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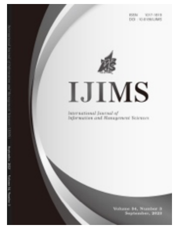

A Fuzzy Evaluation and Improvement Decision-Making Model for E-Learning Systems with a Performance Evaluation Matrix

Chun-Hung Yu[1,2], *Chang-Hsien Hsu*[3,4,*], *And Yen-Po Chen*[5,*]

[1] Language Center, National Chin-Yi University of Technology, Taichung 411030, Taiwan, R. O. C.;
[2] Language Teaching Center, National Chi Nan University, Nantou County 545301, Taiwan, R. O. C.;
[3] Department of Business Administration, Asia University, Taichung 413305, Taiwan, R. O. C.;
[4] Department of Cultural and Creative Industries, National Chin-Yi University of Technology, Taichung 411030, Taiwan, R.O.C.;
[5] Department of English, National Chengchi University, Taipei 116011, Taiwan, R. O. C.

Keywords

E-learning System; Likert Scale; Performance Evaluation Matrix; Fuzzy Evaluation; Fuzzy Coordinate Points

Abstract

After the COVID-19 pandemic, e-learning courses have enabled learners to freely control their learning time and space, making them a mainstream educational channel. As the Internet of Things (IoT) becomes increasingly widespread and developed, numerous e-learning applications have rapidly emerged. Therefore, improving e-learning system performance can boost learner (customer) satisfaction, attract more users from diverse locations, and generate greater economic benefits. By identifying and enhancing the service elements in need of improvement, overall learner satisfaction with the e-learning systems can be increased, thereby attracting more users. Some studies have highlighted that the Performance Evaluation Matrix (PEM) is constructed using a Likert scale, which collects data on both the satisfaction and importance of service elements to develop evaluation indices that form the basis of the matrix. Since these indices involve unknown parameters and businesses often operate under mechanisms requiring rapid responses, the collected sample sizes are usually small. Therefore, this paper derives fuzzy coordinate points for each service element. By identifying the locations of these fuzzy coordinates within the PEM, this paper proposes an evaluation and improvement decision-making model. Additionally, a case study of a computer-assisted language learning system is employed in demonstrating the implementation of the proposed model.

1. Introduction

With the growing prevalence and maturity of the Internet of Things (IoT), a wide variety

*corresponding author. Email: pci@asia.edu.tw (C.-H. Hsu); y92200b@gmail.com (Y.-P. Chen)

of e-learning applications have sprung up in the wake of the COVID-19 pandemic (Chang et al. 2021). As online courses allow learners to control when and where they study, e-learning has become a mainstream mode of learning. Numerous studies have indicated that improving the operational performance of e-learning systems can raise learner (customer) satisfaction, attract a broader and more geographically dispersed user base, and ultimately yield greater economic benefits (Chen et al. 2023; As et al. 2024). Several studies have suggested that developing an effective decision-making model for evaluating and improving e-learning systems enables system administrators to more accurately assess learner satisfaction with the various service elements offered (Chen et al. 2024; Lin et al. 2023; Liu et al. 2023). Enhancing service elements identified as needing improvement can, in turn, increase overall learner satisfaction with e-learning systems, thereby attracting a larger user base. In addition, the Performance Evaluation Matrix (PEM) utilizes a Likert scale to gather data on the satisfaction and importance of service elements. This data is then used to develop evaluation indices, which ultimately form the PEM (Chen et al. 2020; Wang et al. 2015; Yu et al. 2017; Yu et al. 2023; Yu et al. 2020a). To maintain generality, this study assumes that the questionnaire contains q service elements. Using an R-point Likert scale to assess both satisfaction and importance, respondents are required to complete a total of $2q$ items. Let the random variable X_h represent the satisfaction level of the h^{th} service item, and let the random variable Y_h represent the importance level of the h^{th} service item. Next, the Satisfaction Index and Importance Index for the h^{th} service items are defined as follows:

$$\text{Satisfaction Index: } \Theta_h = \frac{\mu_h - 1}{R-1}, \quad h = 1, 2, \ldots, q \tag{1}$$

and

$$\text{Importance Index: } \Gamma_h = \frac{\mu_h' - 1}{R-1}, \quad h = 1, 2, \ldots, q, \tag{2}$$

