^k Z_{jq}^{(g,k)}} \leq 1 \quad j = 1, 2, \dots, n; k = 1, 2, \dots, K \quad (9)$$

$$u_r^k, v_i^k, \mu_h^k, \omega_g^k, w_k \geq \varepsilon > 0, \alpha_k \text{ unrestricted in sign} \quad , \quad (k, h), (g, k) \in L$$

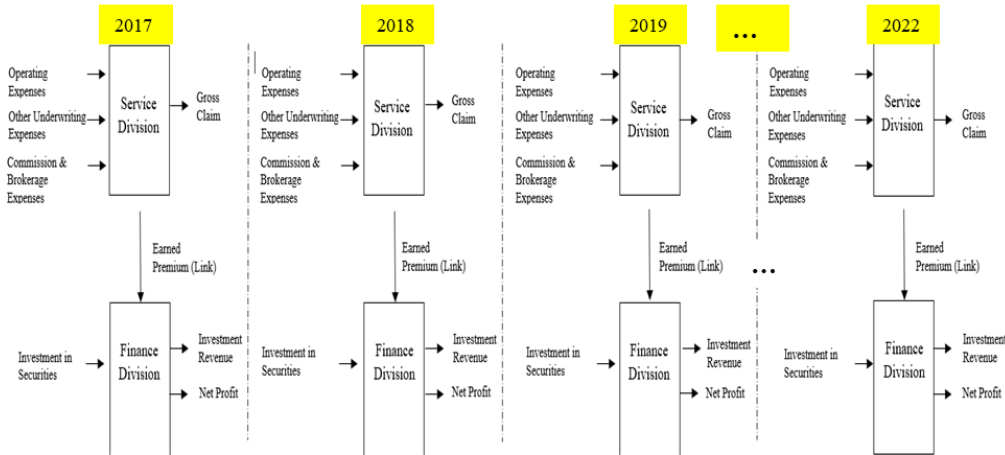


Fig 2 The Static Network Data Envelopment Analysis (SNDEA) Model

3.3 Dynamic Network Data Envelopment Analysis (DNDEA)

Dynamic DEA considers the internal activity relationship of the decision-making unit in the same period and considers the transition between the previous and later periods. The DNDEA model designed inputs, intermediate (link), outputs and carry-over, and the DNDEA structure are illustrated in Fig 3 and formulated as (10)-(13).

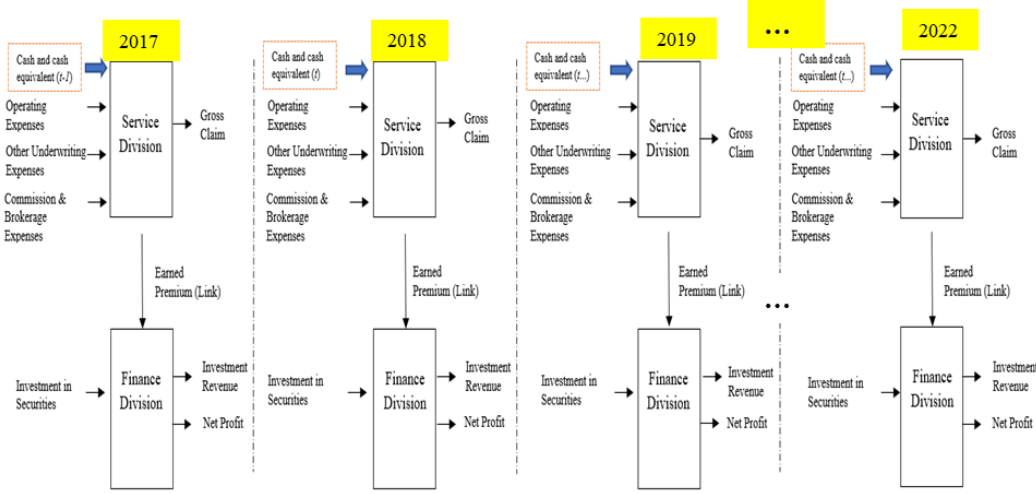


Fig 3 The Dynamic Network Data Envelopment Analysis (DNDEA) Model

$$\text{Max } E_0^{k,t}, \quad k = 1, 2, \dots, K; t = 1, 2, \dots, T \quad (10)$$

s. t.

