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Effects of Carbon Tariffs on Optimal Production-Inventory Decisions for Deteriorating Items under Carbon Tax Policy

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Keywords

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Abstract.

This study considers a global supply chain system included a single manufacturer without a mature carbon market and a single retailer where its government plans to impose carbon tariffs on imported goods from the manufacturer. A production-inventory model for a three-stage supply chain that includes material supply, manufacturer's production and delivery, and retailer's ordering and sales is developed for deteriorating items. Carbon tax policy for the retailer is considered in this study. The purpose is to determine the optimal material supply, production and delivery strategies for the manufacturer, and the optimal pricing and replenishment strategies for the retailer, so as the joint total profit of the entire supply chain is maximized. By using mathematical programming, the optimal solutions for the manufacturer and retailer are obtained. Further, numerical examples are presented to demonstrate the solution procedure. Through numerical analysis, it is expected to provide enterprise or supply chain decision makers, especially in multinational enterprises to understand the impact of carbon tariffs on supply chain inventory and pricing decisions and respond accordingly.

1. Introduction

In response to climate change, major economies have successively made clear carbon emission reduction commitments. Under the circumstances that developed countries generally set strict carbon reduction targets, the idea of using trade policy as an indirect control over foreign emission sources has gained many supporters in regions considering unilateral climate policies (Kuik & Hofkes [21]). However, the stringency of emissions

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regulation varies across regions, raising concerns over carbon leakage - an outcome where stringent regulation in one region shifts production to regions with weaker regulation (Drake [8]). From the perspective of global supply chain, carbon leakage increases global emissions, offsetting most of the regulation's emission improvements (Demailly & Quirion [7], Fowlie et al. [12]). Carbon tariffs may provide a way for climate-concerned nations to reduce carbon leakage and regulate the emissions embodied in imported consumption goods (Weber & Matthews [27], Veel [26], Böhringer et al. [3, 4]). The EU carbon market is mature and world leading. The European Commission issued a formal draft of the Carbon Border Adjustment Mechanism (CBAM) on July 14, 2021, and will be officially implemented in 2026. Importers are required to pay carbon tariffs by paying "CBAM certificates". As a result, carbon reduction will no longer be an independent event, the carbon markets and carbon policies of various countries will be constraining each other. Such tariffs could substantially expand the scope of unilaterally formulated climate policy by covering both foreign and indirect sources of emissions (Böhringer et al. [5]).

At present, there are studies on the impact of the domestic carbon policies on carbon reduction and profits in supply chains (e.g., Konur [20], Mishra et al. [22], Shen et al. [25], and Rout et al. [24]), but the impact of carbon tariffs on multinational supply chains is an emerging topic. Zhou et al. [31] explored the impacts of carbon tariff imposition on the supply chain network design. Fang et al. [10] proposed a global supply chain model consisting of a retailer in an emission-regulated country and supplier in a non-emission-regulated country. Regarding the important concept of carbon tariff and its inevitably wide implementation prospects worldwide, there is currently no multinational supply chain production-inventory research that takes the carbon tariff into consideration. Therefore, if we can explore the impact of carbon tariffs on the optimal production, delivery, ordering, and pricing decisions of the production-inventory models in a multinational supply chain system, it will not only help the supply chain members to optimize their profitability and carbon reduction capabilities, but also provide references for the manufacturer's country to construct its domestic carbon policies.

Inventory management is a core activity of supply chain operations management, spanning the production, storage and sales processes of the entire supply chain from manufacturer to retailer. For joint inventory research, Goyal [14] first developed an integrated inventory model of a vendor and a buyer. Banerjee [2] designed that a vendor produces to order for a buyer on a lot-for-lot basis in a joint model. Ha and Kim [16] recognized that products can be delivered during production to comply with the spirit of just-in-time (JIT) in a production-inventory model. Kelle et al. [18] proposed the concept of multiple shipments for a batch delivery. Researchers (e.g., Ho et al. [17], Wu & Chen [29], Du & Lei [9], Kogan [19], and Goodarzian et al. [13]) have continued to develop strategies for integrated production-inventory models. However, few studies have considered three-stage production-inventory models that include the ordering and inventory of raw materials from the perspective of integrated supply chain. In reality, most types of goods have deteriorating phenomenon. For example, due to environmental factors such as humidity and temperature during storage, deterioration or corruption will result in reduced inventory or poor quality. It is important to control and maintain the inventories of deteriorating items for the modern corporation (Wu et al. [28]). Most of previous inventory studies only consider the deterioration of the finished products (Goyal & Giri [14], Bakker et al. [1], Yang et al. [30], Feng et al. [11], and Pervin et al. [23]). Our model collectively considers the deterioration of the raw materials and finished products.

Scholars have discussed supply chain inventory issues related to carbon reduction measures or deteriorating items; however, the supply chain production-inventory model with deteriorating materials is rarely considered, and there is no research on discussing multinational supply chains. This study considers a global supply chain system included a single manufacturer without a mature carbon market and a single retailer where its government plans to impose carbon tariffs on imported goods and explores multinational three-stage supply chain production-inventory models which include material supply, foreign manufacturer's production and delivery, and retailer's ordering and sales. The exchange rate of transaction between supply chain members is also taken into consideration. We first establish the retailer's total profit per unit time in domestic currency, the manufacturer's total profit per unit time in foreign currency, and their carbon emission functions. Then, we consider two situations in this study: (1) the manufacturer's country has no carbon emission reduction policy; (2) the manufacturer's country adopts carbon tax policy. From the integration of the supply chain, the joint total profit function in domestic currency is established under cross-border and domestic carbon policies. The purpose is to determine the production, delivery, replenishment, and pricing strategies of the integrated production-inventory model, to maximize the joint total profit of the supply chain system. Numerical examples are provided to clarify the solution procedure and compare carbon policy schemes. In addition, managerial implications and decision-making considerations are presented.

