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International Journal of Information and Management Sciences

International Journal of
Information and Management Sciences

35 (2024), 99-118. DOI:10.6186/IJIMS.202406_35(2).0001

Profit Analysis of Two-Dimensional State Markovian Queuing Model with Correlated Servers, Multiple Vacation, Catastrophes, Feedback and Balking

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Keywords

Correlated servers; Multiple vacation; Catastrophes; Feedback; Balking.

Abstract

The present paper studies the queuing system with correlated servers, multiple vacation, catastrophes, feedback and balking. Inter arrival times follow an exponential distribution with parameters λ and service times follow Bivariate exponential distribution BVE (μ, μ, ν) where μ is the service time parameter and ν is the correlation parameter. Both the servers go on vacation with probability one when there are no units in the system. Occurrence of catastrophes follow Poisson distribution with rate ξ. Laplace transform approach has been used to find the time-dependent solution. The model estimates the total expected cost, total expected profit and obtained the optimal values by varying time for cost and profit. The best optimal value is at t=1 when joining probability is taken as 0.75 and t=4 when feedback probability is taken as 0.30 for minimum cost and maximum profit, respectively. These important key measures give a greater understanding of the model behaviour. Numerical analysis and graphical representations have been done by using Maple software.

1. Introduction

The main purpose to carrying out the present research is the use of correlated servers in day to day life. From literature survey, it was found that not much work has been done with correlated servers. In earlier studies, all authors studied the correlated server Markovian queue with finite waiting space capacity. However, our study focuses on correlated servers with infinite waiting space capacity, including different parameters. Correlated servers are closer to real-world scenarios and may require more sophisticated modeling approaches.

A system of queues in series or in parallel should ordinarily be studied taking into account the interdependence of servers, but this leads to very complicated mathematics even in very simple case of systems. So to reduce such complications of analysis the servers are considered to be independent. But this independence of servers cause impact in time bound operations such as

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vehicle inspection counters, toll booths, large bars and cafeterias *etc.* where for efficient system functioning the correlation between the servers contributes significantly. Nishida et al. (1974) investigated a two-server Markovian queue assuming the correlation between the servers and obtained steady-state results for a limited waiting space capacity of two units. Sharma (1990) investigated the transient solution to this problem again using only two units waiting spaces capacity. Sharma & Maheswar (1994) developed a computable matrix approach to study a correlated two-server Markovian queue with finite waiting space. They also derived waiting time distribution for steady-state and obtained the transient probabilities through steady-state by using a matrix approach and Laplace transform approach.

Various studies have been conducted to evaluate different performance measures to verify the robustness of the system in which a server takes a break for a random period of time *i.e.* vacation. When the server returns from a vacation and finds the empty queue, it immediately goes on another vacation and continuous taking vacation *i.e.* multiple vacation and if it finds at least one waiting unit, then it will commence service according to the prevailing service policy. Different queuing systems with multiple vacation have been extensively investigated and effectively used in several fields including industries, computer & communication systems, telecommunication systems *etc.* Different types of vacation policies are available in literature such as single vacation, multiple vacation and working vacations. Researches on multiple vacation systems have grown tremendously in the last several years. Cooper (1970) was the first to study the vacation model and determined the mean waiting time for a unit arrive at a queue served in cyclic order. Sharda & Indra (1996) obtained explicit time-dependent probabilities for a queuing system where the server takes multiple vacation and also serves the units intermittently. Sharda & Indra (1999) studied a two-state queuing model by utilizing the intermittently available time and the vacation time. The vacation and the intermittently time are having the general distribution where as service time is exponentially distributed. Ke & Pearn (2004) developed closed-form solutions for analysing the management policy of an M/M/1 queuing system with server breakdowns and multiple vacation by using the probability generating function approach. Indra & Bansal (2010) used the supplementary variable and Laplace transform techniques to derive explicit probabilities of the exact number of arrivals and departures by a given time as well as reliability and availability of the server. Upadhyaya (2016) and Panta *et al.* (2021) carried out a brief survey of the research on vacation queuing systems with different techniques. Niranjan *et al.* (2019) analysed a bulk arrival and batch service retrial queuing system with server failure and multiple vacation and also obtained optimum cost by using the Supplementary variable technique. Sharma & Indra (2022) studied the effect of balking and reneging on a two-dimensional state queuing model with multiple vacation by using the Laplace transform approach.

Queuing systems with catastrophes are also getting a lot of attention nowadays and may be used to solve a wide range of real-world problems. Catastrophes may occur at any time, resulting in the loss of units and the deactivation of the service centre, because they are totally unpredictable in nature. Such type of queues with catastrophes plays an important role in computer programs, telecommunication and ticket counters *etc.* For example, virus or hacker attacking a computer system or program causing the system fail or become idle. Kumar & Arivudainambi (2000) derived transient solution of an M/M/1 queuing model with catastrophes by using the Laplace transform approach. Kumar & Madheswari (2005) analysed transient solution of an M/M/1 queuing system with the possibility of catastrophes and server failures by using the Laplace transform approach. Tarabia (2011) carried out an analysis of infinite-buffer queuing system with single server, balking and catastrophes. Kumar (2017) considered Markovian multi-server queuing system with balking and catastrophes and obtained transient solution by using the probability generating function along with Bessel function properties. Sampath (2020) considered an M/M/1 queue with balking, catastrophes, server failure and repairs and obtained an explicit expression for the time-dependent system size probabilities in terms of the modified Bessel function of first kind. Ammar et al. (2022) investigated a stationary fluid queue operated by a state-dependent birth-death process with catastrophes by using the Laplace transform approach.

Units may be served repeatedly for many reasons, *e.g.* when a unit is unsatisfied with a service, the service might be retried until receive satisfactorily service. For example, we visit to the online shopping store and order a full-sleeve jacket but when we receive the order it turn out to be half-sleeve jacket. Since we are unsatisfied with the service, so we go to the return policy or exchange policy provided by the shopping store and received satisfactory service. Many researchers have been attracted to the study of queues with feedback as large number of applications have been found in many areas including production systems, post offices, supermarkets, hospital management, financial sectors, ticket offices, grocery stores, ATMs and so forth. The concept of feedback was first introduced by (Finch, 1959). Queuing system with feedback have been followed by many researchers including (Takacs, 1963; D'Avignon & Disney, 1976; Sharda et al., 1986; Garg & Singla, 2005; Thangaraj & Vanitha, 2010; Nazarov et al., 2021). The retrial feedback queues were also studied widely by researchers including (Ayyappan & Sathiya, 2013; Chang et al., 2018).