$$E_o^{k,t} = \frac{\sum_{r=1}^{r_k} u_r^{k,t} Y_{ro}^{k,t} + \sum_{\forall(k,h)} \sum_{p=1}^{\phi(k,h)} \mu_h^k Z_{op}^{(k,h)t} + \sum_{\forall k} \sum_{p=0}^{\phi(t,t+1)} \mu_{t+1}^t Z_j^{k(t,t+1)} - \alpha_{k,t}}{\sum_{i=1}^{m_k} v_i^{k,t} X_{io}^{k,t} + \sum_{\forall(g,k)} \sum_{q=1}^{\phi(g,k)} \omega_g^k Z_{oq}^{(g,k)t} + \sum_{\forall k} \sum_{q=0}^{\phi(t-1,t)} \omega_{t-1}^t Z_j^{k(t-1,t)}} \quad (11)$$

$$k = 1, 2, \dots, K; t = 1, 2, \dots, T$$

$$\sum_{i=1}^{m_k} v_i^{k,t} X_{io}^{k,t} + \sum_{\forall(g,k)} \sum_{q=1}^{\phi(g,k)} \omega_g^k Z_{oq}^{(g,k)t} + \sum_{\forall k} \sum_{q=0}^{\phi(t-1,t)} \omega_{t-1}^t Z_j^{k(t-1,t)} = 1 \quad (12)$$

$$\frac{\sum_{r=1}^{r_k} u_r^{k,t} Y_{ro}^{k,t} + \sum_{\forall(k,h)} \sum_{p=1}^{\phi(k,h)} \mu_h^k Z_{op}^{(k,h)t} + \sum_{\forall k} \sum_{p=0}^{\phi(t,t+1)} \mu_{t+1}^t Z_j^{k(t,t+1)} - \alpha_{k,t}}{\sum_{i=1}^{m_k} v_i^{k,t} X_{io}^{k,t} + \sum_{\forall(g,k)} \sum_{q=1}^{\phi(g,k)} \omega_g^k Z_{oq}^{(g,k)t} + \sum_{\forall k} \sum_{q=0}^{\phi(t-1,t)} \omega_{t-1}^t Z_j^{k(t-1,t)}} \leq 1 \quad (13)$$

$$j = 1, 2, \dots, n; k = 1, 2, \dots, K; t = 1, 2, \dots, T$$

$$u_r^k, v_i^k, \mu_h^k, \omega_g^k, w_k \geq \varepsilon > 0, \alpha_k \quad \text{unrestricted in sign}$$

$$\text{all } (k, h), (g, k) \in L$$

3.4 Multi-Objective Programming (MOP)

In this study, dynamic black-box DEA (DBDEA), static network DEA (SNDEA), and dynamic network DEA (DNDEA) are formulated by applying the multi-objective programming method. Multi-objective programming (MOP) was solved by Zimmermann (1978) with the fuzzy approach by searching for the optimal solution with the highest degree of membership (satisfaction) in the feasible region.

$$\text{Max } E_0^k = \frac{\sum_{r=1}^{r_k} u_r^k Y_{r0}^k - u_0}{\sum_{i=1}^{m_k} v_i^k X_{i0}^k} \quad (14)$$

s.t.

$$\frac{\sum_{r=1}^{r_k} u_r^k Y_{rj}^k - u_0}{\sum_{i=1}^{m_k} v_i^k X_{ij}^k} \leq 1 \quad (15)$$

$$j = 1, 2, \dots, n; k = 1, 2, \dots, K$$

$$u_r^k, v_i^k \geq \varepsilon > 0, \alpha_k \quad \text{unrestricted in sign}$$

$$r = 1, 2, \dots, r_k; i = 1, 2, \dots, m_k$$

The following is the multi-objective programming execution process:

Step 1: Obtain the ideal solution of every objective.

To obtain the ideal solution, every objective is optimized independently of other objectives. For DMU_0 , we maximize $E_0^k, k = 1, \dots, K$ individually to determine their ideal objective values, $E_0^{k*}, k = 1, \dots, K$.

Step 2: Obtain the anti-ideal solution of every objective.

To obtain the anti-ideal solution, every objective is computed in the opposite way regardless of other objectives. Now, we minimize $E_0^k, k = 1, \dots, K$ to determine their ideal anti-ideal solutions $E_0^{k-}, k = 1, \dots, K$.

Step 3: Define the membership function of every objective by its ideal and anti-ideal solutions

$$\text{as } \mu_{E_0^k}(E_0^k) = \frac{E_0^k - E_0^{k-}}{E_0^{k*} - E_0^{k-}} \quad (16)$$

The membership function estimates the fulfilment level of every objective.