Where μ_h represents the expected value of the random variable X_h, and μ_h' represents the expected value of the random variable Y_h. As evidenced by Yu et al. (2018), since both the satisfaction index and the importance index contain unknown parameters, it is necessary to collect samples through customer interviews and then estimate these parameters using the sample data. Modern businesses strive for rapid response mechanisms, often requiring decisions to be made with limited sample sizes. Several studies have shown that when sample sizes are small, directly using confidence intervals of indices for statistical inference can increase the risk of misjudgment due to the relatively large expected lengths of the confidence intervals (Chen 2019, 2022; Chen et al. 2019a). To enhance evaluation accuracy under such conditions, some studies have proposed confidence interval-based fuzzy testing as a more reliable alternative (Chen and Yu, 2020; Chen et al. 2024; Lin et al. 2023). Accordingly, this study develops a PEM for assessing e-learning systems, with the Satisfaction Index on the horizontal axis and the Importance Index on the vertical axis. The PEM is initially categorized into four evaluation quadrants based on the mean of the estimated observed values for all satisfaction indices and all importance indices. Then, the upper confidence limits for the satisfaction and importance indices are derived, and a fuzzy test is derived from the confidence limits of these two indices to determine the fuzzy evaluation coordinate points. Finally, depending on the quadrant in which each service element's fuzzy evaluation coordinate falls within the PEM, corresponding rules for fuzzy evaluation and improvement decision are formulated.

Following the recommendation of Yu et al. (2018), this study adopts the questionnaire designed by Shee and Wang (2008) to assess users' perceptions toward the satisfaction and importance levels across various service elements provided by e-learning systems. The questionnaire comprises four dimensions: (1) Learner Interface, (2) Learning Community, (3) System Content, and (4) Personalization. These four dimensions comprise a total of 13 items, making the questionnaire both concise and comprehensive.

The subsequent parts of this paper are organized as follows. Section 2 derives the upper confidence limits for the satisfaction and importance indices, respectively. Next, the upper confidence limits for the satisfaction and importance indices are calculated separately. Section 3 presents the construction of a PEM for e-learning systems. The matrix is first divided into an acceptance zone and an improvement zone based on the average estimated observed values for all satisfaction indices. The improvement zone is then further partitioned into a high-priority improvement zone and a secondary improvement zone using the average estimated observed values of all importance indices. Subsequently, fuzzy evaluation coordinate points are established for all service elements. Based on the positions of the service elements within the matrix, fuzzy evaluation and improvement decision rules are formulated in accordance with the fuzzy evaluation method. Section 4 provides a real-world case study to exemplify the implementation of the proposed model. Finally, Section 5 concludes the paper.

2. Upper Confidence Limits of Indices

As previously mentioned, to maintain generality, this study assumes that the questionnaire contains q service elements. Therefore, q satisfaction indices and q importance indices are established, as shown in Equations (1) and (2), respectively. Subsequently, the upper confidence limits for the satisfaction and importance indices are derived, and fuzzy membership functions are also constructed.

(1) Satisfaction Index

Since these satisfaction indices involve unknown parameters, they must be estimated using sample data. Let $(X_{h,1}, X_{h,2}, ..., X_{h,n})$ be a random sample of the satisfaction index X_h for the h^{th} service item. Then, the sample mean and sample standard deviation are then given as follows:

Sample mean: $\bar{X}_h = \dfrac{1}{n}\sum_{j=1}^{n} X_{h,j}$; (3)

Sample standard deviation: $S_h = \sqrt{\dfrac{1}{n-1}\sum_{j=1}^{n}\left(X_{h,j} - \bar{X}_h\right)^2}$ (4)

The expected value of $\bar{X}_h$ is μ_h, and the standard deviation of $\bar{X}_h$ is

$$\sigma_{\bar{X}_h} = \frac{\sigma_h}{\sqrt{n}}. \quad (5)$$

The estimator of the satisfaction index for Item h is defined as follows:

$$\hat{\Theta}_h = \frac{X_h - 1}{R - 1}, \ h = 1, 2, ..., q. \quad (6)$$

The expected value and standard deviation of $\hat{\Theta}_h$ are individually expressed as follows:

$$E\left[\hat{\Theta}_h\right]=\frac{E\left[\bar{X}_h\right]-1}{R-1}=\frac{\mu_h-1}{R-1}=\Theta_h; \tag{7}$$

$$\sigma_{\hat{\Theta}_h}=\sqrt{V_{AR}\left[\hat{\Theta}_h\right]}=\frac{\sigma_h}{n(R-1)}. \tag{8}$$

Let the random variable Z_h be written as follows:

$$Z_h=\frac{\Theta_h-\Theta_h}{S_h/\sqrt{n}(R-1)}. \tag{9}$$

In accordance with the *Central Limit Theorem*, the random variable Z_h approximately follows a standard normal distribution when sample size n is sufficiently large. Thus,

$$\begin{aligned}1-\alpha&=p\left(-z_{\alpha/2}\le Z_h\le z_{\alpha/2}\right)=p\left(-z_{\alpha/2}\le\frac{\hat{\Theta}_h-\Theta_h}{S_h/\sqrt{n}(R-1)}\le z_{\alpha/2}\right)\\&=p\left(\hat{\Theta}_h-z_{\alpha/2}\frac{S_h}{\sqrt{n}(R-1)}\le\Theta_h\le\hat{\Theta}_h+z_{\alpha/2}\frac{S_h}{\sqrt{n}(R-1)}\right).\end{aligned} \tag{10}$$

