International Journal of
Information
and
Management
Sciences

http://ijims.ms.tku.edu.tw/main.php

International Journal of
Information and Management Sciences

33 (2022), 13-33. DOI:10.6186/IJIMS.202203_33(1).0002

An Economic Order Quantity Model with Credit-Dependent Demand under Two-Level Trade Credit and Supplier Credits Linked to Order Quantity

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Keywords

Inventory
Credit-dependent demand
Trade credit

Abstract.

A supplier is usually willing to provide the retailer a permissible delay of payments if the retailer orders a large quantity under a business trading environment. Similarly, a retailer might offer a credit period to its customers to motivate customers to increase their demand. In order to reflect these real life phenomena, this research establishes an appropriate inventory model with credit-dependent demand under two-level trade credit and permissible delays in payments linked to order quantity. The objective is to determine the optimal customer's trade credit period, order quantity, and replenishment time to maximize the total profit of the retailer. An easy-to-use algorithm to find the optimal solutions is to provide and implement with a graphical user interface. Numerical examples are given to illustrate the theoretical results and the sensitivity analysis of parameters on the optimal solutions. Some managerial insights are obtained. For instance, the retailer should shorten the replenishment cycle and reduce the order quantity if the interest charged is large.

1. Introduction

In the traditional inventory economic order quantity (EOQ) model, it is tacitly assumed that the supplier is paid for the items as the items are received. However, in practice, the supplier may provide a permissible delay to the customer if the total amount is paid within the permitted fixed settlement period. The permissible delay in payments may also be seen as an alternative to a price discount because it does not provoke competitors to reduce their prices, and thus introduces lasting price reductions. Goyal [11] first derived an EOQ model under the conditions of permissible delay in payments. Aggarwal and Jaggi [1] then extended Goyal's model to allow for deteriorating items. Next, Jamal et al. [17] further generalized the model to allow for shortages. Teng [33] amended Goyal's model by identifying the difference between unit price and unit cost. Ouyang et al. [29] proposed a general EOQ model with trade credit and partial backlogging for a retailer to determine its optimal shortage interval and replenishment cycle. Goyal et al. [12] developed an appropriate EOQ model for a retailer where the supplier offers a progressive interest charge. Roy and Samanta [31] proposed an inventory model with two rates of production for deteriorating items with a permissible delay in payment. Min et al. [25] established a replenishment model for deteriorating items with trade credit and finite replenishment rate. Tsao [41] studied a joint location, inventory and preservation decision-making problem for non-instantaneous deteriorating items with delays in payments. Chang et al. [4] discussed a model to study. Impacts of inspection errors and trade credits on the economic order quantity model for items with imperfect quality. Yu [42] developed a two-warehouse system under shortage backordering, trade credit, and decreasing rental conditions. There are several interesting and relevant studies related to trade credits, such as Ouyang et al. [28], Chang et al. [7], Teng [35], Liang and Zhou [19], Ouyang and Chang [27], Bhunia et al. [2] Guchhait et al. [13].

Huang [14] first established an EOQ model under two levels of trade credit in which the supplier provides the retailer a trade credit period M, and the retailer also offers its customer a trade credit period N (with $N < M$). Teng and Goyal [37] amended Huang's model and a relaxed dispensable assumption, $N < M$. Jaggi et al. [16] proposed an EOQ model with credit-linked demand under a permissible delay in payments. Liao [21] developed an EOQ model with exponentially deteriorating items under two-level trade credit. Thangam and Uthayakumar [40] extended the model of Jaggi et al. [16] and developed an EOQ model with selling price and credit-linked demand for deteriorating items. Chang et al. [6] proposed an EOQ model for exponentially deteriorating items with both upstream and downstream trade credits. Lou and Wang [24] developed an EOQ model for a manufacturer when its supplier offers an up-stream trade credit, while it, in turn, provides its buyers with a down-stream trade credit. Numerous interesting and relevant papers related to two-level trade credit have been published, including Huang [15], Teng and Goyal [37], Teng and Chang [35], Teng et al. [36], Giri and Maiti [10], Liao et al. [22], Thangam [39], Liao et al. [23], Mukherjee and Mahata [26].

