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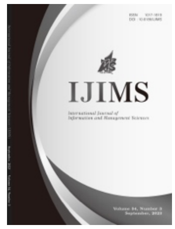

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Optimization Analysis of Priority Queue with Negative Customers and Balking Policy

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Keywords

Priority queue; Negative customers; Preemptive; Balking; Cost function

Abstract

This paper conducts theoretical modeling and performance optimization research on an M/M/1 queueing system with negative customers and balking strategies under preemptive priority. Firstly, based on the continuous-time Markov framework theory, the steady-state probabilities under two-dimensional states are established using quasi-birth-death processes. Secondly, the key performance indicators such as the steady-state probability distribution of positive customers under different priority levels are derived by matrix iterative methods. To validate the theoretical model, a collaborative transmission model for voice and data signals under interference is constructed in the context of multi-service transmission in wireless communication networks. Numerical calculations and performance analyses are implemented by using Matlab, and the variations in voice signal arrival rates that affect the expected loss rate of different signal types are heavily studied. Finally, a cost function is formulated for optimization analysis to determine the optimal balking function that maximizes the expected revenue of the system per unit time. The research outcomes expand the research domain of priority queuing systems and provide theoretical guidance for optimizing the design of communication network queuing systems.

1. Introduction

Since its inception, priority queue has been widely used in many fields, such as emergency patients in hospitals, VIP customers in banks, output and supply of warehouses, etc. In addition, with the development of Internet technology, it is widely used in communication systems, computer interrupt systems, and electronic countermeasures systems. Miller (1981) studied the M/M/1 queue with preemptive priority and non-preemptive priority by matrix analytic method, and gave the iterative formula for solving stationary distribution. Kapadia et al. (1984) analyzed

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the non-preemptive priority queue with limited capacity space, and obtained the balking probability of two types of customers. Jouini & Roubos (2014) considered the multi-server Markovian queues with two types of impatient customers, and numerically compared two cases where the discipline of service within each customer type is first-come first-served (FCFS) or last-come first-served (LCFS). Aibatov (2016) considered a queuing system with unreliable servers and priority customers, and obtained the limit distribution of low priority customers in the system. Devos et al. (2020) analyzed a priority queueing system with a regular queue and an orbit. Subsequently, Xu et al. (2023) studied an M/G/1 retrial queueing system with priority service and derived the Laplace-Stieltjes transform of sojourn time through the supplementary variable method. Considering that dividing packets into different priorities can improve the service quality of the network, Xu et al. (2019) constructed a priority wireless sensor network node queuing model. Recently, Zhang et al. (2024) proposed an active queue management method to schedule differential packets and developed a preemptive priority M/M/1/c vacation queueing model to evaluate the proposed method.

Balking policy also is the research hotspot of queuing theory. For example, in the emergency room of the hospital, when the critical patients arrive at the hospital, they will decide whether to transfer the hospital according to the hospital bed resources. By using the generalized eigenvalue method, Drekic & Woolford (2005) studied the priority queue with balking policy. Ilyashenko et al. (2017) analyzed a retrial queueing system with preemptive priority and randomized push-out mechanism. Adan et al. (2019) studied a queuing model of two types of customers with different distribution of service time and impatient time, and obtained main performance indicators.

Due to the infection of Trojan virus, the network is interrupted or even the system is paralyzed in the computer network. In the signal transmission process, part of the transmitted data will also be offset due to the interference of external signals. Considering the balking behavior of customers, Gelenbe (1991) introduced the concept of negative neurons into the neural network queuing model, and proposed the concept of negative customers. Kim (2012) took as an example an opportunistic spectrum access in cognitive radio networks and derived the waiting-time distributions of each class in the M/G/1 priority queue with multiple classes of customers under the proposed T-preemptive priority discipline. Based on the consideration of the effect of the number of P2P online players on energy consumption, Ma et al. (2024) developed an M/M/c random variations queues with negative customers and preemptive priority policies. The matrix-geometric solution method and Gauss-Seidel iterative method are used to derive the performance measures of the system at steady state for two types of contents.