2. Notation and Assumptions

To establish multinational supply chain production-inventory models under carbon tariffs with domestic carbon reduction policies, definitions of parameters and variables as shown in Table 1 are required.

Table 1: Parameters and variables of the proposed model.

δ	Foreign currency exchange rate.
P	Manufacturer's production rate.
A_R	Retailer's order cost/order (original currency).
A_M	Manufacturer's order cost for materials/order (original currency).
r	Amount of materials required to produce one unit of finished products.
S	Manufacturer's setup cost /time (original currency).
s	Retailer's inspection cost of finished products/unit.
c_1	Manufacturer's material cost/unit (original currency).
c_2	Manufacturer's production cost/unit (original currency).
v	Retailer's purchasing cost/unit (original currency).
h_r	Retailer's holding cost of finished products/unit/unit time (original currency).
h_1	Manufacturer's holding cost of materials/unit/unit time (original currency).
h_2	Manufacturer's holding cost of finished products/unit/unit time (original currency).

F_R	Retailer's fixed shipping cost/time.
f_R	Retailer's variable shipping cost/unit.
F_M	Manufacturer's fixed shipping cost/time.
f_M	Manufacturer's variable shipping cost/unit.
p_c	The amount of tax levied per unit of carbon emissions for the manufacturer.
p_e	The amount of tax levied per unit of carbon emissions for the retailer (foreign currency).
p_t	The carbon tariff to be paid per unit of carbon emissions (foreign currency).
$\hat{A}_R$	Carbon emissions from the retailer's ordering of finished products/order.
$\hat{A}_M$	Carbon emissions from the manufacturer's ordering of materials/order.
$\hat{S}$	Carbon emissions from the manufacturer's setup/time.
$\hat{s}$	Carbon emissions from the retailer's inspection of finished products/unit.
$\hat{c}_1$	Carbon emissions from the manufacturer's materials procurement/unit.
$\hat{c}_2$	Carbon emissions from the manufacturer's production of finished products/unit.
$\hat{v}$	Carbon emissions from the retailer's finished products procurement/unit.
$\hat{h}_R$	Carbon emissions from the retailer's storage of finished products/unit/unit time.
$\hat{h}_1$	Carbon emissions from the manufacturer's storage of finished products/unit/unit time.
$\hat{h}_2$	Carbon emissions from the manufacturer's storage of materials/unit/unit time.
$\hat{F}_R$	Fixed carbon emissions from the retailer's shipping/shipment.
$\hat{f}_R$	Variable carbon emissions from the retailer's shipping/unit.
$\hat{F}_M$	Fixed carbon emissions from the manufacturer's shipping/shipment.
$\hat{f}_M$	Variable carbon emissions from the manufacturer's shipping/unit.
θ_1	Deterioration rate of materials.
θ_2	Deterioration rate of finished products.
λ	Defective rate of finished products, where $\lambda \in (0, 1)$.
ρ	The rate of tariff relief agreed by the retailer when the manufacturer has been levied carbon-related charges by the country of origin.
p	Retailer's selling price/unit.
$D(p)$	Retailer's demand rate, a function of selling price.
Q	Retailer's order quantity of finished products.
Q_M	Manufacturer's order quantity of raw materials.
T_p	Length of period for the manufacturer to produce and deliver the first batch of finished products to the retailer.
T_b	Length of the retailer's replenishment cycle.
T_v	Length of the manufacturer's production cycle.
T_s	Length of the manufacturer's production period in a production cycle.
n	Number of shipments from the manufacturer to the retailer in a production cycle.
q	Quantity of non-defective products shipped from the manufacturer to the retailer per shipment.

2.1. Assumptions

(1) A global supply chain system where members come from different countries, including a single manufacturer and a single retailer, a single raw material and a single finished product is considered.

(2) The retailer is faced carbon tax policies, and the country where the retailer is located will impose carbon tariffs on imported goods from manufacturers in countries that lack carbon emission reduction mechanisms to encourage other countries to establish carbon market mechanisms.

(3) The production rate of the manufacturer is finite and greater than the demand rate of the retailer.

(4) The retailer orders numerous size Q units and allows the manufacturer to divide the order into n consignments. Because the finished products the manufacturer produced contain defective products at a rate of λ, the manufacturer may ship $q/(1-\lambda)$ units to the retailer to ensure that the retailer receives q units of non-defective products in each shipment.

(5) Due to frequent changes in exchange rates, this model uses the average exchange rate to facilitate the establishment of the model.

(6) Shortages are not allowed regardless of the manufacturer or retailer.

3. Model Formation and Solution

This research develops a multinational multi-stage supply chain integrated production-inventory model, including three stages of raw material supply, production delivery, and order sales. A single manufacturer and a single retailer are considered in the supply chain inventory system. At the beginning, the retailer makes an order with Q units and asks the manufacturer to ship in n times. The manufacturer also places an order with the raw material supplier and purchase rQ units of raw materials for processing and production upon receiving the retailer's order. Because the finished products produced by the manufacturer contain defective products at the rate of λ, the manufacturer may ship $q/(1-\lambda)$ units to the retailer to ensure that the retailer receives q units of non-defective products in each shipment. Thus, the total shipping quantity in a production cycle (length of the period is T_v) is $Q/(1-\lambda)$. The manufacturer begins shipping to the retailer as the production quantity reaches $q/(1-\lambda)$ units for the first time (length of the period is T_p). After that it will ship q units to the retailer every regular interval (length of the period is T_b). Because the manufacturer's production rate is higher than the retailer's demand rate, the manufacturer may stop producing when the inventory level reaches $I_{\max}$ (length of the period is T_S), but continue to ship regularly until all the ordered quantity has be shipped.