The lower and upper confidence limits of Θ_h are defined as follows, respectively:

$$L\Theta_h=\hat{\Theta}_h-z_{\alpha/2}\frac{S_h}{\sqrt{n}(R-1)} \tag{11}$$

and

$$U\Theta_h=\hat{\Theta}_h+z_{\alpha/2}\frac{S_h}{\sqrt{n}(R-1)}, \tag{12}$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ quintile of the standard normal distribution. Let $(x_{h,1},x_{h,2},\ldots,x_{h,n})$ be the observed values of $(X_{h,1},X_{h,2},\ldots,X_{h,n})$, then the observed values of $\hat{\Theta}_h$, $L\Theta_h$ and $U\Theta_h$ are separately written as follows:

$$\hat{\theta}_h=\frac{\bar{x}_h-1}{R-1}, \tag{13}$$

$$L\theta_h=\hat{\theta}_h-z_{\alpha/2}\frac{s_h}{\sqrt{n}(R-1)}, \tag{14}$$

and

$$U\theta_h=\hat{\theta}_h+z_{\alpha/2}\frac{s_h}{\sqrt{n}(R-1)}, \tag{15}$$

where s_h is the observed value of S_h as follows:

$$s_h=\sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(x_{h,j}-\bar{x}_h\right)^2}. \tag{16}$$

(2) Importance Index

Similar to the satisfaction indices, the importance indices also involve unknown parameters and must therefore be estimated using sample data. Let $(Y_{h,1}, Y_{h,2}, \ldots, Y_{h,n})$ be a random sample of the importance index Y_h for the h^{th} service item. The sample mean and sample standard deviation are then given as follows, respectively:

Sample mean: $\bar{Y}_h = \frac{1}{n}\sum_{j=1}^{n} Y_{h,j}$; (17)

Sample standard deviation: $S'_h = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(Y_{h,j} - \bar{Y}_h\right)^2}$. (18)

The expected value of $\bar{Y}_h$ is μ'_h, and the standard deviation of $\bar{Y}_h$ is

$$\sigma_{\bar{Y}_h} = \frac{\sigma'_h}{\sqrt{n}}. \tag{19}$$

The estimator of the importance index for Item h is defined as follows:

$$\hat{\Gamma}_h = \frac{Y_h - 1}{R-1}, \quad h = 1, 2, \ldots, q. \tag{20}$$

The expected value and standard deviation of $\hat{\Gamma}_h$ are individually written as follows:

$$E\left[\hat{\Gamma}_h\right] = \frac{E\left[\bar{Y}_h\right]-1}{R-1} = \frac{\mu'_h - 1}{R-1} = \Gamma_h; \tag{21}$$

$$\sigma_{\hat{\Gamma}_h} = \sqrt{V_{AR}\left[\hat{\Gamma}_h\right]} = \frac{\sigma'_h}{n(R-1)}. \tag{22}$$

Let the random variable Z'_h be expressed as follows:

$$Z'_h = \frac{\Gamma_h - \Gamma_h}{S'_h / \sqrt{n}(R-1)}. \tag{23}$$

Based on the *Central Limit Theorem*, the random variable Z'_h approximately follows a standard normal distribution when the sample size n is sufficiently large. Thus,

$$\begin{aligned} 1-\alpha &= p\left(-z_{\alpha/2} \le Z'_h \le z_{\alpha/2}\right) = p\left(-z_{\alpha/2} \le \frac{\hat{\Gamma}_h - \Gamma_h}{S'_h/\sqrt{n}(R-1)} \le z_{\alpha/2}\right) \\ &= p\left(\hat{\Gamma}_h - z_{\alpha/2} \times \frac{S'_h}{\sqrt{n}(R-1)} \le \Gamma_h \le \hat{\Gamma}_h + z_{\alpha/2} \times \frac{S'_h}{\sqrt{n}(R-1)}\right). \end{aligned} \tag{24}$$

The lower and upper confidence limits of Γ_h are defined as follows, respectively:

$$L\Gamma_h = \hat{\Gamma}_h - z_{\alpha/2}\frac{S'_h}{\sqrt{n}(R-1)} \tag{25}$$

and

$$U\Gamma_h = \hat{\Gamma}_h + z_{\alpha/2}\frac{S'_h}{\sqrt{n}(R-1)}, \tag{26}$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ quintile of the standard normal distribution. Let $(y_{h,1}, y_{h,2}, ..., y_{h,n})$ be the observed values of $(Y_{h,1}, Y_{h,2}, ..., Y_{h,n})$, then the observed values of $\hat{\Gamma}_h$, $L\Gamma_h$ and $U\Gamma_h$ are individually written as follows:

$$\hat{\tau}_h = \frac{\overline{y}_h - 1}{R-1}, \tag{27}$$

$$L\tau_h = \hat{\tau}_h - z_{\alpha/2}\frac{s'_h}{\sqrt{n}(R-1)}, \tag{28}$$

and $$U\tau_h = \hat{\tau}_h + z_{\alpha/2}\frac{s'_h}{\sqrt{n}(R-1)}, \tag{29}$$

where s'_h is the observed value of S'_h as follows:

$$s'_h = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(y_{h,j} - \overline{y}_h\right)^2}. \tag{30}$$