Step 4: Two types of solutions will be obtained according to the decision setting: compensatory and non-compensatory solutions.

Compensatory solution: the model's objective function is to maximize the sum of the membership function

$$\text{Max } \sum_{k=1}^k \mu_{E_0^k}(E_0^k) \quad (17)$$

s.t.

$$\mu_{E_0^k}(E_0^k) = \frac{E_0^k - E_0^{k-}}{E_0^{k*} - E_0^{k-}} \quad (18)$$

$$\frac{\sum_{r=1}^{r_k} u_r^k Y_{rj}^k}{\sum_{i=1}^{m_k} v_i^k X_{ij}^k} \leq 1 \quad (19)$$

$$j = 1, 2, \dots, n; k = 1, 2, \dots, K$$

$$u_r^k, v_i^k \geq \varepsilon > 0, \alpha_k \text{ unrestricted in sign}$$

$$r = 1, 2, \dots, r_k; i = 1, 2, \dots, m_k$$

Non-Compensatory solution: the model's objective function is to minimize the membership function

$$\text{Max } \lambda \quad (20)$$

$$\text{s.t. } \lambda \leq \mu_{E_0^k}(E_0^k)$$

$$\mu_{E_0^k}(E_0^k) = \frac{E_0^k - E_0^{k-}}{E_0^{k*} - E_0^{k-}} \quad (21)$$

$$\frac{\sum_{r=1}^{r_k} u_r^k Y_{rj}^k}{\sum_{i=1}^{m_k} v_i^k X_{ij}^k} \leq 1 \quad (22)$$

$$j = 1, 2, \dots, n; k = 1, 2, \dots, K$$

$$u_r^k, v_i^k \geq \varepsilon > 0, \alpha_k \text{ unrestricted in sign}$$

$$r = 1, 2, \dots, r_k; i = 1, 2, \dots, m_k, \text{ where } \lambda \text{ is the minimum of all departmental efficiencies to be maximized.}$$

Step 5: Obtain the overall efficiencies as the average of the departmental efficiencies computed above.

4. Results and Discussion

Based on the models and steps outlined in Section 3, the efficiencies were computed by LINGO 12. The average DMU overall efficiency scores are shown in Table 3 and Fig 4. The DBDEA when calculated as (3)-(5) does not consider department units, and focuses on the decision-making units though a dynamic structure, as shown in Fig 1. The inputs are cash and cash equivalents (carry-over from previous year), operating expenses, other underwriting

expenses, commission and brokerage expenses, and investments in security. The outputs comprise gross claim, earned premium, investment revenue, net profit. Cash and cash equivalents is the carry-over transferred to the next period.

The SNDEA model formulated as (6)-(9) considers the service department and finance department, and is calculated by the structure shown in Fig 2. The efficiencies of SNDEA are solved by SNDEA-Fixed and SNDEA-Free links, separately. The inputs of the service department are operating expenses, other underwriting expenses, and commission and brokerage expenses; the outputs are gross claims and earned premiums (outgoing link). The inputs in the finance department are earned premium (incoming link) and investment in securities; the outputs are investment revenues and net profits.

The efficiencies of DNDEA calculated as (10)-(13) was calculated from the service and finance departments, following the dynamic structure as shown in Fig 3. The special variables included in this model are cash and cash equivalents, which is the carry-over from the previous year ($t-1$) and it can be turned to output for the current period (t). The inputs of the service department are operating expenses, other underwriting expenses, commission and brokerage expenses; the outputs are gross claim and earned premium; earned premium was determined to be the link to the finance department. The inputs of finance department are investment in securities and the outputs are investment revenue and net profit.

From Table 3 and Fig 4, we see that efficiencies from all models reveal similar patterns, declining in 2020, with a shift to rising in 2021, when DNDEA models changed most significantly. The DBDEA does not consider internal processes, and consequently generates higher scores than other models. Looking into the internal processes of the DMUs in the Thai non-life insurance industry, the departmental efficiencies are shown in Table 4-5 and Fig 5-6. In terms of the service departments, DNDEA (both fixed-link and free-link) models obtain consistently higher efficiencies. However, SNDEA reveals different trends from the fixed-link and free-link models over the period 2017-2021, but then converge in 2022. In terms of the finance department, all models manifest similar patterns, with the DNDEA free-link presenting greater variation compared with other models. Notably, the efficiencies of both departments trended downward in 2020, during the COVID -19 epidemic. Consequently, the DMU overall efficiencies showed a downward trend in 2020.