Based on the above notation and assumptions, the total profit of the supply chain members can be established as follows:

3.1. Retailer's total profit

The retailer's inventory level of finished products at time t during the replenishment cycle changes because of the demand and the deterioration, and is represented by the following differential equation:

$$dI_R(t)/dt + \theta_2 I_R(t) = -D(p), \qquad 0 \le t \le T_b. \tag{1}$$

Solving (1) with the boundary condition $I_R(T_b) = 0$ gives the retailer's inventory of finish products in each replenishment cycle as

$$I_R(t) = \frac{D(p)}{\theta_2}[e^{\theta_2(T_b-t)} - 1], \qquad 0 \le t \le T_b. \tag{2}$$

Thus, the retailer's quantity of finish products per replenishment cycle can be obtained as

$$\frac{q}{1-\lambda} = I_R(0) = \frac{D(p)}{(1-\lambda)\theta_2}(e^{\theta_2 T_b} - 1). \tag{3}$$

The total profit of the retailer denominated in original currency (denoted by $TP_b(p, T_b)$) includes sales revenue, ordering cost, inspection cost, purchase cost, holding cost, and shipping cost as follows:

$$\begin{aligned} TP_b(p, T_b) = &\frac{1}{T_b}\Big\{pD(p)T_b - A_R - F_R - \frac{sD(p)(e^{\theta_2 T_b} - 1)}{(1-\lambda)\theta_2} - \frac{\delta v D(p)}{\theta_2}(e^{\theta_2 T_b} - 1) \\ &- \frac{h_R D(p)}{\theta_2^2}(e^{\theta_2 T_b} - \theta_2 T_b - 1) - \frac{f_R D(p)}{(1-\lambda)\theta_2}(e^{\theta_2 T_b} - 1)\Big\}. \end{aligned} \tag{4}$$

3.2. Retailer's carbon emissions

The retailer's carbon emissions per replenishment cycle are related to the ordering, inspection, procurement, delivery and storage of finished products, which means we can obtain the retailer's carbon emissions per unit time (denoted by $E_b(p, T_b)$) as follows.

$$\begin{aligned} E_b(p, T_b) = &\frac{1}{T_b}\Big\{\hat{A}_R + \hat{F}_R + \frac{\hat{s}D(p)(e^{\theta_2 T_b} - 1)}{(1-\lambda)\theta_2} + \frac{\hat{v}D(p)}{\theta_2}(e^{\theta_2 T_b} - 1) \\ &+ \frac{\hat{h}_R D(p)}{\theta_2^2}(e^{\theta_2 T_b} - \theta_2 T_b - 1) + \frac{\hat{f}_R D(p)}{(1-\lambda)\theta_2}(e^{\theta_2 T_b} - 1)\Big\}. \end{aligned} \tag{5}$$

3.3. Manufacturer's total profit

For a production cycle, once receiving a retailer's order (Q units), the manufacturer places orders with the original raw material supplier for processing and production. Since each unit of finished product requires r units of raw materials with deterioration properties during storage, the manufacturer's inventory level of raw materials fluctuates due to material usage for production and its deterioration during the time interval $[0, T_s]$.

The manufacturer's inventory level of raw materials changes during the time interval $[0, T_s]$ can be represented by the following differential equation:

$$dI_M(t)/dt + \theta_1 I_M(t) = -rP, \qquad 0 \leq t \leq T_s. \tag{6}$$

The manufacturer's inventory level of raw materials $I_M(t)$ can be obtained by solving (6) based on the boundary condition $I_M(T_s) = 0$ as follows:

$$I_M(t) = \frac{rP}{\theta_1}(e^{\theta_1(T_s - t)} - 1), \qquad 0 \leq t \leq T_s. \tag{7}$$

From (7), it can get the total order quantity of raw materials per production cycle $Q_M = I_M(0)$ as follows:

$$Q_M = I_M(0) = \frac{rP}{\theta_1}(e^{\theta_1 T_s} - 1). \tag{8}$$

As to the manufacturer's inventory level of finished products, it changes because of the production and the deterioration during the time interval $[0, T_s]$. Because the manufacturer's production rate for non-defective products is greater than the demand rate, the manufacturer stops production if the inventory reaches a certain level. Hence, the manufacturer's inventory level of finished products can be discerned through the following differential equation:

$$dI_p(t)/dt + \theta_2 I_p(t) = P, \qquad 0 \leq t \leq T_s. \tag{9}$$

Based on the boundary condition $I_p(0) = 0$, the manufacturer's inventory level of finished products during the time interval $[0, T_s]$ can be obtained:

$$I_p(t) = \frac{P}{\theta_2}(1 - e^{-\theta_2 t}), \qquad 0 \leq t \leq T_s. \tag{10}$$

When the manufacturer produces the first shipment of $q/(1-\lambda) = Q/[n(1-\lambda)]$ units (length of the period is T_p), it ships to the retailer immediately. After that, the number of fixed shipments ($q/(1-\lambda)$ units) is repeated at every interval T_b. Thus, it can be found that $I_p(T_p) = D(p)(e^{\theta_2 T_b} - 1)/(1-\lambda)\theta_2 = P(1 - e^{-\theta_2 T_p})/\theta_2$, which implies

$$T_p = \frac{1}{\theta_2} \ln \left[\frac{(1-\lambda)P}{(1-\lambda)P - D(p)(e^{\theta_2 T_b} - 1)} \right]. \tag{11}$$