3. Establishing a Performance Evaluation Matrix and Fuzzy Evaluation Rules

In this section, the satisfaction index Θ_h is used as the horizontal axis and the importance index Γ_h as the vertical axis to construct a performance evaluation matrix for the e-learning system, as shown in Figure 1. Then, building on the research by Chen et al. (2019b) and the notion of continuous enhancement in Total Quality Management (TQM), the mean θ_0 of the estimated observed values of all satisfaction indices is applied to divide the PEM of the e-learning system into two zones: an improvement zone and an acceptance zone, where

$$\theta_0 = \frac{1}{q}\sum_{h=1}^{q}\hat{\theta}_h. \tag{31}$$

Next, the mean τ_0 of the estimated observed values of all importance indices is employed to divide the improvement zone into a high-priority improvement zone and a secondary improvement zone, as shown in Figure 1, where

$$\tau_0 = \frac{1}{q}\sum_{h=1}^{q}\hat{\tau}_h. \tag{32}$$

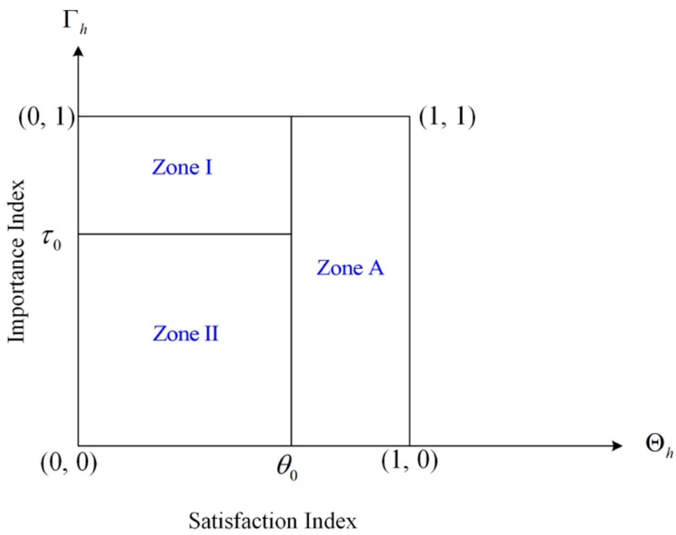

Fig. 1 *PEM for the E-learning System*

Subsequently, the high-priority improvement zone (Zone I), the secondary improvement zone (Zone II), and the acceptance zone (Zone A) are defined as follows:

$$\text{Zone A} = \{(x_h, y_h) \mid \theta_0 \le x_h \le 1, 0 \le y_h \le 1\};$$
$$\text{Zone I} = \{(x_h, y_h) \mid 0 \le x_h < \theta_0, \tau_0 \le y_h \le 1\};$$
$$\text{Zone II} = \{(x_h, y_h) \mid 0 \le x_h < \theta_0, 0 \le y_h < \tau_0\}.$$

Moreover, when the satisfaction index of Service Item h is higher than the mean ($\Theta_h \ge \theta_0$), Item h does not require improvement, which is equivalent to performing the following hypothesis test:

H_0: $\Theta_h \ge \theta_0$ (There is no need to improve Service Item h.)

H_1: $\Theta_h < \theta_0$ (Improvement of Service Item h is necessary to enhance satisfaction.)

According to Chen et al. (2019c), the α-cuts of triangular-shaped fuzzy number $\tilde{\theta}_h$ is defined as

$$\tilde{\theta}_{\hbar}[\alpha] = \begin{cases} \left[\theta_{\hbar,1}(\alpha), \theta_{\hbar,2}(\alpha)\right] = \left[\hat{\theta}_{\hbar} - z_{\alpha/2}\frac{s_{\hbar}}{\sqrt{n(R-1)}}, \hat{\theta}_{\hbar} + z_{\alpha/2}\frac{s_{\hbar}}{\sqrt{n(R-1)}}\right], 0.01 \le \alpha \le 1 \\ \left[\theta_{\hbar,1}(\alpha), \theta_{\hbar,2}(\alpha)\right] = \left[\hat{\theta}_{\hbar} - z_{0.005}\frac{s_{\hbar}}{\sqrt{n(R-1)}}, \hat{\theta}_{\hbar} + z_{0.005}\frac{s_{\hbar}}{\sqrt{n(R-1)}}\right], 0 \le \alpha \le 0.01 \end{cases}. \quad (33)$$