In the DBDEA model, most of the insurers were able to achieve high efficiency scores, and the yearly average was found to be higher than in both the SNDEA and DNDEA models. In addition, when SNDEA (fixed and free-link) and DNDEA (fixed and free-link) were assessed by separating departments, we found that low-efficiency companies are in the same rank, or close, in both models. The poor performance could be attributed to discriminating power (Nourani et al., 2018). Otherwise, the DNDEA is preferable to the dynamic black-box model because the dynamic black-box model could not allow us to explore the related activities in this study's model. The static network DEA perspective is favored in the industry because its activities do not involve carry-overs. Among the three structures, DNDEA can be most applicable in the Thai insurance industry because it most closely resembles the relevant insurance business structural models. In summary, the non-life insurers have undergone a significant challenge from 2020 because of the of pandemic insurance business, including service and financial performance and especially financial. The efficiencies revealed recovery in 2021, and then a more significant declining presented in 2022 afterward, including that can be resulted from the effects of lags due to pandemic insurance claims.

This study also analyzes the discriminating power of all models by calculating the discrimination index, as shown in Table 6, where P_k denotes the k -th percentile. We use $P_{80}-P_{20}$ and $P_{80}+P_{60}-P_{10}-P_{20}$ for measuring the discriminating power. The indices show that both SNDEA models have the most discriminating power in evaluating overall efficiencies, and DBDEA has the least discriminating power. Finally, all DMUs are ranked by the overall efficiencies obtained from all models, as shown in Table 7. The companies occupying the first to eighth ranks are DMU 35 (Viriyah insurance), 47 (AIA nonlife insurance), 41 (HINA , CHUBB), 1 (Bangkok insurance), 6 (Tokiomarine), 32 (Mitsui Sumitomo insurance), 15 (Dhipaya insurance) and 3 (Krungthai Panich insurance), respectively. Notably, the DMU 16 (Tune Protect insurance), ranked 45th, went bankrupt and was delisted in 2021, so its efficiencies in 2021-2022 were estimated as lost data in the models. The distribution of the overall efficiencies of insurance firms from 2017-2022 are shown in Fig 6.

Table 3 Average DMU Overall Efficiency Scores

overall	2017	2018	2019	2020	2021	2022
DNDEA-fixed	0.6342	0.6872	0.6790	0.5808	0.6466	0.5104
DNDEA-free	0.6616	0.7280	0.7125	0.6047	0.7941	0.5285
SNDEA-fixed	0.5695	0.6427	0.5965	0.5098	0.5300	0.4673
SNDEA-free	0.5960	0.6933	0.6482	0.5466	0.6868	0.6048
DBDEA	0.9644	0.9545	0.9638	0.9167	0.9353	0.8834

Table 4 The Service Efficiency Scores

	2017	2018	2019	2020	2021	2022
DNDEA-fixed	0.8453	0.7361	0.8724	0.8044	0.8652	0.7531
DNDEA-free	0.8551	0.7627	0.8839	0.8140	0.8711	0.7633
SNDEA-fixed	0.7002	0.6521	0.6861	0.6549	0.6545	0.5891
SNDEA-free	0.6977	0.7496	0.6935	0.7083	0.6575	0.5998

Table 5 The Finance Efficiency Scores

	2017	2018	2019	2020	2021	2022
DNDEA-fixed	0.4230	0.6384	0.4857	0.3572	0.4281	0.2678
DNDEA-free	0.4682	0.6932	0.5411	0.3954	0.7170	0.2938
SNDEA-fixed	0.4387	0.6333	0.5069	0.3647	0.4055	0.3454
SNDEA-free	0.3954	0.5468	0.6932	0.4837	0.7160	0.6099

Table 6 The Discrimination Index (overall)¹

	P_{20}	P_{40}	P_{60}	P_{80}	$P_{80}-P_{20}$	$P_{80}+P_{60}-P_{10}-P_{20}$
DNDEA-fixed	0.4280	0.5425	0.6738	0.8565	0.4284	0.5598
DNDEA-free	0.4869	0.5792	0.7275	0.9046	0.4177	0.5660
SNDEA-fixed	0.3122	0.4553	0.6003	0.7742	0.4620	0.6070
SNDEA-free	0.4176	0.5544	0.6987	0.8478	0.4303	0.5746
DBDEA	0.8737	0.9863	1.0000	1.0000	0.1263	0.1400

¹ P_k denotes the k -th percentile.