Queues with balking have numerous applications in everyday life. Balking occurs if units avoid joining the queue, when they perceive the queue to be too long. Long queues at cash counters, ticket booths, banks, barber shops, grocery stores, toll plaza *etc.* The concept of balking was first introduced by (Haight, 1957; 1960). After that, many researchers focused on the study of Markovian queuing system with balking including different parameters using different techniques (Abou-El-Ata et al., 1992; Goswami, 2014; Ke et al., 2017; Kumar & Soodan, 2019). Jain et al. (2022) studied Markovian retrial queue by considering the concept of working vacation, balking behaviour of the customers and imperfect service.

With above concepts in mind, we analyse a two-dimensional state M/M/2 queuing model with correlated servers, multiple vacation, catastrophes, feedback, balking.

A two-dimensional state model has been used to deal with complicated transient analysis of some queuing problem. This model is used to examine the queuing system for exact number of arrivals and departures by given time t. In case of a one-dimensional state model, it is difficult to determine how many units have entered, left or waiting units in the system, while the two-dimensional state model exactly identifies the numbers of units that have entered, left, or waiting in the system. The idea of two-dimensional state model for the M/M/1 queue was first given by (Pegden & Rosenshine, 1982). After that, the two-dimensional state model has attracted the attention of a lot of researchers. Indra & Sharda (2004) analysed a first-come, first-served, single channel queuing system in which probabilities of arrivals in batches of different sizes at a transition mark depend upon the latest arrival run and obtained the Laplace transforms of probabilities of (i) number of units in the system (ii) exact number of arrivals and (iii) exact number of departures.

During the COVID-19 pandemic, hospitals and emergency services relied heavily on correlated servers to manage patient admissions record critical information such as symptoms, test results, treatment plans, healthcare resources, *etc.* Consider the situation of two hospitals that work independently, that is, do not share any information about the coronavirus patients, symptoms of patients, medications they used, *etc.* with each other and do not help each other, due to this communication gap, the medical staff will take much more time to fight the coronavirus. But if both of them work interdependently on each other, *i.e.* share all the information about the virus and help each other, then there is a greater possibility that it will take less time to deal with the coronavirus and more lives may be saved. Therefore, interdependent servers are more reliable than independent servers. When servers work interdependently, they are called correlated servers. Some of the hospital staff, after dealing with the coronavirus patients, may take leave when they find there are no more coronavirus patients in the emergency room, be considered as vacation period. But if, due to the shortening of oxygen cylinders or PPE kits, the hospital facilities were disturbed and all the patients left the hospital, be considered as major catastrophe. Due to the sudden increase in the number of coronavirus patients in the hospital, it became difficult to get emergency services, due to which patients became impatient and did not stand in the entry queue for medical facility or in the queue for ventilator booking, be called balking. Sometimes patients who did not receive good medical services during the epidemic immediately contacted hospital staff to receive satisfactory medical services termed as feedback.

The present paper has been structured as follows. In section 1 introduction and in section 2 the model assumptions, notations and description are given. Section 3 contains recursive solution by using the differential-difference equations to find out the time-dependent solution and section 4 describes important performance measures. Section 5 investigates the total expected cost function and total expected profit function for the given queuing system. In section 6, we present the numerical results in the form of tables and section 7 contains the tables and graphs to illustrate the impact of various factors on performance measures. The last section contains discussion on the findings and suggestions for further work.

2. Assumptions

- Arrivals follow Poisson distribution with parameter λ.
- There are two servers and the service times follow Bivariate exponential distribution BVE* (μ, μ, ν) where μ is the service time parameter and ν is the correlation parameter.
- The vacation time of the server follow exponential distribution with parameter w.
- On arrival a unit either decides to join the queue with probability β or not to join the queue with probability $1-\beta$.
- After completion of the service, the units rejoin at the early end of the queue to receive service with probability q.
- Occurrence of catastrophes follows Poisson distribution with parameter ξ.
- Various stochastic processes involved in the system are statistically independent of each other.

*introduced by (Marshall & Olkin, 1967)

Initially, the system starts with zero units and the server is on vacation, *i.e.*

$$P_{0,0,V}(0)=1 \quad ; \quad P_{0,0,B}(0)=0 \tag{2.1}$$

2.1. Notations

$$\delta_{i,j} = \begin{cases} 1 & ; for \quad i = j \\ 0 & ; for \quad i \neq j \end{cases} \quad ; \quad \sum_{i}^{j} = 0 \quad for \quad j < i \tag{2.2}$$

2.2. The Two-Dimensional State Model

$P_{i,j,V}(t)$ = " The probability that there are exactly i arrivals and j departures by time t and the server is on vacation".

$P_{i,j,B}(t)$ = " The probability that there are exactly i arrivals and j departures by time t and the server is busy in relation to the queue ".

$P_{i,j}(t)$ = " The probability that there are exactly i arrivals and j departures by time t ".

3. The Differential-Difference Equations for the Queuing Model under Study

A consideration of variations in a continuous variable often results in a differential equation and, in the same way, a consideration of variations in a discrete variable leads to a difference equation. In queuing theory, when we deal with functions like $P_n(t)$, where n is a discrete variable and t a continuous one, and consider changes in both n and t, the results is a differential difference equation. By using the Laplace transform, the 'differential' aspect of the equation is done away with, thus yielding a pure difference equation. However If we use Laplace transforms, the differential aspect is got rid of and we are left with a difference equation, as, for example, in Srivastava and Kashyap (1982), Eq (3.1) to (3.4). These resulting differential or difference equations may be solved by usual methods. However, usually the Laplace transforms is used this reducing the equation to a simple algebraic equation, as in done in (3.6) to (3.12). Thus, the use of the Laplace transforms results in a considerable simplification of the queue equations.