Therefore, the triangular-shaped fuzzy number is denoted as $\Delta\tilde{\theta}_{\hbar} = [L\theta_h, \hat{\theta}_h, R\theta_h]$, where

$$L\theta_h = \hat{\theta}_h - z_{0.005}\frac{s_h}{\sqrt{n(R-1)}}, \quad (34)$$

and

$$R\theta_h = \hat{\theta}_h + z_{0.005}\frac{s_h}{\sqrt{n}(R-1)}. \tag{35}$$

Then the fuzzy membership function of $\tilde{\theta}_h$ is written as follows:

$$\xi(x) = \begin{cases} 0, \ if \ x < L\theta_h \\ a, \ if \ L\theta_h \le x < \hat{\theta}_h \\ 1, \ if \ x = \hat{\theta}_h \\ b, \ if \ \hat{\theta}_h < x \le R\theta_h \\ 1, \ if \ R\theta_h < x \end{cases}, \tag{36}$$

where a and b are determined by

$$\hat{\theta}_h - z_{a/2}\frac{s_h}{\sqrt{n}(R-1)} = x, \tag{37}$$

and

$$\hat{\theta}_h + z_{b/2}\frac{s_h}{\sqrt{n}(R-1)} = x. \tag{38}$$

Based on Figure 2, we have $d_{Th} = R\theta_h - L\theta_h$ and $d_{Rh} = R\theta_h - \theta_0$.

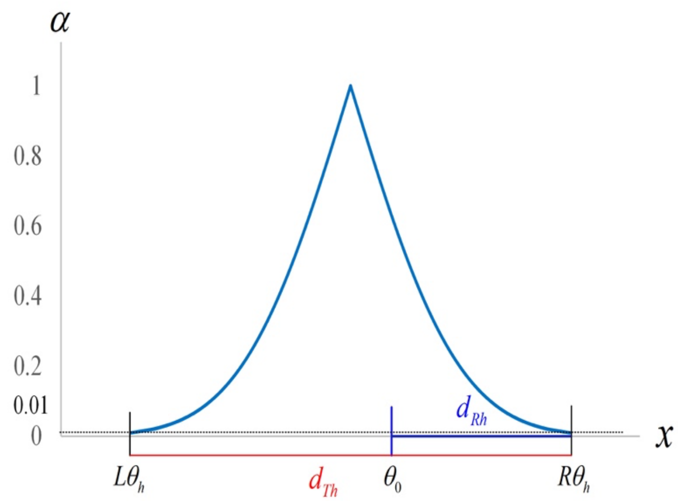

Fig. 2 *The Fuzzy Membership Function of* $\tilde{\theta}_h$

Then

$$\frac{d_{Rh}}{d_{Th}} = \frac{R\theta_h - \theta_0}{R\theta_h - L\theta_h}. \tag{39}$$

As noted by Lo et al. (2024) and Yu et al. (2020b), we can take $\phi \in (0.\ 0.5)$ and let the fuzzy decision value θ_{Fh} of the h^{th} service item satisfy the following equation:

$$\frac{d_{Rh}}{d_{Th}} = \frac{R\theta_h - \theta_{Fh}}{R\theta_h - L\theta_h} = \phi . \tag{40}$$

Therefore, we have

$$\theta_{Fh} = (1-\phi)R\theta_h + \phi L\theta_h = \hat{\theta}_h + (1-2\phi) z_{0.005} \frac{s_h}{\sqrt{n}(R-1)} . \tag{41}$$

Consistent with the findings of Lo et al. (2024) and Chen and Yu (2020), the decision-making rules for the fuzzy tests are defined as follows:

(1) If $\theta_{Fh} \geq \theta_0$, then do not reject H_0 and conclude that $\Theta_h \geq \theta_0$. Thus, the h^{th} service item does not need improvement.

(2) If $\theta_{Fh} < \theta_0$, then reject H_0 and conclude that $\Theta_h < \theta_0$. Thus, the h^{th} service item requires improvement.

When the fuzzy test result for the satisfaction index of Service Item h indicates that improvement is needed, a subsequent fuzzy test is conducted on the importance index to ascertain the priority for improvement. At this point, the hypotheses for the test are proposed as follows:

H_0: $\Gamma_h \geq \tau_0$ (High priority for improvement)

H_1: $\Gamma_h < \tau_0$ (Low priority for improvement)

According to Chen et al. (2019c), the α-cuts of triangular shaped fuzzy number $\tilde{\tau}_{\hbar}$ are defined as follows:

$$\tilde{\tau}_{\hbar}[\alpha] = \begin{cases} [\tau_{\hbar,1}(\alpha), \tau_{\hbar,2}(\alpha)] = \left[\hat{\tau}_{\hbar} - z_{\alpha/2}\frac{s'_{\hbar}}{\sqrt{n}(R-1)}, \hat{\tau}_{\hbar} + z_{\alpha/2}\frac{s'_{\hbar}}{\sqrt{n}(R-1)}\right], 0.01 \leq \alpha \leq 1 \\ [\tau_{\hbar,1}(0.01), \tau_{\hbar,2}(0.01)] = \left[\hat{\tau}_{\hbar} - z_{0.005}\frac{s'_{\hbar}}{\sqrt{n}(R-1)}, \hat{\tau}_{\hbar} + z_{0.005}\frac{s'_{\hbar}}{\sqrt{n}(R-1)}\right], 0 \leq \alpha \leq 0.01 \end{cases} . \tag{42}$$