Table 7 Overall Ranks 2017-2022

Rank	Score	DMU	Rank	Score	DMU	Rank	Score	DMU
1	0.9207	35	18	0.6864	12	35	0.5118	7
2	0.8916	47	19	0.6859	17	36	0.5061	31
3	0.8671	41	20	0.6805	34	37	0.5006	48
4	0.8538	1	21	0.6636	4	38	0.4866	40
5	0.8492	6	22	0.6423	14	39	0.4786	44
6	0.8447	32	23	0.6251	26	40	0.4486	23
7	0.8350	15	24	0.6210	22	41	0.4212	19
8	0.8020	3	25	0.6185	33	42	0.4115	46
9	0.7801	45	26	0.6164	8	43	0.4104	10
10	0.7717	2	27	0.5922	20	44	0.3994	30
11	0.7705	24	28	0.5920	39	45	0.3963	16*
12	0.7614	21	29	0.5684	18	46	0.3864	13
13	0.7587	38	30	0.5517	29	47	0.3808	11
14	0.7469	42	31	0.5498	25	48	0.3656	5
15	0.7407	9	32	0.5373	37	49	0.3515	43
16	0.7006	36	33	0.5313	49			
17	0.6942	27	34	0.5308	28			

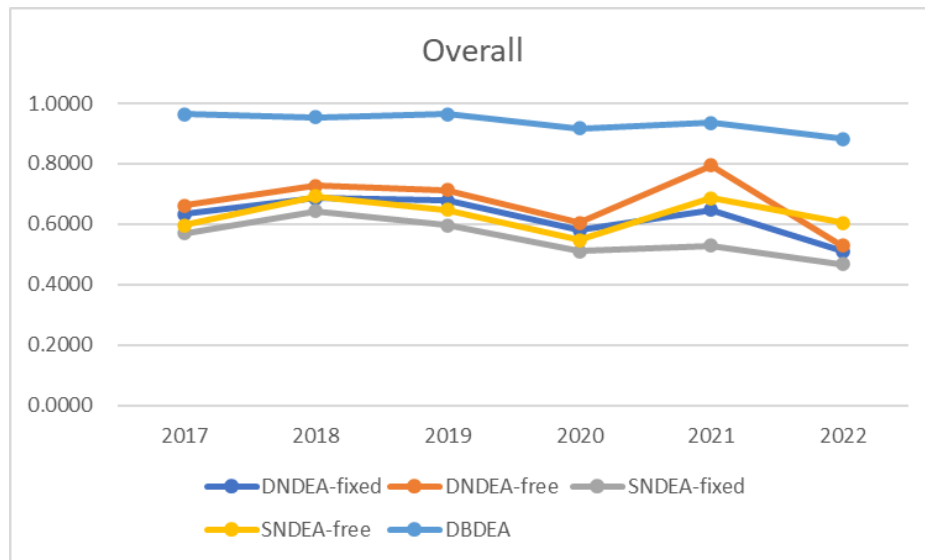


Fig 4 Average Overall Efficiencies of the Insurance Firms

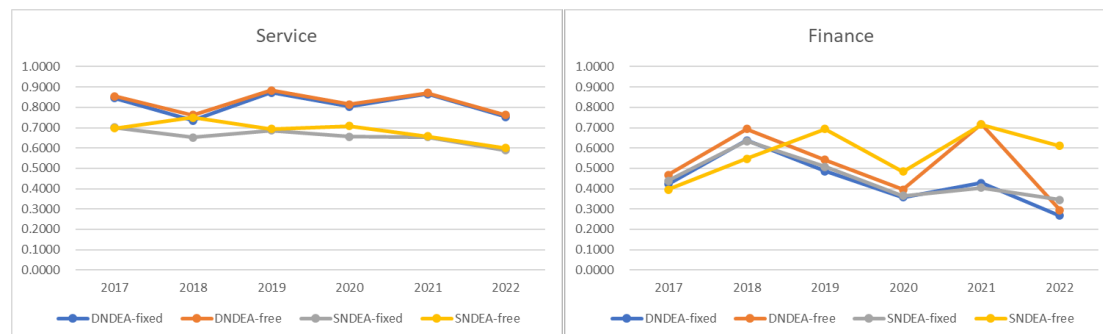


Fig 5 Average Departmental Efficiencies of the Insurance Firms 2017-2022

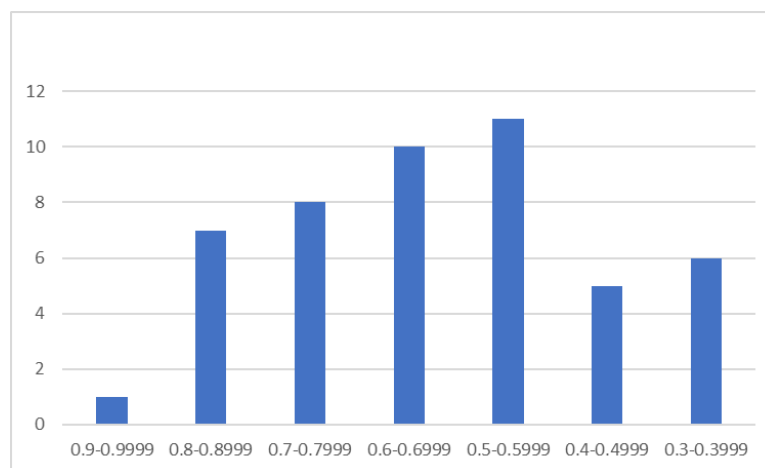


Fig 6 Distribution of Overall Efficiencies of the Insurance Firms 2017-2022

5. Conclusion

This study aimed to evaluate the efficiencies of non-life insurers in Thailand by applying the dynamic and network DEA methods. Three models are developed, including black-box data envelopment analysis (DBDEA), static network data envelopment analysis (SNDEA) and dynamic network data envelopment (DNDEA) by applying a multi-objective programming approach to measure the efficiency values of 49 non-life insurance companies in Thailand. In each insurance company, DMUs were considered via two sub-processes (i.e. departments), namely, the service and financial departments, over the period 2017-2022

In summary, the main findings of this study are as follows:

1. Among the various models used in this study, SNDEA showed the highest discriminatory power, followed by DNDEA, while DBDEA showed the weakest discriminatory power.
2. The patterns of overall efficiencies from various models are consistent in principle; however, the departmental efficiencies show different trends over the period investigated.
3. The overall and departmental scores tended to trend downward during the COVID -19 epidemic (2020) but were marginally higher in the following year (2021).