Following, the manufacturer is no longer producing during $[T_s, T_v]$, and its inventory level of finish products decreases because of deterioration. Similarly, the inventory level of finished products is governed by the following differential equation:

$$dI_d(t)/dt + \theta_2 I_d(t) = 0, \qquad T_s \leq t \leq T_v. \tag{12}$$

Based on the boundary condition $I_d(T_s) = I_{\max}$, the manufacturer's inventory level of finished products during the time interval $[T_s, T_v]$ can be obtained:

$$I_d(t) = I_{\max} e^{\theta_2 (T_s - t)}, \qquad T_s \leq t \leq T_v. \tag{13}$$

It is obvious that $I_p(T_s) = I_d(T_s)$ which implies

$$I_{\max} = \frac{P}{\theta_2}(1 - e^{-\theta_2 T_s}). \tag{14}$$

Substituting (14) into (13), it can be obtained that the manufacturer's inventory level of the finished products during the time interval $[T_s, T_v]$ as follows.

$$I_d(t) = \frac{P}{\theta_2}\Big[e^{\theta_2(T_s - t)} - e^{-\theta_2 t}\Big], \qquad T_s \le t \le T_v. \tag{15}$$

Further, from the fact that $I_d(T_v) = nq/(1-\lambda)$, it can be obtained

$$T_s = \frac{1}{\theta_2}\ln\Big[\frac{P + nD(p)(e^{\theta_2 T_b} - 1)e^{\theta_2 T_v}/(1-\lambda)}{P}\Big]. \tag{16}$$

Therefore, the manufacturer's total profit denominated in foreign currency includes sales revenue, setup cost, ordering cost for materials, materials cost, production cost, shipping cost and holding cost. These components are evaluated as follows:

(a) Sales revenue:
The manufacturer's sales revenue per production cycle is $vQ = \frac{vnD(p)}{\theta_2}(e^{\theta_2 T_b} - 1)$.

(b) Setup cost:
The manufacturer's setup cost per production cycle is S.

(c) Ordering cost for materials
The manufacturer's ordering cost for materials per production cycle is A_M.

(d) Material cost:
The manufacturer's material cost per production cycle is $c_1 q_M = \frac{c_1 rP}{\theta_1}(e^{\theta_1 T_s} - 1)$.

(e) Production cost:
The manufacturer's production cost per production cycle is $c_2 P T_s$.

(f) Shipping cost:
The manufacturer's shipping cost per production cycle is

$$n\Big[F_M + \frac{f_m q}{(1-\lambda)}\Big] = n\Big[F_M + \frac{f_m D(p)}{(1-\lambda)\theta_2}(e^{\theta_2 T_b} - 1)\Big].$$

(g) Holding cost:
The manufacturer's holding cost contains two parts: raw materials and finished products, where the holding cost of the raw materials is

$$h_1 \int_0^{T_s} I_M(t)dt = \frac{h_1 rP}{\theta_1^2}(e^{\theta_1 T_s} - \theta_1 T_s - 1).$$

As to the holding cost of finished products, because its total cumulative inventory per production cycle is the difference between the manufacturer's cumulative inventory and the retailer's cumulative inventory, given by

$$\int_0^{T_s} I_p(t)dt + \int_{T_s}^{T_v} I_d(t)dt - [qT_b/(1-\lambda)][1 + 2 + \cdots + (n-1)],$$

the holding cost of finished products can be calculated as follows.

$$
\begin{aligned}
&h_2\left\{\int_0^{T_s} I_p(t)dt+\int_{T_s}^{T_v} I_d(t)dt-[qT_b/(1-\lambda)][1+2+\cdots+(n-1)]\right\}\\
&=h_2\left\{\int_0^{T_s}\frac{P}{\theta_2}(1-e^{-\theta_2 t})dt+\int_{T_s}^{T_v}\frac{P}{\theta_2}[e^{\theta_2(T_s-t)}-e^{-\theta_2 t}]dt-\frac{n(n-1)D(p)T_b}{2(1-\lambda)\theta_2}(e^{\theta_2T_b}-1)\right\}\\
&=h_2\left\{\frac{PT_s}{\theta_2}-\frac{P[e^{-\theta_2T_v}(e^{\theta_2T_s}-1)]}{\theta_2^2}-\frac{n(n-1)D(p)T_b(e^{\theta_2T_b}-1)}{2(1-\lambda)\theta_2}\right\}
\end{aligned}
$$

Consequently, the total profit per unit time for the manufacturer, denoted by $TP_v(p,T_v,T_s,T_b,n)$, is

$$
\begin{aligned}
&TP_v(p,T_v,T_s,T_b,n)\\
&=\frac{1}{T_v+T_b}\left\{\frac{vnD(p)}{\theta_2}(e^{\theta_2T_b}-1)-S-A_M-nF_M-\frac{c_1rP}{\theta_1}(e^{\theta_1T_s}-1)\right.\\
&-c_2PT_s-\frac{nf_mD(p)}{(1-\lambda)\theta_2}(e^{\theta_2T_b}-1)-\frac{h_1rP}{\theta_1^2}(e^{\theta_1T_s}-\theta_1T_s-1)\\
&\left.-h_2\left\{\frac{TP_s}{\theta_2}-\frac{P[e^{-\theta_2T_v}(e^{\theta_2T_s}-1)]}{\theta_2^2}-\frac{n(n-1)D(p)T_b(e^{\theta_2T_b}-1)}{2(1-\lambda)\theta_2}\right\}\right\}. \qquad (17)
\end{aligned}
$$

From (11) and (16) and the fact that $T_v=T_p+(n-1)T_b$, $TP_v(p,T_v,T_s,T_b,n)$ shown as in (17) can be reduced to $TP_v(p,T_b,n)$.