$$\frac{d}{dt}P_{i,i,V}(t) = -\lambda\beta P_{i,i,V}(t) + (q\mu+\nu)P_{i,i-1,B}(t)(1-\delta_{i,0}) + \nu P_{i,i-2,B}(t)(1-\delta_{i,0}-\delta_{i,1}) + \xi(1-P_{i,i,V}(t)) \qquad i \geq 0 \tag{3.1}$$

$$\frac{d}{dt}P_{i+1,i,B}(t) = -(\lambda\beta+q\mu+\nu+\xi)P_{i+1,i,B}(t) + 2q\mu P_{i+1,i-1,B}(t)(1-\delta_{i,0}) + \nu P_{i+1,i-2,B}(t)(1-\delta_{i,0}-\delta_{i,1}) + wP_{i+1,i,V}(t) \qquad i \geq 0 \tag{3.2}$$

$$\frac{d}{dt}P_{i,j,V}(t) = -(\lambda\beta+w+\xi)P_{i,j,V}(t) + \lambda\beta P_{i-1,j,V}(t) \qquad i>j\geq 0 \tag{3.3}$$

$$\begin{aligned}\frac{d}{dt}P_{i,j,B}(t) = &-(\lambda\beta+2q\mu+\nu+\xi)P_{i,j,B}(t) + \lambda\beta P_{i-1,j,B}(t)(1-\delta_{i-1,j}) + 2q\mu P_{i,j-1,B}(t)(1-\delta_{j,0}) \\ &+ \nu P_{i,j-2,B}(t) + wP_{i,j,V}(t)\end{aligned} \qquad i>j+1 \tag{3.4}$$

Clearly,

$$P_{i,j}(t)=P_{i,j,V}(t)+P_{i,j,B}(t)(1-\delta_{i,j}) \qquad i\geq j\geq 0 \quad (3.5)$$

The preceding equations (3.1) to (3.4) are solved by taking the Laplace transforms together with initial condition

$$\overline{P}_{0,0,V}(s)=\frac{(\xi+s)}{s(s+\lambda\beta+\xi)} \quad (3.6)$$

$$\overline{P}_{i,0,V}(s)=\frac{(\lambda\beta)^i(\xi+s)}{s(s+\lambda\beta+\xi)(s+\lambda\beta+w+\xi)^i} \qquad i>0 \quad (3.7)$$

$$\overline{P}_{i,i,V}(s)=\left(\frac{q\mu+\nu}{s+\lambda\beta+\xi}\right)P_{j,j\text{-}1,B}(s)+\left(\frac{\nu}{s+\lambda\beta+\xi}\right)P_{j,j\text{-}2,B}(s) \qquad i>0 \quad (3.8)$$

$$\overline{P}_{i,0,B}(s)=\frac{w(\lambda\beta)^i(\xi+s)}{s(s+\lambda\beta+\xi)(s+\lambda\beta+w+\xi)(s+\lambda\beta+q\mu+\nu+\xi)(s+\lambda\beta+2q\mu+\nu+\xi)^{i\text{-}1}}$$
$$+w(\lambda\beta)^i\sum_{m=1}^{i\text{-}1}\frac{(\xi+s)}{s(s+\lambda\beta+\xi)(s+\lambda\beta+w+\xi)^{m+1}(s+\lambda\beta+2q\mu+\nu+\xi)^{i\text{-}m}} \qquad i\geq 1 \quad (3.9)$$

$$\overline{P}_{i+1,i,B}(s)=\left(\frac{2q\mu}{s+\lambda\beta+q\mu+\nu+\xi}\right)P_{i+1,i\text{-}1,B}(s)+\left(\frac{\nu}{s+\lambda\beta+q\mu+\nu+\xi}\right)P_{i+1,i\text{-}2,B}(s)+$$
$$\left(\frac{(q\mu+\nu)w\lambda\beta}{(s+\lambda\beta+\xi)(s+\lambda\beta+w+\xi)(s+\lambda\beta+q\mu+\nu+\xi)}\right)P_{i\text{-}1,i\text{-}1,B}(s) \qquad i>0 \quad (3.10)$$

$$\overline{P}_{i,j,V}(s)=\left(\frac{q\mu+\nu}{s+\lambda\beta+\xi}\right)\left(\frac{\lambda\beta}{s+\lambda\beta+w+\xi}\right)^{i\text{-}j}P_{j,j\text{-}1,B}(s)+\left(\frac{\lambda\beta}{s+\lambda\beta+w+\xi}\right)^{i\text{-}j}\left(\frac{\nu}{s+\lambda\beta+\xi}\right)P_{j,j\text{-}2,B}(s) \qquad i>j\geq 0 \quad (3.11)$$

$$\overline{P}_{i,j,B}(s)=\left(\frac{\lambda\beta}{s+\lambda\beta+2q\mu+\nu+\xi}\right)P_{i\text{-}1,j,B}(s)+\left(\frac{2q\mu}{s+\lambda+2q\mu+\nu+\xi}\right)P_{i,j\text{-}1,B}(s)+\left(\frac{\nu}{s+\lambda\beta+2q\mu+\nu+\xi}\right)P_{i,j\text{-}2,B}(s)$$
$$+\left(\frac{q\mu+\nu}{s+\lambda\beta+\xi}\right)\left(\frac{w}{s+\lambda\beta+2q\mu+\nu+\xi}\right)\left(\frac{\lambda\beta}{s+\lambda\beta+w+\xi}\right)^{i\text{-}j}\left[P_{j,j\text{-}1,B}(s)+P_{j,j\text{-}2,B}(s)\right] \qquad i>j+1,\ j>0 \quad (3.12)$$

It is seen that

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left[\overline{P}_{i,j,V}(s)+\overline{P}_{i,j,B}(s)(1-\delta_{i,j})\right]=\frac{1}{s} \qquad \text{and hence} \quad (3.13)$$

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\left[P_{i,j,V}(t)+P_{i,j,B}(t)(1-\delta_{i,j})\right]=1 \qquad \text{a verification.} \quad (3.14)$$

4. Performance Measures

(a) The Laplace transform of $P_{i.}(t)$ of the probability that exactly $\boldsymbol{i}$ units arrive by time $\boldsymbol{t}$; when initially there are no unit in the system is given by

$$\overline{P}_{i.}(s)=\sum_{j=0}^{i}\left[\overline{P}_{i,j,V}(s)+\overline{P}_{i,j,B}(s)(1-\delta_{i,j})\right]=\sum_{j=0}^{i}\overline{P}_{i,j}(s)=\frac{(\lambda\beta)^i}{(s+\lambda\beta)^{i+1}} \quad (4.1)$$

And its inverse Laplace transform is $P_{i.}(t)=\frac{e^{-\lambda\beta t}(\lambda\beta t)^i}{i!}$ (4.2)

The arrivals follow a Poisson distribution as the probability of the total number of arrivals is not affected by vacation time of the server.