The aforementioned studies all implicitly assumed that the trade credit period offered by suppliers is absolute. However, a supplier may be willing to offer a permissible delay of payments if a retailer orders a large quantity (which is greater than or equal to a predetermined quantity, say Q_d), in order to encourage the retailer to order more. Further, a supplier does not offer the trade credit period to his/her retailer if the order quantity is less than the predetermined quantity. Chang et al. [5] established an EOQ model with deteriorating items where a supplier provides a permissible delay of payments for a large order that is greater than or equal to the predetermined quantity. Ouyang et al. [30] developed an EOQ model for deteriorating items with permissible partial delay in payment linked to order quantity. Teng et al. [38] proposed an inventory model for increasing demand in a supply chain with upstream and downstream trade credits. Chen et al. [9] studied a retailer EOQ model where the supplier offers a conditionally permissible delay in payments linked to order quantity. Other studies for trade credit linked to order quantity include Chang [3], Liao [20], Chang et al. [8], Shah and Cárdenas-Barrón [32], and Lashgari et al. [18].

Based on the above statements, we establish an EOQ model with credit-dependent demand under a permissible delay in payments linked to order quantity. In this study, we focus on two levels of trade credit - the retailer's trade credit period M and the customer's trade credit period N. We assume the retailer's trade credit is linked to order quantity and the demand is dependent on the customer's trade credit period. The retailer's trade credit is offered by the supplier, and thus it is a parameter for the retailer. The customer's trade credit period is provided by the retailer to the customers. It impacts the demand rate and the retailer's total profit. Hence, we consider that it is a decision variable and is determined by the retailer. The objective of this study is to determine the optimal solutions for the customer's trade credit period, order quantity and replenishment time in order to maximize the total profit of retailer. The optimality conditions are derived to find these solutions. We also provide an easy-to-use algorithm which is implemented as an application with a graphical user interface to quickly obtain the optimal solutions. Finally, using the proposed algorithm, several numerical examples are presented to illustrate the theoretical results and the sensitivity analysis of parameters. Some managerial insights are obtained. For instance, the retailer should shorten the replenishment cycle and reduce the order quantity if the interest charged is large.

2. Notations and Assumptions

The following notations are used throughout this study:

- A : The ordering cost per order.
- c : The purchasing cost per unit.
- p : The selling price per unit, with $p > c$.
- h : The unit holding cost per unit time excluding interest charge.
- I_e : The interest earned per dollar per unit time.
- I_c : The interest charged per dollar per unit time.
- M : The retailer's trade credit period offered by the supplier.
- N : The customer's trade credit period offered by the retailer, where N is a positive integer and decision variable.
- D : The demand rate per unit time where the demand rate is an increasing function of the customer's trade credit period(N) offered by the retailer; that is, $D = D(N)$, where $D(N) \leq D^{\max}$ and $D^{\max}$ is the maximum demand per unit time. For notational simplicity, $D(N)$ and D will be used interchangeably in this article.
- T : The replenishment cycle time (decision variable).
- Q : The order quantity.
- Q_d : The minimum order quantity for which the delay in payments offered by the supplier is permitted, that is, Q_d is a predetermined quantity.
- T_d : The time interval in which Q_d units are depleted to zero due to demand.
- $I(t)$: The level of inventory at time t, $0 \leq t \leq T$.

$TP(T, N)$: the total profit per unit time, which is a function of T and N where the total profit per unit time consists of the (a) sales revenue, (b) cost of purchasing, (c) cost of placing orders, (d) cost of carrying inventory (excluding interest charges), (e) cost of interest charged for unsold items at the initial time or after the permissible delay M and sold items before the customer's trade credit period N, and (f) interest earned from sales revenue during the permissible period interval $[N, M]$ (with $N < M$).

In addition, the following assumptions are used throughout this paper:

(1) Shortages are not allowed.
(2) Replenishment is instantaneous.
(3) If the order quantity is less than Q_d, the payment for the items received must be made immediately. That is, the supplier does not offer the trade credit period to the retailer.
(4) If the order quantity is greater than or equal to Q_d, the delay in payments up to M is permitted. During the trade credit period, the account is not settled and the generated sale revenue is deposited in an interest bearing account. At the end of the permissible delay, the retailer pays off all units ordered and starts paying for the interest charges on the items in stock. The retailer can accumulate revenue in an account and obtain interest earned when $M \geq N$. There is no interest earned for the retailer when $M < N$.
(5) The ending inventory is zero.