3.4. Manufacturer's carbon emissions

The manufacturer's carbon emissions per cycle are related to the setup, ordering for materials, production, procurement for materials, transportation and storage of raw materials and finished products, which means we can obtain the manufacturer's carbon emissions per unit time (denoted by $E_v(p,T_v,T_s,T_b,n)$) as follows.

$$
\begin{aligned}
&E_v(p,T_v,T_s,T_b,n)\\
&=\frac{1}{T_v+T_b}\left\{\hat{S}+\hat{A}_M+n\hat{F}_M+\frac{\hat{c}_1rP}{\theta_1}(e^{\theta_1T_s}-1)+\hat{c}_2PT_s+\frac{\hat{h}_1rP}{\theta_1^2}(e^{\theta_1T_s}-\theta_1T_s-1)\right.\\
&+\frac{n\hat{f}_mD(p)}{(1-\lambda)\theta_2}(e^{\theta_2T_b}-1)\\
&\left.+\hat{h}_2\left(\frac{PT_s}{\theta_2}-\frac{P[e^{-\theta_2T_v}(e^{\theta_2T_s}-1)]}{\theta_2^2}-\frac{n(n-1)D(p)T_b(e^{\theta_2T_b}-1)}{2(1-\lambda)\theta_2}\right)\right\}. \qquad (18)
\end{aligned}
$$

Similarly, $E_v(T_v,T_s,T_b,n)$ shown as in (18) can be reduced to $E_v(p,T_v,T_sT_b,n)$ based on (11) and (16) and the fact that $T_v=T_p+(n-1)T_b$.

The aim of this study is to jointly determine the optimal ordering, production and delivery strategies of the manufacturer and retailer under different carbon emission policy combinations so that the joint total profit of supply chain is maximized. The optimization problem of this study can be expressed as follows according to the different carbon emission reduction policy combinations faced by supply chain members:

Situation I. *The retailer faces carbon tax policy and the manufacturer has not yet faced carbon reduction policies*

In this situation, government agency where the retailer is located imposes a carbon tax on the total amount of carbon emissions under carbon tax policy. Suppose that p_e represents the amount of tax levied per unit of carbon emissions (domestic currency), the retailer's total profit function with carbon tax policy (denoted by $TP_b^{\mathrm{I}}(p,T_b)$) is

$$TP_b^{\mathrm{I}}(p,T_b) = TP_b(p,T_b) - p_e E_b(p,T_b). \tag{19}$$

Although the manufacturer has not yet faced carbon reduction policies, when its finished products are imported into the retailer's country, they are subject to carbon tariffs. The carbon tariff is calculated by multiplying the amount of carbon emissions by a given rate. Assume that p_t represents the rate of carbon tariff, the manufacturer's total profit function with carbon tariff (denoted by $TP_v^{\mathrm{I}}(p,T_b,n)$) can be calculated as follows.

$$TP_v^{\mathrm{I}}(p,T_b,n) = TP_v(p,T_b,n) - (p_t/\delta) \times E_v(p,T_b,n). \tag{20}$$

Since the total profit function of the manufacturer and the retailer are denominated in different currencies, the total profit of the manufacturer needs to be converted into domestic currency (the exchange rate is δ) as the two parties are in a state of integration. Therefore, the joint total profit per unit time in the multinational supply chain system (denoted by $\Pi_{\mathrm{I}}(p,T_b,n)$) for Situation I is

$$\Pi_{\mathrm{I}}(p,T_b,n) = \delta TP_v^{\mathrm{I}}(p,T_b,n) + TP_b^{\mathrm{I}}(p,T_b). \tag{21}$$

Situation II. *Both the retailer and manufacturer face carbon tax policy.*

In this situation, the retailer's total profit function with carbon tax policy is the same as (19). As to the manufacturer, he/she is already taxed on carbon in the country of origin and thus receives a reduction in carbon tariffs with the rate ρ. Suppose that p_c represents the amount of tax levied per unit of carbon emissions, and p_t represents the rate of carbon tariff, the manufacturer's total profit function with carbon tariff (denoted by $TP_v^{\mathrm{II}}(p,T_b,n)$) can be calculated as follows.

$$\Pi_{\mathrm{II}}(p,T_b,n) = TP_v(p,T_b,n) - p_c \times E_v(p,T_b,n) - [(p_t/\delta) - \rho p_c] \times E_v(p,T_b,n). \tag{22}$$

Similarly, since the total profit function of the manufacturer and the retailer are denominated in different currencies, the total profit of the manufacturer needs to be converted into domestic currency (the exchange rate is δ) as the two parties are in a state of integration. Therefore, the joint total profit per unit time in the multinational supply chain system (denoted by $\Pi_{\mathrm{II}}(p,T_b,n)$) for Situation II is

$$\Pi_{\mathrm{II}}(p,T_b,n) = \delta TP_v^{\mathrm{II}}(p,T_b,n) + TP_b^{\mathrm{I}}(p,T_b). \tag{23}$$

The purpose of this study is to determine the optimal length of replenishment cycle, the numbers of shipments, and length of product cycle such that the joint total profits per unit time are maximum. Due to the complexity of the model and because n is a integer, finding the closed-form solutions for p, T_b and n and directly checking the concavity of profit function is difficult. Thus, alternatively, the concavity will be verified by numerical analysis in the next section and an algorithm to obtain the solutions for the joint total profit per unit time is developed.

Algorithm.

Step 1. For Situation j, let $n_j = 1$, where $j =$ I, II.