Based on actual queuing processes, the aforementioned literature investigates queuing service systems with different types of customers. However, in queuing systems with priority customers or negative customers, the impact of balking strategies on the steady-state performance cannot be ignored. Currently, relatively few studies have been conducted on queueing systems that integrate consideration of balking strategies, negative customers, and priority policies.

Inspired by the aforementioned literature, this paper integrates negative customers and priority rights into an M/M/1 queuing system with a customer balking strategy, aiming to fill the research gap in this domain. The introduction of negative customers, priority customers, and balking strategy extends the two-dimensional Markov process, which makes it more challenging to solve using classical queuing theory. This paper solved the queuing problem using a quasi-birth-and-death process and obtained the system stationary equations by analyzing a two-dimensional continuous-time Markov process. Utilizing the matrix-geometric solution and iterative algorithms, explicit expressions for the joint stationary distribution between priority customers and server states in the queuing system are derived, then the matrix-geometric representation of the steady-state probability vector can be obtained. Based on the analytical derivation of the steady-state probability vector, explicit expressions are derived for performance metrics, including expected queue length, marginal distribution, expected balking rates, expected loss rates and expected offset rates of the negative customer. The impacts of system parameters on key performance indicators are analyzed through numerical simulation experiments. The reasonable cost function provides a reliable theoretical basis for the design and optimization of queueing systems. This paper investigated the impacts of priority policies and negative customer strategies on queuing system performance, and further controlled the performance level based on numerical analysis results. The findings can provide theoretical support for the optimal design of priority queuing systems.

This model has significant theoretical value and wide application scenarios in the field of communication network resource optimization. For example, in an information transmission system, voice signals have a higher priority than data signals because of the timeliness of audio signals. During the transmission process in networks, the interference signals are not received upon arrival, and cause attenuation to the normal signals, which is equivalent to the negative customers. Furthermore, when the transmission signal arrives, delayed reception may cause channel congestion due to the untimely reception of the system, which reflects the application of the balking strategy.

The remaining sections of this paper are organized as follows: Section 2 presents the system model description and related assumptions. Section 3 analyzes the stationary distribution of the queueing system. Section 4 investigates the algorithmic solution for steady-state probability vectors. Section 5 derives computational formulas for system performance metrics in steady-state conditions. Section 6 conducts numerical analysis to examine how parameter variations affect key performance indicators. Section 7 constructs a cost function and analyzes parameter evolution trends under different balking functions. Section 8 summarizes the research findings and proposes potential future research directions.

2 Model description

Assume that there are two kinds of positive customers in the system, and the system can accommodate at most $N-1$ positive customers of type $i(i=1,2)$, and satisfy the following assumptions

(1) The arrival processes of the two types of positive customers are independent Poisson processes with parameters $\lambda_i, i=1,2$, respectively.

(2) The service time of the server for the two types of positive customers follows the exponential distribution with parameters μ_i, $i=1,2$, respectively.

(3) Balking process: If the system has i positive customers of type 1 and j positive customers of type 2, the arriving type 1 positive customer will enter the system with probability

a_i, or balk with probability $1-a_i$, and the arriving type 2 positive customer will enter the system with probability b_j, or balk with probability $1-b_j$, where $0 \le a_{i+1} \le a_i, 0 \le b_{j+1} \le b_j$, $i, j = 0,1,2...N-2$, specially, $a_{N-1} = b_{N-1} = 0$. Therefore, the arrival rates of the type 1 positive customers and the arrival rate of the type 2 positive customers respectively are $\lambda_{1,i} = \lambda_1 a_i$ and $\lambda_{2,j} = \lambda_2 b_j$, $i, j = 0,1,2...N-1$.

(4) Priority policy: The type 1 positive customer has a preemptive priority. When there is no type 1 positive customer in the system, the type 2 positive customer can accept the service. During the service of the type 2 positive customer, the arriving type 1 positive customer will preempt the service of the type 2 positive customer, and forced type 2 positive customer returns to original queuing head to wait again.