3. Mathematical Formulations

The inventory level $I(t)$ is depleted due to demand. Hence, the rate of change of inventory can be expressed as

$$\frac{dI(t)}{dt} = -D, \qquad 0 \leq t \leq T, \tag{3.1}$$

with the boundary conditions $I(0) = Q$ and $I(T) = 0$. The solution of (1) can be easily derived as

$$I(t) = D(T - t), \qquad 0 \leq t \leq T, \tag{3.2}$$

and the order quantity is given by

$$Q = DT. \tag{3.3}$$

From (3.3), we can obtain the time interval T_d in which Q_d units are depleted to zero due to demand, which can be expressed as

$$Q_d = DT_d. \tag{3.4}$$

Thus, we know that T_d can be determined uniquely from (3.4), and the inequality $Q < Q_d$ holds if and only if $T < T_d$.

The total profit per unit time consists of the following elements.

(a) Sales revenue $= pD$. (3.5)

(b) Cost of purchasing $= cD$. (3.6)

(c) Cost of placing orders $= A/T$. (3.7)

(d) Cost of carrying inventory $= h\int_0^T I(t)dt/T = hDT/2$. (3.8)

Regarding interest charged and earned (i.e. costs of (e) and (f)), there are two cases based on the values of $\mathcal{T}$ and T_d, namely Case I: $T \leq T_d$ and Case II: $T \geq T_d$. The details of the cases are discussed below.

Case I: $T < T_d$

In this case, the order quantity is less than Q_d, and so the delay in payments is not permitted. The supplier must be paid for the items as the retailer receives them. However, the retailer promises to offer the trade credit period N to his customer. That is, the supplier does not offer the trade credit period M to the retailer, but the retailer offers the trade credit period N to the customer. Note that the retailer must close accounts with all items ordered at the initial time with interest charged at I_c per dollar per unit time, and start to pay off the loan after time N. Thus, we obtain the interest payable unit time as $cI_cD(N+T/2)$. Then, the total profit per unit time $TP(T,N)$ can be derived as

$$\begin{aligned} TP_1(T,N) =&(p-c)D - \frac{A}{T} - h\frac{DT}{2} - cI_cD(N+\frac{T}{2}) \\ =&(p-c-cI_cN)D - \frac{A}{T} - \frac{(h+cI_c)}{2}DT. \end{aligned} \tag{3.9}$$

Case II: $T \geq T_d$

In this case, the order quantity is not less than Q_d and so the delay in payments is permitted. That is, the retailer buys all items at time zero and must pay off the total purchasing cost at time M. In addition, the retailer receives the payment from the first customer at time N and the last customer at time $T+N$. Based on the values of M, N and $T+N$, there are three possible sub-cases to calculate the interest charged and earned, namely, Sub-case II-1: $T \geq T_d$ and $N \leq M \leq T+N$, Sub-case II-2: $T \geq T_d$ and $N \leq T+N \leq M$, and Sub-case II-3: $T \geq T_d$ and $M \leq N \leq T+N$.

Sub-case II-1: $T \geq T_d$ and $N \leq M \leq T+N$.

In this sub-case, the retailer starts getting the money at time N and $N \leq M$. Hence, the retailer accumulates revenue in an account and earns I_e per dollar per unit time starting from N through M. In addition, the retailer pays off the purchasing cost at time M. Since $M \leq T+N$, the retailer cannot pay off the supplier by M. Hence, the retailer must raise funds for the items sold after time $M-N$ with interest charged at I_c per dollar per unit time. Therefore, we obtain the total profit per unit time $TP(T,N)$ as

$$\begin{aligned} TP_{2\text{-}1}(T,N) =&(p-c)D-\frac{A}{T}-h\frac{DT}{2}-\frac{cI_cD}{2T}(T+N-M)^2+\frac{pI_eD}{2T}(M-N)^2 \\ =&[p-c+cI_c(M-N)]D-\frac{(h+cI_c)}{2}DT-\frac{1}{2T}[2A+(cI_c-pI_e)D(M-N)^2]. \end{aligned} \tag{3.10}$$

Sub-case II-2: $T \geq T_d$ and $N \leq T+N \leq M$.