Therefore, the triangular-shaped fuzzy number is written as $\Delta\tilde{\tau}_h = [L\tau_h, \hat{\tau}_h, R\tau_h]$, where

$$L\tau_h = \hat{\tau}_h - z_{0.005} \frac{s'_h}{\sqrt{n}(R-1)}, \tag{43}$$

and

$$U\tau_h = \hat{\tau}_h + z_{0.005} \frac{s'_h}{\sqrt{n}(R-1)} . \tag{44}$$

Then the fuzzy membership function of $\tilde{\tau}_{\hbar}$ is defined as follows:

$$\xi'(x)=\begin{cases}0, & if\ x<L\tau_h\\ a', & if\ L\tau_h\le x<\hat{\tau}_h\\ 1, & if\ x=\hat{\tau}_h\\ b', & if\ \hat{\tau}_h<x\le R\tau_h\\ 1, & if\ R\tau_h<x\end{cases}, \tag{45}$$

where a' and b' are determined by

$$\hat{\tau}_h - z_{a'/2}\frac{s'_h}{\sqrt{n}(R-1)}=x, \tag{46}$$

and

$$\hat{\tau}_h + z_{b'/2}\frac{s'_h}{\sqrt{n}(R-1)}=x. \tag{47}$$

Based on Figure 3, we have $d'_{Th}=R\tau_h-L\tau_h$ and $d'_{Rh}=R\tau_h-\tau_0$.

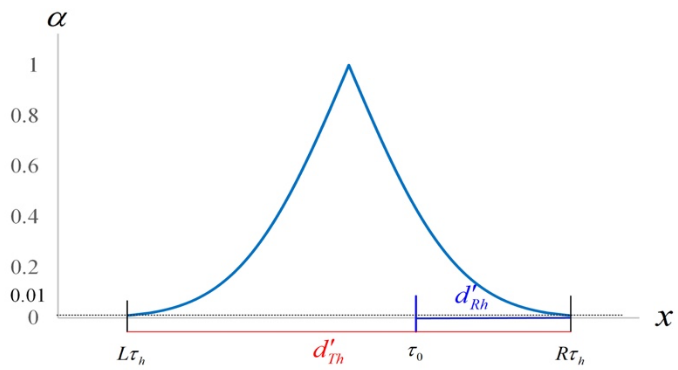

Fig. 3 *The Fuzzy Membership Function of* $\tilde{\tau}_h$

Then

$$\frac{d'_{Rh}}{d'_{Th}}=\frac{R\tau_h-\tau_0}{R\tau_h-L\tau_h}. \tag{48}$$

As demonstrated by Lo et al. (2024) and Yu et al. (2020c), we can take $\phi'\in(0.\ 0.5)$ and let the fuzzy decision value τ_{Fh} of the h^{th} service item satisfy the following equation:

$$\frac{d'_{Rh}}{d'_{Th}}=\frac{R\tau_h-\tau_{Fh}}{R\tau_h-L\tau_h}=\phi'. \tag{49}$$

Therefore, we have

$$\tau_{Fh}=(1-\phi')R\tau_h+\phi' L\tau_h=\hat{\tau}_h+(1-2\phi')z_{0.005}\frac{s'_h}{\sqrt{n}(R-1)}. \tag{50}$$

Based on Lo et al. (2024) and Chen and Yu (2020), the decision-making rules for the fuzzy tests are defined as follows:

(3) If $\tau_{Fh} \geq \tau_0$, then do not reject H_0 and conclude that $\Gamma_h \geq \tau_0$. Then the service item is regarded as a high priority for improvement.

(4) If $\tau_{Fh} < \tau_0$, then reject H_0 and conclude that $\Gamma_h < \tau_0$. Then the service item is regarded as a low priority for improvement.