(5) Remove policy: There are two types of negative customers, and the arrival of the type i negative customers follows the Poisson process with parameter $\theta_i, i = 1,2$. The negative customers do not accept the service and have an offset effect on the positive customers waiting in normal queues. It is assumed that the type i negative customers can only remove the corresponding type i positive customer at the end of the queue according to the RCE policy. If there is no positive customer of the type i in the system, the arriving type i negative customer automatically disappears.

Suppose that all random variables are independent of each other and the same type of positive customer obeys the first-come-first-served service rule.

3 System stationary analysis

Denoted the number of the type i positive customers at time t by $L_i(t)(i=1,2)$, then $\{(L_1(t), L_2(t)), t \ge 0\}$ is a two-dimensional Markov process. Since the capacity of each type positive customer is $N-1$, and the state space is $\Omega = \{(i,j) \mid 0 \le i \le N-1, 0 \le j \le N-1\}$. According to model assumption, the system state transition is shown in Figure 1.

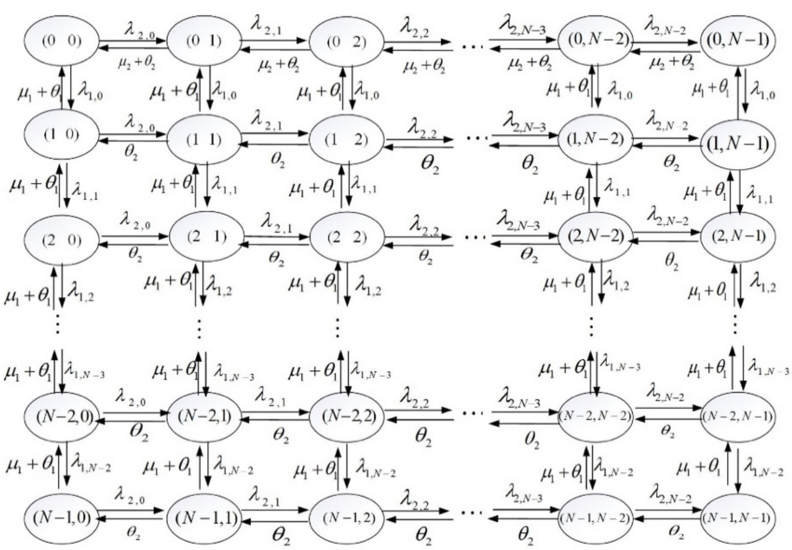

Fig. 1 *State Transition Diagram*

Denoted by

$$p_{10} = -\left(\lambda_{1,0} + \lambda_{2,0}\right), \quad p_{1j} = -\left(\lambda_{1,0} + \lambda_{2,j} + \mu_2 + \theta_2\right), j = 1,2,\ldots,N-2, \quad p_{1N-1} = -(\lambda_{1,0} + \mu_2 + \theta_2),$$

$$q_{i0} = -\left(\lambda_{1,i} + \lambda_{2,0} + \mu_1 + \theta_1\right), \quad q_{ij} = -\left(\lambda_{1,i} + \lambda_{2,j} + \mu_1 + \theta_1 + \theta_2\right), i = 1,2,\ldots,N-1, j = 1,2,\ldots,N-2.$$

$$q_{iN-1} = -(\lambda_{1,i} + \mu_1 + \theta_1 + \theta_2). i = 1,2,...,N-1.$$

In lexicographic order, the infinitesimal generator of the process $\{(L_1(t), L_2(t)), t \geq 0\}$ is as follows

$$Q = \begin{bmatrix} A_0 & C_0 & & & & & \\ B & A_1 & C_1 & & & & \\ & B & A_2 & C_2 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & B & A_{N-3} & C_{N-3} & \\ & & & & B & A_{N-2} & C_{N-2} \\ & & & & & B & A_{N-1} \end{bmatrix}$$

where,

$$A_0 = \begin{bmatrix} p_{10} & \lambda_{2,0} & & & & & \\ \mu_2 + \theta_2 & p_{11} & \lambda_{2,1} & & & & \\ & \mu_2 + \theta_2 & p_{12} & \lambda_{2,2} & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & \mu_2 + \theta_2 & p_{1N-3} & \lambda_{2,N-3} & \\ & & & & \mu_2 + \theta_2 & p_{1N-2} & \lambda_{2,N-2} \\ & & & & & \mu_2 + \theta_2 & p_{1N-1} \end{bmatrix}$$