In this sub-case, since $M \geq T+N$, the retailer receives the total revenue at time $T+N$, and is able to pay the supplier the total purchasing cost at time M. Hence, the retailer has no interest charged. Therefore, the total profit per unit time $TP(T,N)$ is given as

$$\begin{aligned} TP_{2\text{-}2}(T,N) =&(p-c)D-\frac{A}{T}-h\frac{DT}{2}-\frac{pI_eDT}{2}+pI_eD(M-N) \\ =&[p-c+pI_e(M-N)]D-\frac{A}{T}-\frac{(h+pI_e)}{2}DT. \end{aligned} \tag{3.11}$$

Sub-case II-3: $T \geq T_d$ and $M \leq N \leq T+N$.

In this sub-case, the customer's trade credit period N is equal to or larger than the retailer's trade credit period M, that is, $M \leq N$. There is no interest earned for the retailer. We can obtain the total profit per unit time $TP(T,N)$ as

$$\begin{aligned} TP_{2\text{-}3}(T,N) =&(p-c)D-\frac{A}{T}-h\frac{DT}{2}-cI_cD[\frac{T}{2}+(N-M)] \\ =&[p-c+cI_c(M-N)]D-\frac{A}{T}-\frac{(h+cI_c)DT}{2}. \end{aligned} \tag{3.12}$$

4. Optimal Solutions

Our objective is to maximize the total profit per unit time $TP(T,N)$, which is a function of the continuous variable T and the discrete variable N. Hence, the problem is to find the optimal values for T and N maximizing $TP(T,N)$. Now, for a fixed value of N, the optimal value of T which maximizes $TP(T,N)$ can be found as follows.

Case I: $T < T_d$.

For a fixed value of N, taking the first-order and the second-order derivatives of $TP_1(T,N)$ with respect to T, we obtain

$$\frac{\partial TP_1(T,N)}{\partial T}=\frac{A}{T^2}-\frac{(h+cI_c)}{2}D, \tag{4.1}$$

and

$$\frac{\partial^2 TP_1(T,N)}{\partial T^2} = -\frac{2A}{T^3} < 0. \tag{4.2}$$

Using (4.1) and (4.2), we know that $TP_1(T,N)$ is a concave function of $\mathcal{T}$. The optimal value of $\mathcal{T}$ (denoted by T_1) which maximizes $TP_1(T,N)$ can be obtained by solving the equation $\partial TP_1(T,N)/\partial T = 0$. The solution is computed as

$$T_1 = \sqrt{\frac{2A}{(h+cI_c)D}}. \tag{4.3}$$

The optimal total profit per unit time is given by

$$TP_1(T,N) = TP_1(T_1,N) = (p-c-cI_cN)D - \sqrt{2AD(h+cI_c)}. \tag{4.4}$$

To ensure that $T_1 < T_d$, we substitute (4.3) into the inequality $T_1 < T_d$, and find that if $2A < T_d^2 D(h+cI_c)$, we have $T_1 < T_d$. On the other hand, if $2A \geq T_d^2 D(h+cI_c)$, then we have

$$\frac{\partial TP_1(T,N)}{\partial T} = \frac{2A - T^2(h+cI_c)D}{2T^2} \geq \frac{D(h+cI_c)(T_d^2 - T^2)}{2T^2} > 0, \text{ for } T \in (0,T_d).$$

Thus, $TP_1(T,N)$ is a strictly increasing function for $T \in (0,T_d)$. The value which maximizes $TP_1(T,N)$ does not exist.

Based on the above results, we have the following lemmas:

Lemma 1. *For a fixed value of N, let $\Delta_0 = T_d^2 D(h+cI_c)$. Then, we have the following:*
(1) *if $2A < \Delta_0$, then $T_1^* = T_1$ is the optimal value which maximizes $TP_1(T,N)$.*
(2) *if $2A \geq \Delta_0$, then the value of T which maximizes $TP_1(T,N)$ does not exist.*

Case II: $T \geq T_d$.