(b) $P_{.j}(t)$ is the probability that exactly j units have been served by time t. In terms of $P_{i,j}(t)$ we have

$$P_{.j}(t)=\sum_{i=j}^{\infty}P_{i,j}(t) \tag{4.3}$$

(c) The probability of exactly n units in the system at time t, denoted by $P_n(t)$, can be expressed in terms $P_{ij}(t)$ as

$$P_n(t)=\sum_{j=0}^{\infty}P_{j+n,j}(t) \tag{4.4}$$

(d) The Laplace transform of mean number of arrival by time t is

$$\sum_{i=0}^{\infty}i\overline{P}_{i.}(s)=\frac{\lambda\beta}{(s^2)} \tag{4.5}$$

And inverse of the mean number of arrivals by time t is

$$\sum_{i=0}^{\infty}iP_{i.}(t)=\lambda\beta t \tag{4.6}$$

(e) The mean number of units in the queue is calculated as follows

$$Q_L(t)=\sum_{n=1}^{\infty}nP_V(t)+\sum_{n=2}^{\infty}(n-2)P_B(t) \tag{4.7}$$

Where $n = i-j$, $P_V(t)=\sum_{j=0}^{i}P_{i,j,V}(t)$ and $P_B(t)=\sum_{j=0}^{i-1}P_{i,j,B}(t)$

5. Cost Function and Profit Function

For the given queuing system, the following notations have been used to represent various costs to find out the total expected cost and total expected profit per unit time

Let

C_H: "Cost per unit time for unit in the queue".
C_B: "Cost per unit time for a busy server".
C_μ: Cost per service per unit time.
C_V: "Cost per unit time when the server is on vacation".
$C_{\mu-q}$: "Cost per unit time when a unit rejoins at the early end of the queue as a feedback unit".

If I is the total expected amount of income generated by delivering a service per unit time then

(i)Total expected cost per unit at time $\boldsymbol{t}$ is given by

$$TC(t)=C_H*Q_L(t)+C_B*P_B(t)+C_V*P_V(t)+\mu*(C_\mu+C_{\mu-q}) \tag{5.1}$$

(ii)Total expected income per unit at time $\boldsymbol{t}$ is given by

$$TE_I(t)=I*\mu*(1-P_V(t))=I*\mu*P_B(t) \tag{5.2}$$

(iii)Total expected profit per unit at time t is given by

$$TE_P(t) = TE_I(t) - TC(t) \tag{5.3}$$

6. Numerical Results (Numerical Validity Check)

1. For the state when the server is on vacation i.e., $P_V(t)$
2. For the state when the server is busy in relation to the queue i.e., $P_B(t)$
3. The probability $P_{i.}(t)$ that exactly i units arrive by time t is

$$P_{i.}(t) = \sum_{j=0}^{i} P_{i,j}(t) \tag{6.1}$$

4. A numerical validity check of inversion of $\overline{P}_{i,j}(s)$ is based on the relationship

$$\Pr\{i \text{ arrivals in } (0, t)\} = \frac{e^{-\lambda\beta t}(\lambda\beta t)^i}{i!} = \sum_{j=0}^{\infty} P_{i,j}(t) = P_{i.}(t) \tag{6.2}$$

The probabilities of this model shown in last column of Table1 given below are consistent to the last column of (Pegden & Rosenshine, 1982) by keeping constant values of w=1, q=0.5, ξ=0, β=1 and ν=0.25 shown in table

Table 1 *Numerical Validity Check of Inversion of* $\overline{P}_{i,j}(s)$

λ	μ	t	i	$\frac{e^{-\lambda t} * (\lambda t)^i}{i!}$	$\sum_{j=0}^{i} P_{i,j,V}(t)$	$\sum_{j=0}^{i-1} P_{i,j,B}(t)$	$\sum_{j=0}^{i} P_{i,j}(t)$
1	2	3	1	0.149361	0.1156997	0.0336614	0.1493612
1	2	3	3	0.224042	0.1203817	0.1036600	0.2240418
1	2	3	5	0.100819	0.0367587	0.0640601	0.1008188
2	2	3	1	0.014873	0.0115207	0.0033518	0.0148725
2	2	3	3	0.089235	0.0479476	0.0412874	0.0892350
2	2	3	5	0.160623	0.0585637	0.1020595	0.1606232
1	2	4	1	0.073263	0.0595569	0.0137055	0.0732625
1	2	4	3	0.195367	0.1174540	0.0779127	0.1953668
1	2	4	5	0.156293	0.0681580	0.0881354	0.1562934
2	2	4	1	0.002684	0.0021816	0.0005020	0.0026837
2	2	4	3	0.028626	0.0172099	0.0114161	0.0286261
2	2	4	5	0.091604	0.0399474	0.0516562	0.0916036
2	4	4	5	0.091604	0.0591946	0.0324089	0.0916036
1	2	4	4	0.195367	0.1005530	0.0948137	0.1953668
1	2	3	6	0.050409	0.0149712	0.0354381	0.0504094

7. Sensitivity Analysis

This part focuses on the impact of the arrival rate (λ), service rate (μ), correlation parameter (ν), vacation rate (w), catastrophes rate (ξ), feedback probability (q) and balking probability (1-β) on the probability when the server is on vacation ($P_V(t)$), probability when the server is busy ($P_B(t)$), expected queue length ($Q_L(t)$), total expected cost ($TC(t)$), total expected income ($TE_I(t)$) and total expected profit ($TE_P(t)$) at time t. We examine the behaviour of the queuing system using measures of effectiveness along with cost and profit analysis by varying one parameter, while keeping all other parameters fixed *i.e.* λ=1, μ=2, ν=0.25, w=3, ξ=0.0001, q=0.5 and β=1 and taking cost per unit time for unit in the queue=10, cost per unit time for a busy server=8, cost per unit time when the server is on vacation=5, cost of service per unit time=4, cost per unit time when a unit rejoins at the early end of the queue=2, total expected amount of income=100 and number of units in the system=8.