$$A_i = \begin{bmatrix} q_{i0} & \lambda_{20} & & & & & \\ \theta_2 & q_{i1} & \lambda_{21} & & & & \\ & \theta_2 & q_{i2} & \lambda_{22} & & & \\ & & \mathrm{O} & \mathrm{O} & \mathrm{O} & & \\ & & & \theta_2 & q_{iN-3} & \lambda_{2N-3} & \\ & & & & \theta_2 & q_{iN-2} & \lambda_{2N-2} \\ & & & & & \theta_2 & q_{iN-1} \end{bmatrix}, i = 1,2,\ldots,N-1.$$

$$B = \begin{bmatrix} \mu_1 + \theta_1 & & & \\ & \mu_1 + \theta_1 & & \\ & & \mathrm{O} & \\ & & & \mu_1 + \theta_1 \end{bmatrix}, C_i = \begin{bmatrix} \lambda_{1i} & & & \\ & \lambda_{1i} & & \\ & & \mathrm{O} & \\ & & & \lambda_{1i} \end{bmatrix}, i = 0,1,2\ldots,N-2$$

and $A_i(0 \le i \le N-1), B, C_i(0 \le i \le N-2)$ are all square matrices of order- N .

It can be seen from Figure 1 that the state transition in the system can only occur in adjacent states, and the state space is limited, therefore, the system exists the steady-state probability distribution if and only if $\rho = \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} < 1$.

Assume that $\rho < 1$, the steady-state distribution of the system is

$$\pi_{i,j} = \lim_{t\to\infty} P\{L_1(t) = i, L_2(t) = j\}, (i, j) \in \Omega.$$

The steady-state probability is written in vector form

$$\pi = (\pi_0, \pi_1, \pi_2, \ldots, \pi_{N-1}),$$

where $\pi_i = (\pi_{i0}, \pi_{i1}, \pi_{i2}, \ldots, \pi_{iN-1}), 0 \le i \le N-1.$

Then the equilibrium equation satisfied by the stationary distribution can be written in the form of block matrix

$$\begin{cases} \pi_0 A_0 + \pi_1 B = 0 \\ \pi_{i-1} C_{i-1} + \pi_i A_i + \pi_{i+1} B = 0, 1 \le i \le N-2 \\ \pi_{N-2} C_{N-2} + \pi_{N-1} A_{N-1} = 0 \end{cases} \tag{3.1}$$

and with the normalization conditions

$$\sum_{i=0} \pi_i e_N = 1 \tag{3.2}$$

where e_N is a N dimensional column vector with all elements of 1.

4. Steady state probability solving

The steady state probability distribution satisfied the following results

Theorem 1. The steady-state probability has the following calculation formula:

$$\pi_i = \pi_0 R_i, \ 0 \le i \le N-1 \tag{4.1}$$

and

$$\begin{cases} \pi_0 \left(R_{N-2} C_{N-2} + R_{N-1} A_{N-1} \right) = 0 \\ \sum_{i=0}^{N-1} \pi_0 R_i e_N = 1 \end{cases} \tag{4.2}$$

where

$R_0 = I_0$ is a N- order dimensional unit matrix.

$R_1 = -A_0 B^{-1}$,

$$R_i = -\left(R_{i-2} C_{i-2} + R_{i-1} A_{i-1} \right) B^{-1}, i = 2, 3, \ldots, N-1. \tag{4.3}$$

Proof. When $i = 0$, obviously, Eq. (4.1) is true. When $i = 1$, from the first formula of Eq. (3.1), we get $\pi_1 = -\pi_0 A_0 B^{-1} = \pi_0 R_1$, that is, Eq. (4.1) is true for $i = 1$. Assuming that Eq. (4.1) is true for both $i=k$ and $i=k-1$, with the second formula of Eq. (3.1), we have

$$\pi_{i+1}=-\left(\pi_{i-1}C_{i-1}+\pi_i A_i\right)B^{-1}=-\left(\pi_0 R_{i-1}C_{i-1}+\pi_0 R_i A_i\right)B^{-1}=\pi_0 R_{i+1}$$

Therefore, Eq. (4.1) is true for $i=k+1$. Based on mathematical induction, Eq. (4.1) is proved. Substitute $\pi_{N-2}=\pi_0 R_{N-2}$ and $\pi_{N-1}=\pi_0 R_{N-1}$ into the third formula of Eq. (3.1), and substitute $\pi_i=\pi_0 R_i$ into Eq. (3.2), we can get Eq. (4.2) and π_0, respectively.