In this case, we consider three sub-cases:

Sub-case II-1: $T \geq T_d$ and $N \leq M \leq T + N$.

Similarly, taking the first-order and the second-order derivatives of $TP_{2\text{-}1}(T,N)$ with respect to $T \in [T_d, \infty)$, we obtain

$$\frac{\partial TP_{2\text{-}1}(T,N)}{\partial T} = \frac{1}{2T^2}[2A + (cI_c - pI_e)D(M-N)^2] - \frac{(h+cI_c)}{2}D, \tag{4.5}$$

and

$$\frac{\partial^2 TP_{2\text{-}1}(T,N)}{\partial T^2} = \frac{-1}{T^3}[2A + (cI_c - pI_e)D(M-N)^2]. \tag{4.6}$$

If $2A \geq T_d^2 D(h+cI_c) - (cI_c - pI_e)D(M-N)^2$, we have $2A + (cI_c - pI_e)D(M-N)^2 \geq T_d^2 D(h+cI_c) > 0$ which implies $\partial^2 TP_{2\text{-}1}(T,N)/\partial T^2 < 0$. Thus, there exists a unique value of $\mathcal{T}$ (denoted by $T_{2\text{-}1}$) which maximizes $TP_{2\text{-}1}(T,N)$ and can be obtained by solving $\partial TP_{2\text{-}1}(T,N)/\partial T=0$. The solution is calculated as

$$T_{2\text{-}1}=\sqrt{\frac{2A+(cI_c-pI_e)D(M-N)^2}{(h+cI_c)D}}. \tag{4.7}$$

The optimal total profit per unit time is given by

$$\begin{aligned} TP^*_{2\text{-}1}(T,N)=&TP_{2\text{-}1}(T_{2\text{-}1},N)\\ =&[p-c+cI_c(M-N)]D-\sqrt{[2A+(cI_c-pI_e)D(M-N)^2]D(h+cI_c)}. \end{aligned} \tag{4.8}$$

To ensure that $T_{2\text{-}1}\geq T_d$, we substitute (4.7) into the inequality $T_{2\text{-}1}\geq T_d$, and find that if $2A\geq T_d^2D(h+cI_c)-(cI_c-pI_e)D(M-N)^2$, we have $T_{2\text{-}1}\geq T_d$. Otherwise, if $2A<T_d^2D(h+cI_c)-(cI_c-pI_e)D(M-N)^2$, we have

$$\frac{\partial TP_{2\text{-}1}(T,N)}{\partial T}<\frac{D(h+cI_c)(T_d^2-T^2)}{2T^2}<0,\ \text{for } T\in(T_d,\infty).$$

Thus, $TP_{2\text{-}1}(T,\ N)$ is a strictly decreasing function of $T\in[T_d,\infty)$, which implies $TP_{2\text{-}1}(T,N)$ has a maximum value at the boundary point $T=T_d$ and

$$\begin{aligned} TP^*_{2\text{-}1}(T,N)=&TP_{2\text{-}1}(T_d,N)\\ =&[p-c+cI_c(M-N)]D-\sqrt{[2A+(cI_c-pI_e)D(M-N)^2]D(h+cI_c)}\\ &-\frac{D(h+cI_c)}{2T_d}\Big(T_d-\sqrt{\frac{2A+(cI_c-pI_e)D(M-N)^2}{D(h+cI_c)}}\Big)^2. \end{aligned} \tag{4.9}$$

In addition, to ensure that $T_{2\text{-}1}\geq M-N$, we substitute (4.7) into the inequality $T_{2\text{-}1}\geq M-N$, and find that if $2A\geq(h+pI_e)D(M-N)^2$, we have $T_{2\text{-}1}\geq M-N$. Otherwise, if $2A<(h+pI_e)D(M-N)^2$, $TP_{2\text{-}1}(T,N)$ is a strictly decreasing function of $T\in[M-N,\infty)$. This implies that $TP_{2\text{-}1}(T,N)$ has a maximum value at the boundary point $T=M-N$ and

$$\begin{aligned} TP^*_{2\text{-}1}(T,N)=&TP_{2\text{-}1}(M-N,N)\\ =&[p-c+cI_c(M-N)]D-\sqrt{[2A+(cI_c-pI_e)D(M-N)^2]D(h+cI_c)}\\ &-\frac{D(h+cI_c)}{2(M-N)}\Big((M-N)-\sqrt{\frac{2A+(cI_c-pI_e)D(M-N)^2}{D(h+cI_c)}}\Big)^2. \end{aligned} \tag{4.10}$$