4. The distribution of overall efficiencies is approximately slightly left-skewed with the mode in the interval [0.5-0.5999].

The results verify that the developed models are useful and possess discriminating power in evaluating the non-life insurers in Thailand. This study contributes in designing the evaluation modes for Thailand based on an innovative formulation of MOP that optimizes departmental and periodical efficiencies of insurance companies simultaneously. Since operations of insurance industry are highly complex, the scope of related researches can be extended. Further studies may advantage from introducing efficiency factors such as technological investment, customer service quality, digital service and so on. As more factors are included in the evaluation models, correlation or multicollinearity among the efficiency factors can be examined. In-depth case studies for specific non-insurers are also suggested for more managerial insights in future investigations.

Appendix

The samples of non-life industry in this study include Bangkok Insurance, Bangkok Health Insurance, Krungthai Panich Insurance, RVP: Road Accident Victims Protection Company Limited, King Wai Insurance, Tokiomarine, KSK Insurance, Charan Insurance, Genie Insurance, Generali Insurance, Jmart Insurance, CHUBB Insurance, Sampo Insurance, Cigna Insurance, Dhipaya Insurance, Tune Protect Insurance, Deves Insurance, Thai Health Insurance, Thai Patthana Insurance, Thai Paiboon Insurance, Thai reinsurance, Thai Sri Insurance, TSI: Thai Setakij Insurance, Thanachart Insurance, Navakij Insurance, NSI: Namseng Insurance, New India Insurance, Bangkok Union Insurance, Thai Vivat Insurance, Pacific Cross Health Insurance, Falcon Insurance, Mitsui Sumitomo Insurance, Mittare Insurance, Muang Thai Insurance, Viriyah Insurance, Union Insurance Public, The Union Prospers Insurance, SMK: Sin Man Kong, Allianz Ayudhya, Indara Insurance, HINA (CHUBB), FWD Insurance, FPG Insurance, MSIG Insurance, Erawan Insurance, AIG Insurance, AIA Nonlife Insurance, Axa Insurance and LMG Insurance.

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