Furthermore, the steady-state probability $\pi=\left(\pi_0,\pi_1,\pi_2,\ldots,\pi_{N-1}\right)$ can be obtained by iteration of Eq. (4.1). The solution of π_0 in Eq. (4.2) and the solution of R_i in Eq. (4.3) can be obtained using mathematical software. The iterative algorithm is as follows:

Step 1. Given system parameter $\lambda_i,\mu_i,\theta_i, i=1,2$ and $a_i,b_j,i,j=0,1,2,\cdots,N-1,$ then denote the matrices $A_i(0\le i\le N-1)$, B and $C_i(0\le i\le N-2)$.

Step 2. By using the loop statement if-else-if-else, the loop function of R_i is established. The loop steps are as follows:

```
function[y]=R(x)
    if x==0
y=diag([1 1 1 1 1])
    elseif x==1
y=-A_0*inv(B)
    else
y=-(R(x-2)*C(x-2)+R(x-1)*A(x-1))*inv(B)
    end
```

Step 3. According to the R_i obtained above, the fsolve function is used to solve the Eq. (4.2) and π_0, and the program is

```
fun=@F;
x0=[0,0,0,0,0]
x=fsolve(fun,x0)
```

Step 4. Finally, the steady-state distribution of the system can be obtained by substituting π_0 and R_i into Eq. (4.1).

5. Steady-state performance indexes

The marginal distributions of the type 1 and type 2 positive customers are $\pi_{i\cdot}$ and $\pi_{\cdot j}$, respectively, that is,

$$\pi_{i\cdot}=\pi_i e_N, i=0,1,2,\ldots,N-1,\quad \pi_{\cdot j}=\sum_{i=0}\pi_{ij}, j=0,1,2,\ldots,N-1.$$

The performance indexes of the system are as follows.

(1) The expected queue length of the type 1 and type 2 positive customers is

$$E_{L_1}=\sum_{i=1} i\pi_{i\cdot}\qquad\qquad E_{L_2}=\sum_{j=1} j\pi_{\cdot j}$$

(2) The expected waiting queue length of the type 1 and type 2 positive customers respectively is

$$E_{L_{1q}} = \sum_{i=2}(i-1)\pi_{i\cdot} \qquad E_{L_{2q}} = \sum_{j=2}(j-1)\pi_{\cdot j} + \sum_{j=1} j(\pi_{\cdot j} - \pi_{0j})$$

(3) The expected balking rates of the type 1 and type 2 positive customers respectively are

$$B.R.^1 = \sum_{i=0}\lambda_1(1-a_i)\pi_{i\cdot} \qquad B.R.^2 = \sum_{j=0}\lambda_2(1-b_j)\pi_{\cdot j}$$

(4) The expected offset rates of the negative customer for the type 1 and type 2 positive customers respectively are

$$N.R.^1 = \sum_{i=1}\theta_1\pi_{i\cdot} \qquad N.R.^2 = \sum_{j=1}\theta_2\pi_{\cdot j}$$

(5) The expected loss rates of the type 1 and type 2 positive customers respectively are

$$L.R.^1 = B.R.^1 + N.R.^1 \qquad L.R.^2 = B.R.^2 + N.R.^2$$

(6) The probability that the system is in the idle period is

$$P_I = \pi_{00}.$$

(7) The overflow probabilities of the type 1 and type 2 positive customers respectively are

$$P_{1flow} = \sum_{j=0}^{N-1}\pi_{N-1,j} = \pi_{N-1,\cdot} = \frac{\sum_{i=0}^{N-1}\lambda_{1i}}{(\mu_1+\theta_1)^{N-1}}\pi_{0\cdot} \qquad P_{2flow} = \sum_{i=0}^{N-1}\pi_{i,N-1}$$