7.1 Impact of Arrival Rate (λ)

Table 2 *Measures of Effectiveness versus* λ

t	λ	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	1.00	0.6636668	0.3363319	0.4008312	22.0173012	67.266380	45.2490788
2		0.5290028	0.4707597	0.5208095	23.6191866	94.151940	70.5327534
3		0.4948042	0.5013937	0.5680509	24.1656796	100.27874	76.1130604
4		0.4931346	0.4995092	0.5586669	24.0484156	99.901840	75.8534244
5		0.4920388	0.4938961	0.5019695	23.4310578	98.779220	75.3481622
1	1.10	0.6386224	0.3613750	0.4493757	22.5778690	72.275000	49.6971310
2		0.4978511	0.5016794	0.5928026	24.4307167	100.33588	75.9051633
3		0.4616160	0.5344739	0.6457114	25.0409852	106.89478	81.8537948
4		0.4592345	0.5319747	0.6195506	24.7474761	106.39494	81.6474639
5		0.458359	0.5240416	0.5315082	23.7992098	104.80832	81.0091102
1	1.20	0.6147775	0.3852176	0.4993079	23.1487073	77.043520	53.8948127
2		0.4686003	0.5305378	0.6677619	25.2649229	106.10756	80.8426371
3		0.4303260	0.5680055	0.7233066	25.9287400	113.60110	87.6723600
4		0.4297345	0.5634674	0.6723942	25.3803537	112.69348	87.3131263
5		0.4282751	0.5570253	0.6068353	24.6659309	111.40506	86.7391291

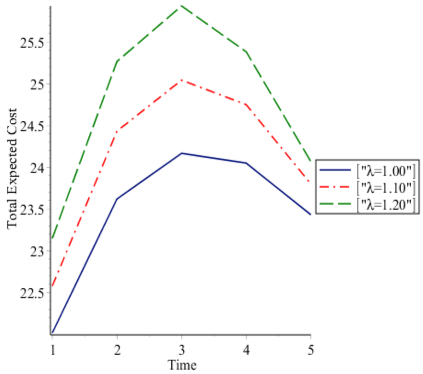

Fig. 1: *Variation of cost with time t by varying arrival rate λ*

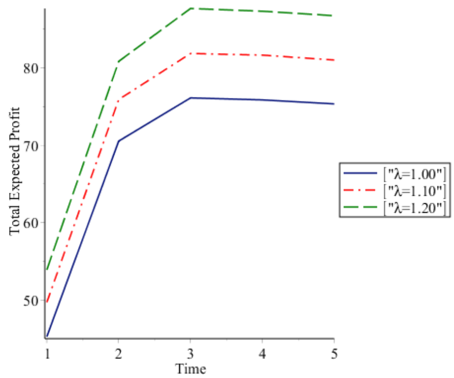

Fig. 2: *Variation of profit with time t by varying arrival rate λ*

Figure 1 & Figure 2 *show the variation of cost and profit respectively with time by varying arrival rate while keeping the other parameters fixed. (As per Table 2)*

7.2 Impact of Service Rate (μ)

Table 3 *Measures of Effectiveness versus* μ

t	μ	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	3.00	0.7031787	0.2968201	0.3734368	27.6248223	89.046030	61.4212077
2		0.6126300	0.3871325	0.4323510	28.4837200	116.13975	87.6560300
3		0.5978089	0.3983890	0.4391134	28.5672905	119.51670	90.9494095
4		0.5963266	0.3953172	0.4185405	28.3295756	118.59516	90.2655844
5		0.5950905	0.3908444	0.3748917	27.8511247	117.25332	89.4021953
1	3.50	0.7207414	0.2792574	0.3631917	30.4696832	97.740090	67.2704068
2		0.6466997	0.3530628	0.4053173	31.1111739	123.57198	92.4608061
3		0.6370993	0.3590986	0.4051939	31.1102243	125.68451	94.5742857
4		0.6360892	0.3565546	0.3853783	30.8866658	124.79411	93.9074442
5		0.6355188	0.3534161	0.3465686	30.4706088	123.69563	93.2250262
1	4.00	0.7369325	0.2630663	0.3547073	33.3362659	105.22652	71.8902541
2		0.6762899	0.3234726	0.3855482	33.8247123	129.38904	95.5643277
3		0.6699756	0.3262223	0.3822621	33.7822774	130.48892	96.7066426
4		0.6687285	0.3239153	0.3638235	33.5731999	129.56612	95.9929201
5		0.6679823	0.3209526	0.3283829	33.1913613	128.38104	95.1896787

7.3 Impact of Correlation Parameter (ν)

Table 4 *Measures of Effectiveness versus* ν

t	ν	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	0.25	0.6636668	0.3363319	0.4008312	22.0173012	67.26638	45.2490788
2		0.5290028	0.4707597	0.5208095	23.6191866	94.15194	70.5327534
3		0.4948042	0.5013937	0.5680509	24.1656796	100.2787	76.1130604
4		0.4921346	0.4995092	0.5586669	24.0434156	99.90184	75.8584244
5		0.4910388	0.4948961	0.5019695	23.4340578	98.97922	75.5451622
1	0.50	0.6887371	0.3112617	0.3911733	21.8455121	62.25234	40.4068279
2		0.5826667	0.4170958	0.4843897	23.0939969	83.41916	60.3251631
3		0.5602065	0.4359914	0.5105643	23.3946067	87.19828	63.8036733
4		0.5591736	0.4324701	0.4934833	23.1904618	86.49402	63.3035582
5		0.5581470	0.4277880	0.4417536	22.6305750	85.55760	62.9270250
1	0.75	0.7110272	0.2889716	0.3828270	21.6951788	57.79432	36.0991412
2		0.6268004	0.3729621	0.4564221	22.6819198	74.59242	51.9105002
3		0.6118155	0.3843824	0.4701202	22.8353387	76.87648	54.0411413
4		0.6102786	0.3813651	0.4504569	22.6068828	76.27302	53.6661372
5		0.6097776	0.3751573	0.4035633	22.0857794	75.03146	52.9456806

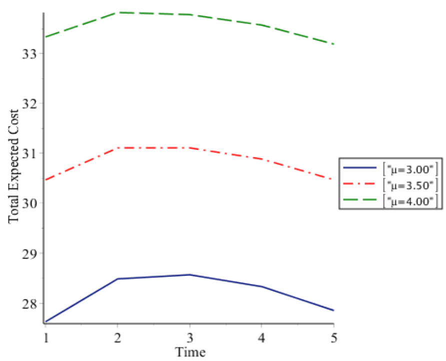

Fig. 3: *Variation of cost with time t by varying service rate μ*

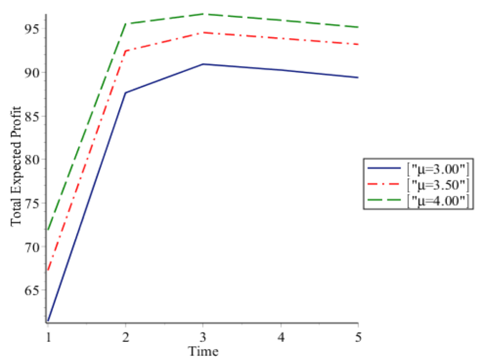

Fig. 4: *Variation of profit with time t by varying service rate μ*

Figure 3 & Figure 4 *show the variation of cost and profit respectively with time by varying service rate while keeping the other parameters fixed. (As per Table 3)*