Let $\Delta_1=T_d^2D(h+cI_c)-(cI_c-pI_e)D(M-N)^2$ and $\Delta_2=(h+pI_e)D(M-N)^2$. We find that (1) if $T_d>M-N$, then $\Delta_1>\Delta_1$; (2) if $T_d<M-N$, then $\Delta_1<\Delta_2$; and (3) if $T_d=M-N$, then $\Delta_1=\Delta_2$. Based on the above results, we have proved the following lemma.

Lemma 2. *For a fixed value of* N,

(1) *if* $T_d>M-N$ *and*

(a) $2A \geq \Delta_2$, *then* $T^*_{2\text{-}1} = T_{2\text{-}1}$ *is the optimal value which maximizes* $TP_{2\text{-}1}(T, N)$.
(b) $2A < \Delta_2$, *then* $T^*_{2\text{-}1} = M - N$ *or* T_d *is the optimal value which maximizes* $TP_{2\text{-}1}(T, N) = \max\{TP_{2\text{-}1}(M - N, N), TP_{2\text{-}1}(T_d, N)\}$.

(2) *if* $T_d < M - N$ *and*
(a) $2A \geq \Delta_1$, *then* $T^*_{2\text{-}1} = T_{2\text{-}1}$ *is the optimal value which maximizes* $TP_{2\text{-}1}(T, N)$.
(b) $2A < \Delta_1$, *then* $T^*_{2\text{-}1} = M - N$ *or* T_d *is the optimal value which maximizes* $TP_{2\text{-}1}(T, N) = \max\{TP_{2\text{-}1}(M - N, N), TP_{2\text{-}1}(T_d, N)\}$.

(3) *if* $T_d = M - N$ *and*
(a) $2A \geq \Delta_2 (= \Delta_1)$, *then* $T^*_{2\text{-}1} = T_{2\text{-}1}$ *is the optimal value which maximizes* $TP_{2\text{-}1}(T, N)$.
(b) $2A < \Delta_2 (= \Delta_1)$, *then* $T^*_{2\text{-}1} = T_d (= M - N)$ *is the optimal value which maximizes* $TP_{2\text{-}1}(T, N)$.

Sub-case II-2: $T \geq T_d$ and $N \leq T + N \leq M$.

Similarly, taking the first-order and the second-order derivatives of $TP_{2\text{-}2}(T, N)$ with respect to $T \in [T_d, \infty)$, we obtain

$$\frac{\partial TP_{2\text{-}2}(T, N)}{\partial T} = \frac{A}{T^2} - \frac{(h + pI_e)}{2} D, \tag{4.11}$$

and

$$\frac{\partial^2 TP_{2\text{-}2}(T, N)}{\partial T^2} = -\frac{2A}{T^3} < 0. \tag{4.12}$$

The optimal value of $\mathcal{T}$ (denoted by $T_{2\text{-}2}$) maximizing $TP_{2\text{-}2}(T, N)$ can be obtained by solving the equation $\partial TP_{2\text{-}2}(T, N)/\partial T = 0$ and yields

$$T_{2\text{-}2} = \sqrt{\frac{2A}{(h + pI_e)D}}. \tag{4.13}$$

The optimal total profit per unit time is given by

$$\begin{aligned} TP^*_{2\text{-}2}(T, N) =& TP_{2\text{-}2}(T_{2\text{-}2}, N) \\ =& [p - c + pI_e(M - N)]D - \sqrt{2AD(h + pI_e)}. \end{aligned} \tag{4.14}$$