Step 2. Find $T_{b(n_j)}$ and $p_{(n_j)}$ by solving the simultaneous equations $\partial\Pi_j(p, T_b, n_j)/\partial T_b = 0$ and $\partial\Pi_j(p, T_b, n_j)/\partial p = 0$, where $j =$ I, II.

Step 3. Substitute n, $p_{(n_j)}$, and $T_{b(n_j)}$ into Equations (21) and (23) to calculate $\Pi_j(p_{(n_j)}, T_{b(n_j)}, n_j)$, where $j =$ I, II.

Step 4. Set $n_j = n_j + 1$, and repeat Steps 2 and 3 to obtain $\Pi_j(p_{(n_j+1)}, T_{b(n_j+1)}, n_j + 1)$.

Step 5. Compare $\Pi_j(p_{(n_j+1)}, T_{b(n_j+1)}, n_j + 1)$ with $\Pi_j(p_{(n_j)}, T_{b(n_j)}, n_j)$.
  (i) If $\Pi_j(p_{(n_j+1)}, T_{b(n_j+1)}, n_j + 1) \leq \Pi_j(p_{(n_j)}, T_{b(n_j)}, n_j)$, then $(p_{(n_j)}, T_{b(n_j)}, n_j)$ is the optimal solution and the process is finished.
  (ii) If $\Pi_j(p_{(n_j+1)}, T_{b(n_j+1)}, n_j + 1) > \Pi_j(p_{(n_j)}, T_{b(n_j)}, n_j)$, return to Step 4.

Once the optimal solution of $(p_{(n_j)}, T_{b(n_j)}, n_j)$ is obtained, the manufacturer's optimal quantity of raw materials, total carbon emission and total profit per unit time and the retailer's quantity of items shipped, optimal order quantity, total carbon emission and total profit per unit time for each situation are determined.

4. Numerical Example

The above theoretical results and algorithm can be applied to the following numerical examples.

Example 1. Consider an inventory system with the following data: $P = 5000$, $S = 800$, $\hat{S} = 30$, $F_M = 200$, $\hat{F}_M = 10$, $f_M = 5$, $\hat{f}_M = 5.0$, $A_M = 150$, $\hat{A}_M = 20$, $v = 300$, $\hat{v} = 2$, $h_1 = 1$, $\hat{h}_1 = 0.2$, $h_2 = 3$, $\hat{h}_2 = 0.6$, $c_1 = 5$, $c_2 = 10$, $\hat{c}_1 = 0.5$, $\hat{c}_2 = 0.5$, $\theta_1 = 0.03$, $\lambda = 0.05$, $\delta = 1/30$, $r = 1$, $D(p) = 2000 - 3p$, $A_R = 5$, $\hat{A}_R = 50$, $F_R = 3$, $\hat{F}_R = 3$, $f_R = 0.3$, $\hat{f}_R = 1$, $s = 0.05$, $\hat{s} = 0.5$, $h_R = 0.03$, $\hat{h}_R = 0.5$, $\theta_2 = 0.05$, $p_t = 1$, $p_e = 1$, $p_c = 15$ and $\rho = 0.9$ in appropriate units. By using the aforementioned algorithm, we obtain the computational results for various situations and present them in Table 2 and Table 3. In Situation I, the optimal number of shipments is 4, the optimal selling price is 336.923, the optimal shipping and order quantity are 235.431 and 941.724, the optimal order quantity of materials is 1,027.54, the optimal carbon emissions of the retailer and manufacturer are 3,844.18 and 1,826.07 and the joint total profit per unit time is 326,033. In Situation II, the optimal number of shipments is 4, the optimal selling price is 336.963, the optimal shipping and order quantity are 233.084 and 932.338, the optimal

Table 2: The solving process and results of the optimal solution for Situation I.

N	p	q	Q	q_m	E_b	E_v	Π_1
1	337.558	498.722	498.722	527.185	3808.43	1442.42	325,535
2	337.151	349.486	698.973	750.406	3807.49	1648.38	325,928
3	336.998	277.76	833.279	902.958	3823.71	1754.16	326,020
4	**336.923**	**235.431**	**941.724**	**1027.54**	**3844.18**	**1826.07**	**326,033**
5	336.881	207.128	1035.64	1136.42	3865.52	1881.84	326,017

Note: Bold indicates the optimal solutions for the model.

Table 3: The solving process and results of the optimal solution for Situation II.

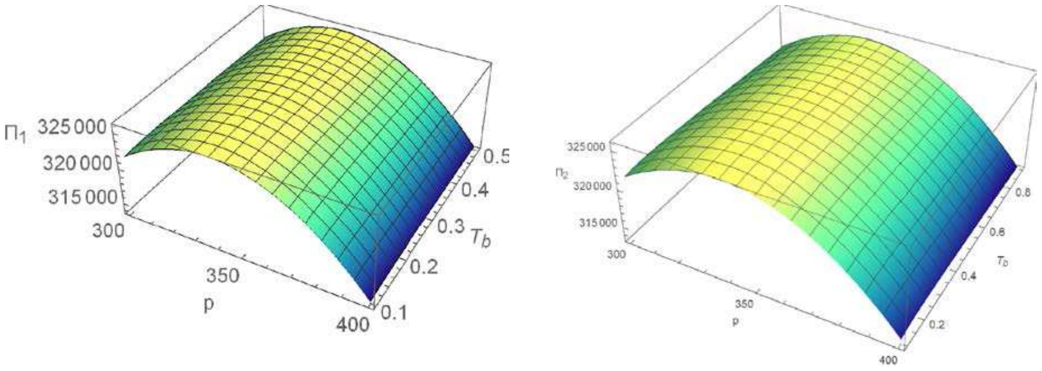
N	p	q	Q	q_m	E_b	E_v	Π_2
1	337.588	500.695	500.695	529.28	3808.35	1442.13	325,463
2	337.186	347.972	695.944	747.092	3807.24	1648.00	325,846
3	337.036	275.539	826.616	895.535	3824.05	1753.05	325,932
4	**336.963**	**233.084**	**932.338**	**1016.94**	**3845.18**	**1824.24**	**325,942**
5	336.922	204.813	1024.06	1123.20	3867.14	1879.35	325,923