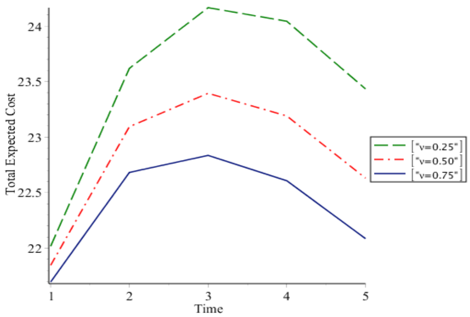

Fig. 5: *Variation of cost with time t by varying correlation parameter v*

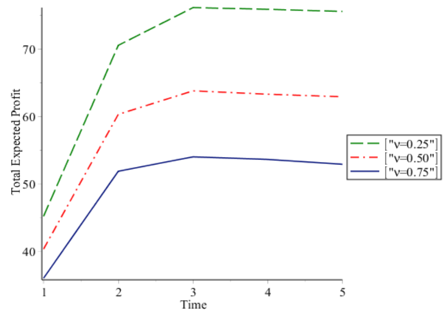

Fig. 6: *Variation of profit with time t by varying correlation parameter v*

Figure 5 & Figure 6 *show the variation of cost and profit respectively with time by varying correlation parameter while keeping the other parameters fixed. (As per Table 4)*

7.4 Impact of Vacation Rate (w)

Table 5 *Measures of Effectiveness versus w*

t	w	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	1.75	0.7305858	0.2694130	0.5336764	23.1449970	53.88260	30.7376030
2		0.5677352	0.4320273	0.7059920	25.3548144	86.40546	61.0506456
3		0.5168946	0.4793033	0.7616904	26.0358034	95.86066	69.8248566
4		0.5145760	0.4770678	0.7464484	25.8539064	95.41356	69.5596536
5		0.5127831	0.4731518	0.6740207	25.0893369	94.63036	69.5410231
1	2.75	0.6732235	0.3267753	0.4208099	22.1884189	65.35506	43.1666411
2		0.5334676	0.4662949	0.5450315	23.8480122	93.25898	69.4109678
3		0.4975294	0.4986685	0.5923112	24.4001070	99.73370	75.3335930
4		0.4965326	0.4951112	0.5819679	24.2632316	99.02224	74.7590084
5		0.4952799	0.4906550	0.5233097	23.6347365	98.13100	74.4962635
1	3.75	0.6420688	0.3579300	0.3537138	21.6109220	71.58600	49.9750780
2		0.5197844	0.4799781	0.4676903	23.1156498	95.99562	72.8799702
3		0.4887060	0.5074919	0.5154592	23.6580572	101.4983	77.8403228
4		0.4875570	0.5040868	0.5081989	23.5524684	100.8173	77.2648916
5		0.4867993	0.4991356	0.4557008	22.9840893	99.82712	76.8430307

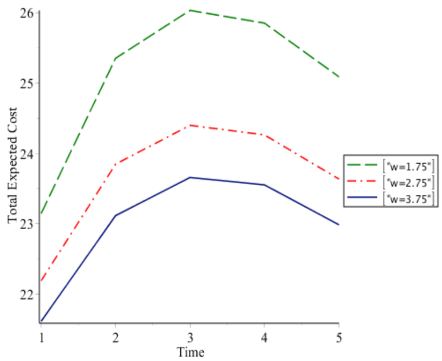

Fig. 7: *Variation of cost with time t by varying vacation rate w*

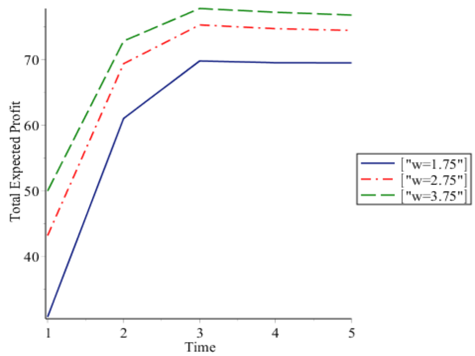

Fig. 8: *Variation of profit with time t by varying vacation rate w*

Figue 7 & Figure 8 *show the variation of cost and profit respectively with time by varying vacation rate while keeping the other parameters fixed. (As per Table 5)*

7.5 Impact of Catastrophes Rate (ξ)

Table 6 *Measures of Effectiveness versus* ξ

t	ξ	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	0.0001	0.6636668	0.3363319	0.4008312	22.0173012	67.26638	45.2490788
2		0.5290028	0.4707597	0.5208095	23.6191866	94.15194	70.5327534
3		0.4948042	0.5013937	0.5680509	24.1656796	100.2787	76.1130604
4		0.4921346	0.4995092	0.5586669	24.0434156	99.90184	75.8584244
5		0.4910388	0.4948961	0.5019695	23.4340578	98.97922	75.5451622
t							
1	0.0002	0.6636841	0.3363147	0.4008169	22.0171071	67.26294	45.2458329
2		0.5290388	0.4707238	0.5207780	23.6187644	94.14476	70.5259956
3		0.4948484	0.5013505	0.5680081	24.1651270	100.2701	76.1049730
4		0.4921834	0.4994677	0.5586278	24.0429366	99.89354	75.8506034
5		0.4910945	0.4948690	0.5019562	23.4339865	98.97380	75.5398135
1	0.0003	0.6637014	0.3362970	0.4008026	22.0169090	67.25940	45.2424910
2		0.5290747	0.4706870	0.5207465	23.6183345	94.13740	70.5190655
3		0.4948925	0.5013070	0.5679653	24.1645715	100.2614	76.0968285
4		0.4922323	0.499426	0.5585888	24.0424575	99.88520	75.8427425
5		0.4911503	0.4948410	0.5019430	23.4339095	98.96820	75.5342905