According to Equations (29) and (37), the evaluation coordinate point of the h^{th} service item can be expressed as follows:

$$\left(x_h, y_h\right) = \left(\theta_{Fh}, \tau_{Fh}\right) = \left(\hat{\theta}_h + (1-2\phi)z_{0.005}\frac{s_h}{\sqrt{n(R-1)}}, \hat{\tau}_h + (1-2\phi')z_{0.005}\frac{s'_h}{\sqrt{n(R-1)}}\right) \tag{51}$$

Next, based on the region in which the fuzzy evaluation coordinate point $\left(x_h, y_h\right)$ falls, the fuzzy evaluation and improvement decision rules are established in accordance with the fuzzy evaluation principles outlined above, as shown below:

1. When the evaluation coordinate point $\left(x_h, y_h\right) \in$ Zone A, the satisfaction level of the h^{th} service item remains unchanged and does not require improvement, in accordance with Evaluation Rule (1).
2. When the evaluation coordinate point $\left(x_h, y_h\right) \in$ Zone I, the satisfaction level of the h^{th} service item requires improvement, in accordance with Evaluation Rule (2). Furthermore, according to Evaluation Rule (3), its priority for improvement is relatively high.
3. When the evaluation coordinate point $\left(x_h, y_h\right) \in$ Zone II, the satisfaction level of the h^{th} service item requires improvement, in accordance with Evaluation Rule (2). Furthermore, according to Evaluation Rule (4), its priority for improvement is relatively low.

4. A Case Study

E-learning, characterized by its independence from time and space constraints, has emerged as a new mode of education. This was especially evident during the COVID-19 pandemic, when it became the only viable learning path. The e-learning approach is commonly implemented in language instruction (Hwang & Tsai, 2011; Wu et al., 2012). According to the Digital Language Learning Global Market Report 2025 (Global information Inc., 2025), the global digital language learning market is expected to experience exponential growth in the coming years. By 2029, the market is projected to reach USD 77.56 billion, with a compound annual growth rate (CAGR) of 21.1%. A survey conducted by the Industrial Development Bureau of Taiwan's Ministry of Economic Affairs on the smart learning industry revealed that Taiwan's smart learning output value reached NT$457.87 billion in 2021, with an impressive annual growth rate of 220.9%. Consequently, this study took two private universities in central Taiwan that utilized the same CALL system as a case study and illustrated the fuzzy hypothesis testing method presented in this paper.

First, this study adopted the Web-Based E-Learning System (WELS) questionnaire developed by Shee and Wang (2008) to investigate students' perceptions of satisfaction and importance regarding the use of the CALL system. The WELS questionnaire is commonly employed to assess user satisfaction in digital learning contexts (Bolliger et al. 2024; Li et al. 2024; Quang et al. 2024). It consists of 13 items (as detailed in Table 1), with each item collecting both the satisfaction and importance ratings from learners. Responses are measured using a five-point Likert scale: 1 = very dissatisfied/very unimportant, 2 = dissatisfied/unimportant, 3 = neutral, 4 = satisfied/important, and 5 = very satisfied/very important. The participants of this study were students from two private universities in central Taiwan who used the CALL system. A total of 367 questionnaires were distributed, with 303 returned, yielding a response rate of 83%. Of the returned questionnaires, 14 were deemed invalid while 289 were valid, resulting in an effective response rate of 79%. Subsequently, the evaluation procedure was conducted based on the fuzzy evaluation rules defined in Section 3, as follows:

Step 1: First, the evaluation coordinate points, denoted as (x_h, y_h), for Service Items 1 to 13 were calculated using Equation (38), as shown in Table 1.

Table 1. *Satisfaction and Importance Indices for the CALL System*

Dimensions	Items	x_h	y_h	Zone	Remark
Learner Interface	1. Ease of use	0.6826	0.7335	A	Acceptance
	2. User-friendliness	0.7031	0.7372	A	Acceptance
	3. Ease of understanding	0.6869	0.7341	A	Acceptance
	4. Operational stability	0.6811	0.7330	A	Acceptance
Learning Community	5.Ease of discussion with other learners	0.6423	0.7080	II	Secondary improvement
	6. Ease of discussion with teachers	0.6440	0.7191	I	Priority improvement
	7. Ease of accessing shared data	0.6218	0.7223	I	Priority improvement
	8. Ease of exchanging learning with the others	0.6417	0.7000	II	Secondary improvement
System Content	9. Up-to-date content	0.6718	0.7267	A	Acceptance
	10. Sufficient content	0.6705	0.7247	A	Acceptance
	11. Useful content	0.6817	0.7308	A	Acceptance
Personalizat ion	12. Capability of controlling learning progress	0.6727	0.7219	A	Acceptance
	13. Capability of recording learning performance	0.6671	0.7253	A	Acceptance
	Average	0.6445	0.7085		

Step 2: Next, based on the fuzzy evaluation coordinate points, denoted as (x_h, y_h), the fuzzy performance evaluation matrix for the CALL system was plotted, as shown in Figure 4.