6. Numerical analysis

In the network information transmission system, the arrival of speech signal and data signal is Poisson process, and the service time obeys exponential distribution. The arrival rate and service rate of speech signal are $\lambda_1 = 2.5$ and $\mu_1 = 3$. The arrival rate and service rate of the data signal are $\lambda_2 = 2$ and $\mu_2 = 3$. The voice signal has a priority over the data signal, that is, only when the voice signal is all received, the data signal is received. In addition, there is also an interference signal in the system. If the arrival of the interference signal obeys the Poisson flow with $\theta_1 = 1$, only the speech signal can be lost; if the arrival of the interference signal obeys the Poisson flow with $\theta_2 = 1$, only the data signal can be lost. The interference signal is canceled according to the RCE policy. In addition, when the new signal arrives, the transmission may fail due to the congestion of the existing signal in the system.

It is assumed that the balking functions corresponding to the speech signal and the data signal are $a_i = \frac{1}{i+1}$ and $b_j = \frac{1}{j+1}$, and the system can receive at most 4 speech and data signals. According to the calculation program in Section 4, the numerical results of the steady-state probability of the system are obtained, as shown in Table 1.

Table 1. *Steady-State Probability Distribution of System*

i \ j	0	1	2	3	4	$\pi_{i\cdot}$
0	0.2705	0.1754	0.0670	0.0186	0.0040	0.5355
1	0.1290	0.1266	0.0575	0.0174	0.0042	0.3347
2	0.0331	0.0410	0.0216	0.0071	0.0019	0.1047
3	0.0059	0.0085	0.0050	0.0018	0.0005	0.0217
4	0.0008	0.0013	0.0008	0.0003	0.0002	0.0034
$\pi_{\cdot j}$	0.4393	0.3528	0.1519	0.0452	0.0108	1.0000

The performance indicators of the system are obtained as follows:

$$E_{L_1} = 0.6228;\ E_{L_2} = 0.8354;\ E_{L_{1q}} = 0.1582;\ E_{L_{2q}} = 0.7289;\ B.R^1 = 0.6421;$$
$$B.R^2 = 0.6448;\ N.R^1 = 0.4645;\ N.R^2 = 0.5607;\ L.R^1 = 1.1066;\ L.R^2 = 1.2055$$
$$P_I = 0.2705;\quad P_{1flow} = 0.0034;\quad P_{2flow} = 0.0108.$$

The delay of the signal in the system can be estimated by the expected queue length of the two types of signals in the system, and the received capacity of the system can be improved by the overflow probability. In order to reasonably allocate resources and effectively improve efficiency, it is also particularly important to study the influence of some parameters on main performance indicators.

Next, by comparing the changes in the expected queue length and expected loss rate of various types of signals under different balking functions, a more convenient strategy is provided for the system. Usually, the commonly used balking function is the following:

$$(a) a_i = 1 - \frac{i}{N}, b_j = 1 - \frac{j}{N} \quad (b) a_i = \frac{1}{i+1}, b_j = \frac{1}{j+1} \quad (c) a_i = \mathrm{e}^{-i}, b_j = \mathrm{e}^{-j}$$

(1) When $\lambda_2=2, \mu_1=\mu_2=3, \theta_1=\theta_2=1, N=5$, the expected queue length of the speech signal E_{L_1} varies with the arrival rate of the speech signal λ_1 is shown in Figure 2. When the arrival rate of the speech signal is the same, the expected queue length of the speech signal is the smallest when the balking function is $a_i = \mathrm{e}^{-i}, b_j = \mathrm{e}^{-j}$, and the expected queue length of the speech signal is the largest when the balking function is $a_i = 1 - \frac{i}{N}, b_j = 1 - \frac{j}{N}$. In addition, when the balking function is the same, the expected queue length of the speech signal E_{L_1} increases with the increase of the arrival rate λ_1, which also reflects the rationality of the iterative method to calculate the steady-state probability.