To ensure that $T_{2\text{-}2} \geq T_d$, we substitute (4.13) into the inequality $T_{2\text{-}2} \geq T_d$, and find that if $2A \geq T_d^2 D(h + pI_e)$, we have $T_{2\text{-}2} \geq T_d$. Otherwise, if $2A < T_d^2 D(h + pI_e)$, $TP_{2\text{-}2}(T, N)$ is a strictly decreasing function of $T \in [T_d, \infty)$. This implies $TP_{2\text{-}2}(T, N)$ has a maximum value at the boundary point $T = T_d$ and

$$\begin{aligned} TP^*_{2\text{-}2}(T, N) =& TP_{2\text{-}2}(T_d, N) \\ =& [p - c + pI_e(M - N)]D - \sqrt{2AD(h + pI_e)} - \frac{D(h + pI_e)}{2T_d}\left(T_d - \sqrt{\frac{2A}{D(h + pI_e)}}\right)^2. \end{aligned} \tag{4.15}$$

In addition, to ensure that $T_{2\text{-}2} \geq M - N$, we substitute (4.13) into the inequality $T_{2\text{-}2} \geq M - N$, and find that if $2A \leq (h + pI_e)D(M - N)^2$, we have $T_{2\text{-}2} \geq M - N$.

Otherwise, if $2A > (h{+}pI_e)D(M-N)^2$, then $T_{2\text{-}2}(T,N)$ is a strictly increasing function of $T \in (0, M-N]$. This implies $TP_{2\text{-}2}(T,N)$ has a maximum value at the boundary point $T = M-N$ and

$$\begin{aligned} TP^*_{2\text{-}2}(T,N) =& TP_{2\text{-}2}(M-N,N) \\ =& [p{-}c{+}pI_e(M-N)]D - \sqrt{2AD(h{+}pI_e)} \\ & - \frac{D(h+pI_e)}{2(M-N)}\Big((M-N) - \sqrt{\frac{2A}{D(h+pI_e)}}\Big)^2. \end{aligned} \tag{4.16}$$

We find that (1) if $T_d < M-N$, then $D(h+pI_e)T_d^2 < D(h+pI_e)(M-N)^2$ and (2) if $T_d = M-N$, then $D(h+pI_e)T_d^2 = D(h+pI_e)(M-N)^2$. Based on the above results, the following lemma is proved.

Lemma 3. *For a fixed value of N,*
(1) *if $T_d < M-N$, then $T^*_{2\text{-}2} = T_{2\text{-}2}$ is the optimal value which maximizes $TP_{2\text{-}2}(T,N)$.*
(2) *if $T_d = M-N$, then $T^*_{2\text{-}2} = T_d = M-N$ is the optimal value which maximizes $TP_{2\text{-}2}(T,N)$.*
Note that it is a contradictory case when $T_d > M-N$.

Sub-case II-3: $T \geq T_d$ and $M \leq N \leq T+N$.

Similarly, taking the first-order and the second-order derivatives of $TP_{2\text{-}3}(T,N)$ with respect to $T \in [T_d, \infty)$, we obtain

$$\frac{\partial TP_{2\text{-}3}(T,N)}{\partial T} = \frac{A}{T^2} - \frac{(h+cI_c)}{2}D, \tag{4.17}$$

and

$$\frac{\partial^2 TP_{2\text{-}3}(T,N)}{\partial T^2} = -\frac{2A}{T^3} < 0. \tag{4.18}$$

The optimal value of $\mathcal{T}$ (denoted by $T_{2\text{-}3}$) maximizing $TP_{2\text{-}3}(T,N)$ can be obtained by solving the equation $\partial TP_{2\text{-}3}(T,N)/\partial T = 0$ and yields

$$T_{2\text{-}3} = \sqrt{\frac{2A}{(h+cI_c)D}}. \tag{4.19}$$

The optimal total profit per unit time

$$TP^*_{2\text{-}3}(T,N) = TP_{2\text{-}3}(T_{2\text{-}3},N) = [p{-}c{+}cI_c(M-N)]D - \sqrt{2AD(h{+}cI_c)}. \tag{4.20}$$

To ensure that $T_{2\text{-}3} \geq T_d$, we substitute (4.19) into inequality $T_{2\text{-}3} \geq T_d$, and find that if $2A \geq T_d^2 D(h{+}cI_c)$, we have $T_{2\text{-}3} \geq T_d$. Otherwise, if $2A \geq T_d^2 D(h{+}cI_c)$, then $TP_{2\text{-}3}(T,N)$ is a strictly decreasing function of $T \in [T_d, \infty)$. This implies $TP_{2\text{-}3}(T,N)$ has a maximum value at the boundary point $T = T_d$ and

$$TP^*_{2\text{-}3}(T,N) = TP_{2\text{-}3}(T_d,N)$$

$$= [p-c+cI_c(M-N)]D - \sqrt{2AD(h+cI_c)} - \frac{D(h+cI_c)}{2T_d}\left(T_d - \sqrt{\frac{2A}{D(h+cI_c)}}\right)^2. \quad (4.21)$$

Based on the above results, the following lemma is proved.