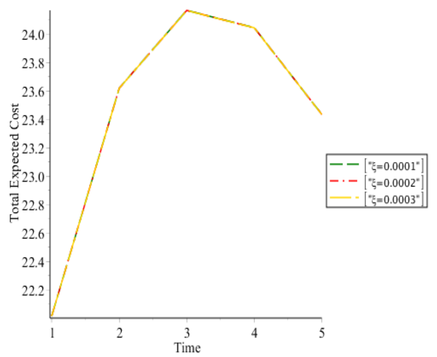

Fig. 9: *Variation of cost with time t by varying catastrophes rate ξ*

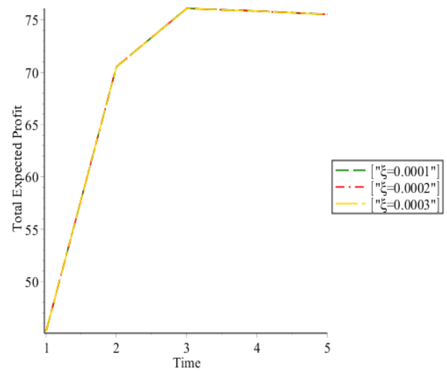

Fig. 10: *Variation of profit with time t by varying catastrophes rate ξ*

Figure 9 & Figure 10 *show the variation of cost and profit respectively with time by varying catastrophes rate while keeping the other parameters fixed. (As per Table 6)*

7.6 Impact of Feedback Probability (q)

Table 7 *Measures of Effectiveness versus q*

t	q	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	0.30	0.6277077	0.3722910	0.43174889	22.4343554	74.458200	52.0238446
2		0.4464499	0.5533126	0.64649784	25.1237287	110.66252	85.5387913
3		0.3857622	0.6104357	0.78761438	26.6884404	122.08714	95.3986996
4		0.3730211	0.6206227	0.83790899	27.2091770	124.12454	96.9153630
5		0.3718642	0.6140707	0.79081275	26.6800141	122.81414	96.1341259
1	0.40	0.6461691	0.3538297	0.41509723	22.2124554	70.765940	48.5534846
2		0.4893921	0.5103704	0.57560330	24.2859567	102.07408	77.7881233
3		0.4431645	0.5530334	0.65932110	25.2333007	110.60668	85.3733793
4		0.4396373	0.5550065	0.66949764	25.3332149	111.00130	85.6680851
5		0.4376427	0.5492922	0.61147746	24.6973257	109.85844	85.1611143
1	0.50	0.6636668	0.3363319	0.40083123	22.0173015	67.266380	45.2490785
2		0.5290028	0.4707597	0.52080953	23.6191869	94.151940	70.5327531
3		0.4948042	0.5013937	0.56805093	24.1656799	100.27874	76.1130601
4		0.4921346	0.4995092	0.55866697	24.0434163	99.901840	75.8584237
5		0.4910388	0.4948961	0.50196951	23.4340579	98.979220	75.5451621

7.7 Impact of Joining Probability (β)

Table 8 *Measures of Effectiveness versus* β

t	β	$P_V(t)$	$P_B(t)$	$Q_L(t)$	$TC(t)$	$TE_I(t)$	$TE_P(t)$
1	0.75	0.7234672	0.2679673	0.2710708	20.4717824	53.59346	33.1216776
2		0.6103502	0.3835847	0.3460438	21.5808666	76.71694	55.1360734
3		0.5853115	0.4120147	0.3800550	22.0232251	82.40294	60.3797149
4		0.5780495	0.4176605	0.3889393	22.1209245	83.53210	61.4111755
5		0.5750761	0.4161542	0.3792918	21.9975321	83.23084	61.2333079
1	0.85	0.6966279	0.2963515	0.3204898	21.0588495	59.27030	38.2114505
2		0.5759314	0.4202513	0.4126541	22.3682084	84.05026	61.6820516
3		0.5473900	0.4501175	0.4527350	22.8652400	90.02350	67.1582600
4		0.5378469	0.4536302	0.4579193	22.8974691	90.72604	67.8285709
5		0.5356416	0.4506271	0.4348521	22.6317458	90.12542	67.4936742
1	0.95	0.6741013	0.3233371	0.3732051	21.6892543	64.66742	42.9781657
2		0.5440601	0.4544922	0.4836768	23.1930061	90.89844	67.7054339
3		0.5117970	0.4851000	0.5289552	23.7293370	97.02000	73.2906630
4		0.5096062	0.4843499	0.5259131	23.6819612	96.86998	73.1880188
5		0.5070839	0.4799586	0.4823194	23.1982823	95.99172	72.7934377

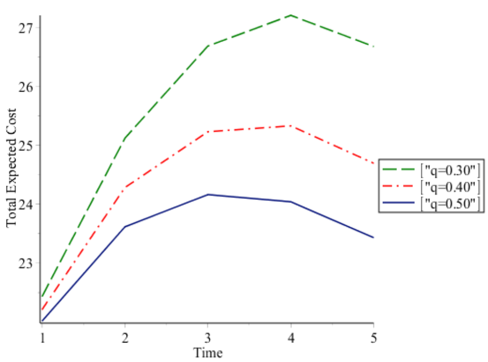

Fig. 11: *Variation of cost with time t by varying feedback probability q*

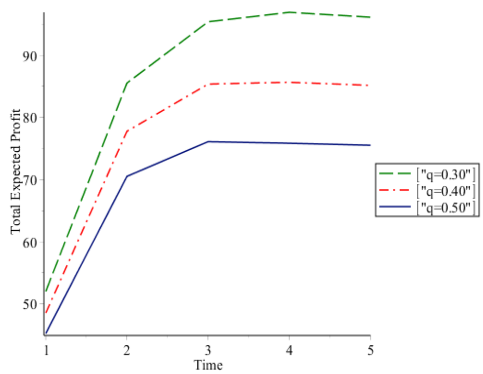

Fig. 12: *Variation of profit with time t by varying feedback probability q*

Figure 11 & Figure 12 *show the variation of cost and profit respectively with time by varying feedback probability while keeping the other parameters fixed. (As per Table 7)*

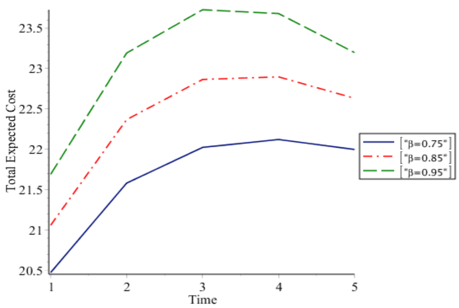

Fig. 13: *Variation of cost with time t by varying joining probability β*

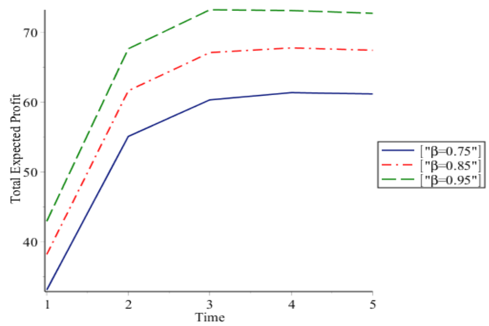

Fig. 14: *Variation of profit with time t by varying joining probability β*

Figure 13 & Figure 14 *show the variation of cost and profit respectively with time by varying joining probability while keeping the other parameters fixed. (As per Table 8)*