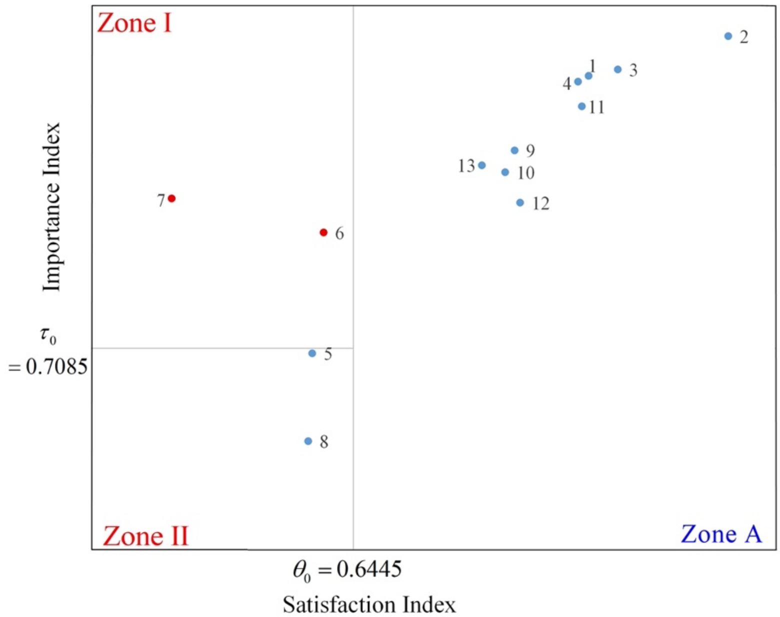

Fig. 4 *Fuzzy Performance Evaluation Matrix for CALL System*

Step 3: Based on the evaluation zones in which the fuzzy evaluation coordinate points, denoted as (x_h, y_h), fall in Figure 2, and consistent with the fuzzy evaluation and improvement decision-making rules established in Section 3, the results are presented as follows:

(1) The evaluation coordinate points for 9 service items - Items 1, 2, 3, 4, 9, 10, 11, 12, and 13 - fall within the acceptance zone (Zone A), that is, $(x_1, y_1),(x_2, y_2),(x_3, y_3),(x_4, y_4),(x_9, y_9),(x_{10}, y_{10}),(x_{11}, y_{11}),(x_{12}, y_{12}),(x_{13}, y_{13}) \in$ Zone A. According to Evaluation Rule (1), the satisfaction levels of these nine service items can be maintained and do not require improvement.

(2) The evaluation coordinate points for Service Items 6 and 7 fall within the high-priority improvement zone (Zone I), that is, $(x_6, y_6),(x_7, y_7) \in$ Zone I. Based on Evaluation Rule (2), the satisfaction levels of these two service items require improvement. Furthermore, according to Evaluation Rule (3), the priority for improving these items is relatively high.

(3) The evaluation coordinate points for Service Items 5 and 8 fall within the secondary improvement zone (Zone II), that is, $(x_5, y_5),(x_8, y_8) \in$ Zone II. Based on Evaluation Rule (2), the satisfaction levels of these two service items require improvement. Moreover, according to Evaluation Rule (4), the priority for improving these items is relatively low.

5. Conclusions

After the COVID-19 pandemic, since nearly all learners have experienced online learning, it has become a major mainstream learning channel. By improving the operational performance of e-learning systems, more learners can be attracted to participate, which can enhance not only learning outcomes but also economic benefits of the digital learning industry. This study utilized a questionnaire constructed upon a five-point Likert scale to elicit learners' satisfaction and importance ratings of key service elements in the e-learning system, thereby establishing satisfaction and importance indices. In consideration of the need for rapid response in business settings, the study derived fuzzy coordinate points from these evaluation indices. These fuzzy points were then mapped onto three major zones of the performance evaluation matrix (Zone A – Acceptance Zone, Zone I – High-priority Improvement Zone, and Zone II – Secondary Improvement Zone) to construct a decision-making model for evaluating and improving the e-learning system. Finally, this study used a computer-assisted language learning (CALL) system as a case study to illustrate how the proposed model can be applied. The results showed that 9 out of 13 service items fell into Zone A – the Acceptance Zone – indicating no need for improvement. In contrast, Service Item 6, "Ease of discussion with teachers," and Service Item 7, "Ease of accessing shared data," fell into Zone I – the High-priority Improvement Zone. Meanwhile, Service Item 5, "Ease of discussion with other learners," and Service Item 8, "Ease of exchanging learning with others," fell into Zone II – the Secondary Improvement Zone.

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Chun-Hung Yu
Language Center, National Chin-Yi University of Technology, Taichung 411030, Taiwan, R. O. C.
Language Teaching Center, National Chi Nan University, Nantou County 545301, Taiwan, R. O. C.
E-mail address: sylvia@ncut.edu.tw
Major area(s): Performance evaluation; English teaching effectiveness evaluation

Chang-Hsien Hsu
Department of Business Administration, Asia University, Taichung 413305, Taiwan, R. O. C.
Department of Cultural and Creative Industries, National Chin-Yi University of Technology, Taichung 411030, Taiwan, R.O.C.
E-mail address: pci@asia.edu.tw
Major area(s): Process capability index; Quality management

Yen-Po Chen
Department of English, National Chengchi University, Taipei 116011, Taiwan, R. O. C.
E-mail address: y92200b@gmail.com
Major area(s): English teaching effectiveness evaluation; Performance evaluation.

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