(2) When $\lambda_2=2, \mu_1=\mu_2=3, \theta_1=\theta_2=1, N=5$, the expected loss rate of the speech signal $L.R^1$ varies with the arrival rate of the speech signal λ_1 is shown in Figure 3. When the arrival rate of the speech signal is the same, the expected loss rate of the speech signal is the smallest when the balking function is $a_i = 1 - \frac{i}{N}, b_j = 1 - \frac{j}{N}$, and the expected loss rate of the speech signal is the largest when the balking function is $a_i = \mathrm{e}^{-i}, b_j = \mathrm{e}^{-j}$. In addition, when the balking function is the same, the expected loss rates of the speech signal $L.R^1$ increases with the increase of the arrival rate λ_1.

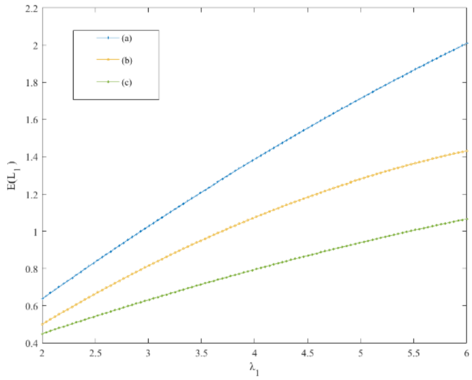

Fig. 2 E_{L_1} **vs.** λ_1

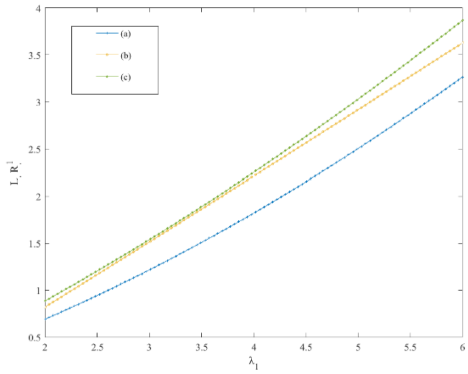

Fig.3 $L_.R_.^1$ **vs.** λ_1

(3) When $\lambda_2=2, \mu_1=\mu_2=3, \theta_1=\theta_2=1, N=5$, the expected queue length of the data signal E_{L_2} varies with the arrival rate of the speech signal λ_1 is shown in Figure 4. When the arrival rate of the speech signal is the same, the expected queue length of the data signal is the smallest when the balking function is $a_i=\mathrm{e}^{-i}, b_j=\mathrm{e}^{-j}$, and the expected queue length of the data signal is the largest when the balking function is $a_i=1-\frac{i}{N}, b_j=1-\frac{j}{N}$. In addition, when the balking function is the same, the expected queue length of the data signal E_{L_2} increases with the increase of the arrival rate λ_1.

(4) When $\lambda_2=2, \mu_1=\mu_2=3, \theta_1=\theta_2=1, N=5$, the expected loss rate of the data signal $L_.R_.^2$ varies with the arrival rate of the speech signal λ_1 is shown in Figure 5. When the arrival rate of the speech signal is the same, the expected loss rate of the data signal is the smallest when the balking function is $a_i=1-\frac{i}{N}, b_j=1-\frac{j}{N}$, and the expected loss rate of the data signal is the largest when the balking function is $a_i=\mathrm{e}^{-i}, b_j=\mathrm{e}^{-j}$. In addition, when the balking function is the same, the expected loss rates of the data signal $L_.R_.^2$ increases with the increase of the arrival rate λ_1.