Note: Bold indicates the optimal solutions for the model.

order quantity of materials is 1,016.94, the optimal carbon emissions of the retailer and manufacturer are 3,845.18 and 1,824.24 and the joint total profit per unit time is 325,942. Comparing the two results, it is found that when the country where the manufacturer is located implements a carbon tax policy, the joint total profit of the entire supply chain system will decrease. For the manufacturer, the numbers of finished products produced and incoming materials are reduced, which in turn reduces carbon emissions. As to the retailer, though the number of finished goods ordered will decrease, the selling price and its carbon emissions will increase.

In addition, Figure 1 displays the graphical illustration of the joint total profit function $\Pi_j(p, T_b, n)$ with respect to p and T_b for $n = 4$, and Figure 2 illustrates the graphical illustration of the joint total profit function $\Pi_j(p, T_b, n)$ versus n for $(p, T_b) = (336.923, 0.2366)$ and $(336.963, 0.2343)$, respectively. That is, the concavity of the joint total profit functions can be verified, and the obtained solutions are optimal for maximizing the joint total profit function for various situations.

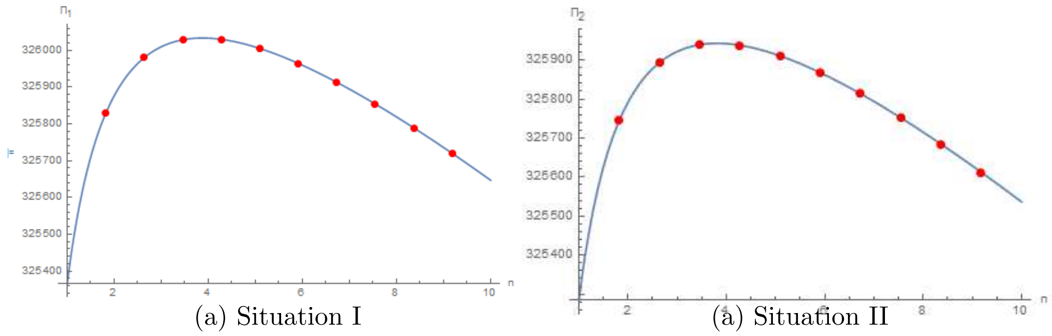
Example 2. This example mainly discusses the impact of changes in the proportion of carbon tariff reductions on the optimal solutions. For convenience, the data are the same as the values used in Example 1 except for $\rho \in \{0, 0.1, 0.2, \ldots, 0.9, 1\}$ under Situation II. The numerous results are obtained as in Table 4 and the changing trends of the optimal solutions are shown as in Figure 3.



(a) Situation I

(a) Situation II

Figure 1: Graphical illustration of $\Pi_j(p, T_b, n)$ with respect to p and T_b for $n = 4$.



(a) Situation I

(a) Situation II

Figure 2: Graphical illustration $\Pi_j(p, T_b, n)$ of versus n for $(p, T_b) = (336.923, 0.2366)$ and (336.963, 0.2343), respectively.

From numerical results of Table 4 and Figure 3, the following management insights can be found:

(1) With the increase in the rate of carbon tariff relief, the retailer's optimal selling price and carbon emissions per unit time decrease while the optimal order quantity increases. As to the manufacturer, both the order quantity of materials and the carbon emissions per unit time increase because of the benefits of carbon tariff relief. Finally, for the multinational supply chain system, tariff relief provided by the country where the retailer is located will contribute to a positive impact on the joint total profit.

(2) When $\rho = 1$ which implies the retailer's country offers 100% carbon tariff relief, the optimal solution for this situation is equivalent to Situation I.

(3) By comparison results in Table 3, it is found that the imposition of carbon tariffs does help manufacturers in importing countries reduce carbon emissions. Further, the rate of carbon tariff relief plays a very critical factor in the entire supply chain profits.

Table 4: The optimal solutions of Situation II for various values of ρ.

ρ	n	p	q	Q	q_m	E_b	E_v	Π_{II}
0	3	337.375	259.461	778.382	841.897	3826.56	1745.07	325,146
0.1	3	337.337	260.964	782.891	846.904	3826.33	1745.81	325,233
0.2	3	337.299	262.527	787.582	852.114	3826.09	1746.58	325,320
0.3	3	337.262	264.155	792.465	857.54	3825.84	1747.39	325,407
0.4	3	337.224	265.852	797.555	863.196	3825.58	1748.23	325,495
0.5	3	337.187	267.621	802.864	869.099	3825.3	1749.1	325,582
0.6	3	337.149	269.469	808.407	875.265	3825.01	1750.02	325,670
0.7	4	337.041	228.736	914.944	997.321	3847.09	1820.87	325,760
0.8	4	337.002	230.856	923.423	1006.88	3846.15	1822.51	325,851
0.9	4	336.963	233.084	932.338	1016.94	3845.18	1824.24	325,942
1	4	336.923	235.431	941.724	1027.54	3844.18	1826.07	326,033