8. Discussion

When the value of arrival rate (λ=1.00, 1.10, 1.20) is varied to study the variation of cost with time t, the value of cost increases with increase in t upto t(=3.00) then decreases slightly. Hence, we get the optimal value at t=1 when λ=1.00 for minimum cost as shown in Fig.1. In Fig. 2 the variation of profit with time t by varying arrival rate (λ=1.00, 1.10, 1.20) is studied. The value of profit increases with increase in t upto t(=3.00) then decreases slightly. Hence, we get the optimal value at t=3 when λ=1.20 for maximum profit.

When the value of service rate (μ=3.00, 3.50, 4.00) is varied to study the variation of cost with time t, the value of cost increases with increase in t upto t(=3.00) when μ=3.00 and t(=2.00) when μ=3.50, 4.00 respectively then decreases slightly. Hence we get the optimal value at t=1 when μ=3.00 for minimum cost as shown in Fig. 3. In Fig. 4 the variation of profit with time t by varying serive rate (μ=3.00, 3.50, 4.00) is studied. The profit increases with increase in time up to t(=3.00) when μ=3.00, 3.50 and 4.00 respectively then decreases slightly. Hence we get the optimal value at t=3 when μ=4.00 for maximum profit.

When the value of correlation parameter (ν=0.25, 0.50, 0.75) is varied to study the variation of cost with time t, the value of cost increases with increase in t upto t(=3.00) then decreases slightly. Hence we get the optimal value at t=1 when ν=0.75 for minimum cost as shown in Fig. 5. In Fig. 6 the variation of profit with time t by varying correlation parameter (ν=0.25, 0.50, 0.75) is studied. The value of profit increases with increase in t upto t(=3.00) then decreases slightly. Hence we get the optimal value at t=3 when ν=0.25 for maximum profit.

When the value of vacation rate (w=1.75, 2.75, 3.75) is varied to study the variation of cost with time t, the value of cost increases with increase in t upto t(=3.00) then decreases slightly. Hence we get the optimal value at t=1 when w=3.75 for minimum cost as shown in Fig. 7. In Fig. 8 the variation of profit with time t by varying vacation rate (w=1.75, 2.75, 3.75) is studied. The value of profit increases with increase in t upto t(=3.00) then decreases slightly. Hence we get the optimal value at t=3 when w=3.75 for maximum profit.

When the value of catastrophes rate (ξ=0.0001, 0.0002, 0.0003) is varied to study the variation of cost with time t, the value of cost increases with increase in t upto t(=3.00) then decreases slightly. Hence we get the optimal value at t=1 when ξ=0.0003 for minimum cost as shown in Fig. 9. In Fig. 10 the variation of profit with time t by varying catastrophes rate (ξ=0.0001, 0.0002, 0.0003) is studied. The value of profit increases with increase in t upto t(=3.00) then decreases slightly. Hence we get the optimal value at t=3 when ξ=0.0001 for maximum profit. Finally, the variation in rate of catastrophes shows the minor effect on cost and profit.

When the value of feedback probability (q=0.30, 0.40, 0.50) is varied to study the variation of cost with time t, the value of cost increases with increase in t upto t(=4.00) when q=0.30, 0.40 and t(=3.00) when q=0.50 respectively then decreases slightly. Hence we get the optimal value at t=1 when q=0.50 for minimum cost as shown in Fig. 11. In Fig. 12 the variation of profit with time t by varying feedback probability (q=0.30, 0.40, 0.50) is studied. The value of profit increases with increase in t upto t(=4.00) when q=0.30, 0.40 and t(=3.00) when q=0.50 respectively then decreases slightly. Hence we get the optimal value at t=4 when q=0.30 for maximum profit.

When the value of joining probability (β=0.75, 0.85, 0.95) is varied to study the variation of cost with time t, the value of cost increases with increase in t upto t(=4.00) when β=0.75, 0.85 and t(=3.00) when β=0.95 respectively then decreases slightly. Hence we get the optimal value at t=1 when β=0.75 for minimum cost as shown in Fig. 13. In Fig. 14 the variation of profit with time t by varying joining probability (β=0.75, 0.85, 0.95) is studied. The profit increases with increase in time up to t(=4.00) when β=0.75, 0.85 and t(=3.00) when β=0.95 then decreases slightly. Hence we get the optimal value at t=3 when β=0.95 for maximum profit.

9. Conclusion and Future Work

The time-dependent solution, for the two-dimensional state M/M/2 queuing system with correlated servers, multiple vacation, balking and catastrophes, has been obtained. This comprehensive model provides valuable insights into the systems behavior by estimating the total expected cost and total expected profit. The findings reveal that the optimal value for minimum cost is achieved when the joining probability is 0.75 at time t=1, while the maximum profit is attained when the feedback probability is 0.30 at time t=4. These key measures give a deeper understanding of the model dynamics. Importantly, the numerical analysis conducted in the study clearly demonstrates the meaningful impact of the correlated servers and multiple vacation on the overall system performance. The findings have significant practical implications for industries such as communication networks, computer networks, supermarkets, hospital administration and the financial sector. By optimizing the joining probability and feedback probability, organizations can minimize costs and maximize profits, leading to more efficient and effective operations.

As part of future study, this model may be examined further for Non-Markovian queue, bulk queue, tandem queues, retrial queues *etc.*

10. Acknowledgment

First and foremost, I am deeply indebted to my supervisor, Prof. (Dr.) Indra Rani for her unwavering guidance and expertise. Her insightful feedback and constant encouragement played a pivotal role in shaping this research. I am grateful to the Department of Statistics and Operational Research, Kurukshetra University, Kurukshetra for providing the resources and research facilities necessary for this study.

Additionally, I would like to acknowledge the Kurukshetra University for providing the University research scholarship which made this research possible.

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(Received January 2024; accepted June 2024)