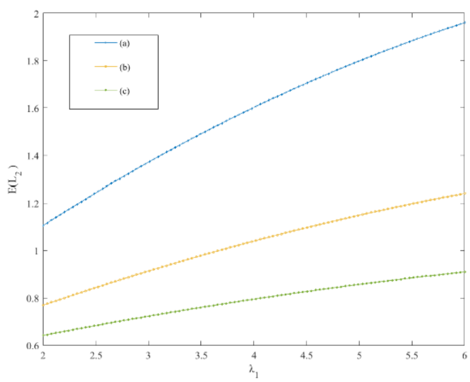

Fig. 4 E_{L_2} vs. λ_1

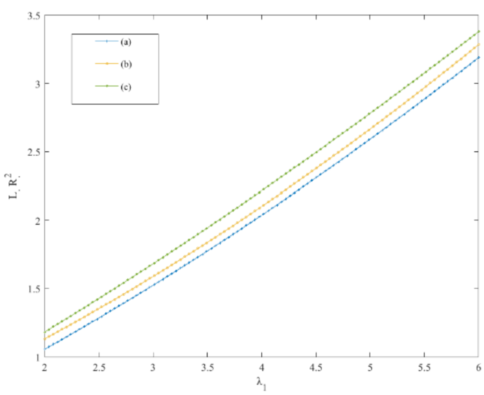

Fig. 5 $L_.R_.^2$ vs. λ_1

It can be seen from the above data that the expected queue length of the two types of signals is the smallest when the balking function is $a_i = \mathrm{e}^{-i}, b_j = \mathrm{e}^{-j}$, which is more advantageous at this time. However, from the perspective of expected loss rate, when the balking function is $a_i = 1 - \frac{i}{N}, b_j = 1 - \frac{j}{N}$, the expected loss rate of the two types of signals is the smallest, which has more advantages. When the system chooses the balking policy, the influence of the expected queue length and the expected loss rate should be considered comprehensively. In addition, when the arrival rate of the speech signal gradually increases, the expected queue length and the expected loss rate also increase, which is also in line with the actual results and has practical significance. Therefore, by setting reasonable parameter values, the system process can be effectively designed to optimize the system.

7. Optimization analysis

According to the performance indexes of the system, the cost-benefit function is constructed to obtain the benefits of the system under different balking functions, and then a more suitable balking scheme is selected for the system. In the above network information transmission system, this section considers the profit of the system in unit time under three balking functions, and assume that the system cost-benefit function is

$$F = (B_1 - E_2)(\pi_{i\cdot} - \pi_{0\cdot}) + (B_2 - E_3)(\pi_{0\cdot} - \pi_{00}) + (B_3 - E_4)E_{L_{1q}} + (B_4 - E_5)E_{L_{2q}}$$
$$- E_1\pi_{00} - E_6 L_{\cdot} R_{\cdot}^1 - E_7 L_{\cdot} R_{\cdot}^2$$

where

$B_1(B_2)$: The profit generated by the system receiving voice (data) signal per unit time

$B_3(B_4)$: The profit generated by the voice (data) signal entering the system waiting for unit time

E_1: The cost of unit time consumed when the server is idle

$E_2(E_3)$: The cost consumed by the system to receive speech (data) signals per unit time

$E_4(E_5)$: The cost of voice (data) signal entering the system waiting for unit time

$E_6(E_7)$: The cost per unit time of speech (data) signal due to balking policy and interference signal offset policy

Assume that $B_1 = 100, B_2 = 90, B_3 = 70, B_4 = 60, E_1 = 10, E_2 = 30, E_3 = 30, E_4 = 20,$ $E_5 = 15, E_6 = 10, E_7 = 10$, the benefit of the system is illustrated in Table 2.

Table 2. *The Benefit of System under Different Balking Policies*

Balking policy	$a_i = 1 - \frac{i}{N}, b_j = 1 - \frac{j}{N}$	$a_i = \frac{1}{i+1}, b_j = \frac{1}{j+1}$	$a_i = \mathrm{e}^{-i}, b_j = \mathrm{e}^{-j}$
F	86.2128	63.2995	51.4018

From Table 2, it is concluded that the system benefit is the largest under the first type of balking policy, so the enterprise can adjust the balking policy and select the appropriate cost function to maximize the benefit.

8. Conclusion

In communication systems and electronic transmission systems, priority queuing greatly accelerates the transmission efficiency of the system. In addition, due to the presence of signal congestion and interference signals, the balking policy and the presence of negative customers are more suitable for practical applications. This paper constructs a priority queue with balking policy and negative customers. The matrix form solution and iterative formula of the steady-state probability of the system are given by matrix iteration method, and the performance indexes of the system are obtained. Finally, the sensitivity analysis of the system is carried out to improve and optimize the efficiency of the server.

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