5. Conclusion

In order to require developing countries to pay attention to the issue of carbon emission reduction, developed countries have planned to impose carbon tariffs on goods imported from developing countries. Therefore, this study developed a multinational supply chain system included a single manufacturer without a mature carbon market mechanism and a single retailer where its government plans to impose carbon tariffs on imported goods by the manufacturer. There are two situations considered in this study: (1) the country where the manufacturer is located does not have any carbon emission reduction policies; (2) the manufacturer's country adopts carbon tax policy. Further, a carbon tax for the retailer is considered in this study. The purpose of this study is to determine the optimal material supply, production and delivery strategies for the manufacturer, and the optimal pricing and replenishment strategies for the retailer, so as the joint total profit of the entire supply chain system is maximized. Due to the complexity of the model and directly checking the concavity of profit function is difficult. Thus, alternatively, the concavity is verified by numerical analysis and an algorithm to obtain the solutions for the joint total profit per unit time is developed. From the numerical analysis, it is found that the imposition of carbon tariffs does help manufacturers in importing countries reduce carbon emissions. When the rate of carbon tariff relief increases, the retailer will increase order quantity and drop the selling price and carbon emissions. On the other hand, the manufacturer increases the order quantity of materials and the carbon emissions as the rate of carbon tariff reduction increases. Further, the rate of carbon tariff relief plays a very critical factor in the entire supply chain profits. It is expected that the results of this study provides enterprise or supply

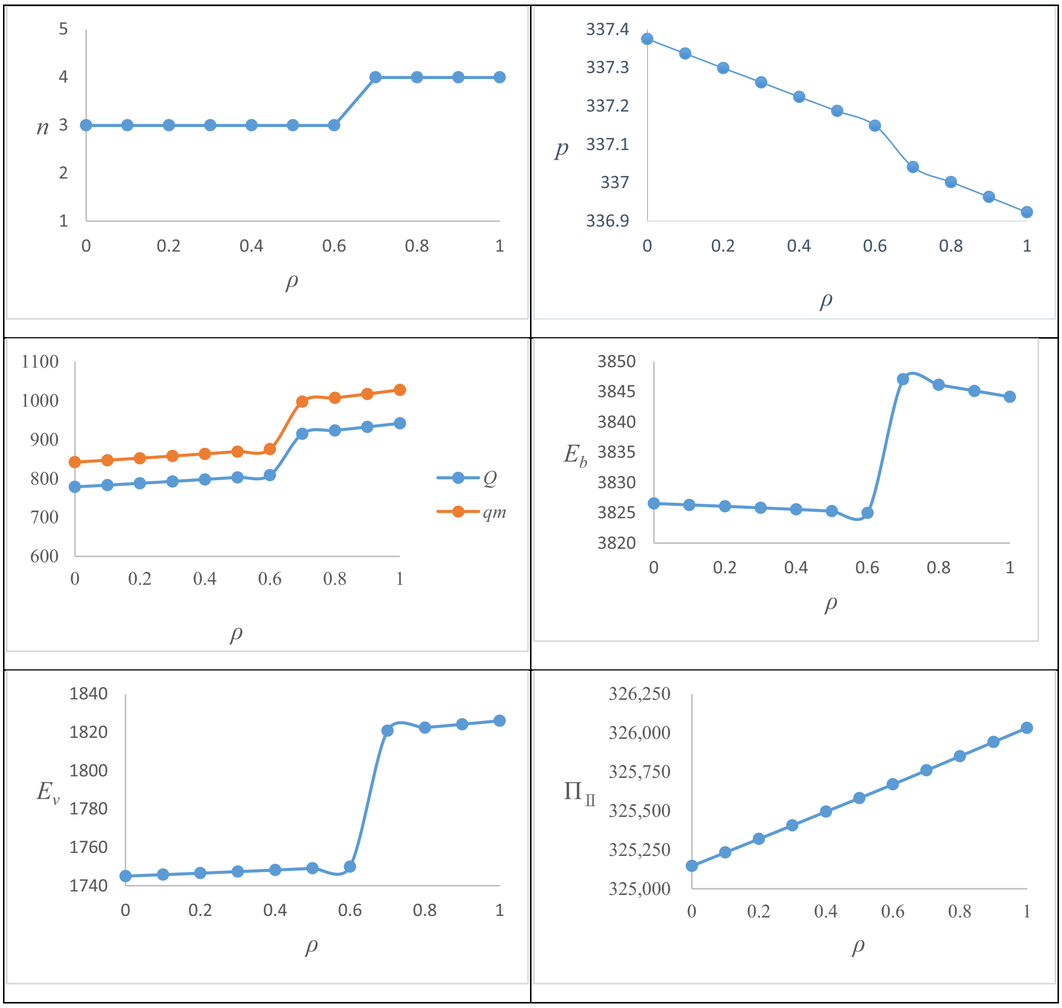


Figure 3: The changing trends of the optimal solutions for various values of ρ.

chain decision makers, especially in multinational enterprises to understand the impact of carbon tariffs on supply chain inventory and pricing decisions and respond accordingly.

The following limitations are identified and future research can be extended as follows. First, only carbon tax policy considered in the proposed models. In reality, different countries have different carbon emission policies such as carbon quota, cap-and-trade and carbon offset, so other combinations of different carbon emission policies can be considered in the future. Furthermore, investments in preservation technology or carbon reduction technology can be added to the model to ensure the sustainable operation of the enterprise. In additions, shortages are not allowed regardless of the manufacturer or retailer. However, in the context of multinational supply chains, the uncertainty of the shipping process is increasing with the times. For example, the new crown pneumonia epidemic or 2021 Suez Canal obstruction. That is, shortage is inevitable and important key issue. Finally, the proposed models can also be extended to more general scenarios such as limited warehouse capacity, trade credit environment or quantity discounts.

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