Lemma 4. *For a fixed value of N,*
(1) *if $2A \geq \Delta_0$, then $T^*_{2\text{-}3} = T_{2\text{-}3}$ is the optimal value which maximizes $TP_{2\text{-}3}(T,N)$.*
(2) *if $2A < \Delta_0$, then $T^*_{2\text{-}3} = T_d$ is the optimal value which maximizes $TP_{2\text{-}3}(T,N)$.*

Using the above lemmas, for a fixed value of N, we obtain the following theorems.

Theorem 1. *For a fixed value of N, if $M > N$, $T_d > M-N$ and $cI_c > pI_e$, then $\Delta_0 > \Delta_1$ and the following results are obtained.*
(1) *If $2A \geq \Delta_0$, then $TP^*(T^*,N) = TP_{2\text{-}1}(T_{2\text{-}1},N)$.*
(2) *If $\Delta_2 < 2A < \Delta_0$, then $TP^*(T^*,N) = \max\{TP_{2\text{-}1}(T_{2\text{-}1},N), TP_1(T_1,N)\}$.*
(3) *If $2A \leq \Delta_2$, then $TP^*(T^*,N) = \max\{TP_{2\text{-}1}(T_d,N), TP_{2\text{-}1}(M-N,N), TP_1(T_1,N)\}$.*

Proof. It immediately follows from Lemma 1 and Lemma 2-(1).

Theorem 2. *For a fixed value of N, if $M > N$, $T_d > M-N$ and $cI_c \leq pI_e$, then $\Delta_0 < \Delta_1$ and the following results are obtained.*
(a) *For $\Delta_2 < \Delta_0 < \Delta_1$, and*
  (1) *If $2A \geq \Delta_0$, then $TP^*(T^*,N) = TP_{2\text{-}1}(T_{2\text{-}1},N)$.*
  (2) *If $\Delta_2 < 2A < \Delta_0$, then $TP^*(T^*,N) = \max\{TP_{2\text{-}1}(T_{2\text{-}1},N), TP_1(T_1,N)\}$.*
  (3) *If $2A \leq \Delta_2$, then $TP^*(T^*,N) = \max\{TP_{2\text{-}1}(T_d,N), TP_{2\text{-}1}(M-N,N), TP_1(T_1,N)\}$.*
(b) *For $\Delta_0 < \Delta_2 < \Delta_1$, and*
  (1) *If $2A \geq \Delta_2$, then $TP^*(T^*,N) = TP_{2\text{-}1}(T_{2\text{-}1},N)$.*
  (2) *If $\Delta_0 < 2A < \Delta_1$, then $TP^*(T^*,N) = \max\{TP_{2\text{-}1}(T_d,N), TP_{2\text{-}1}(M-N,N)\}$.*
  (3) *If $2A \leq \Delta_0$, then $TP^*(T^*,N) = \max\{TP_{2\text{-}1}(T_d,N), TP_{2\text{-}1}(M-N,N), TP_1(T_1,N)\}$.*

Proof. It immediately follows from Lemma 1 and Lemma 2-(1).

Theorem 3. *For a fixed value of N, if $M > N$ and $T_d < M-N$, then $\Delta_2 (= \Delta_4) > \Delta_3$ and the following results are obtained.*
(1) *If $2A \geq \Delta_1$, then $TP^*(T^*,N) = \max\{TP_{2\text{-}1}(T_{2\text{-}1},N), TP_{2\text{-}2}(T_{2\text{-}2},N)\}$.*
(2) *If $2A < \Delta_1$, then $TP^*(T^*,N) = \max\{TP_{2\text{-}1}(T_d,N), TP_{2\text{-}1}(M-N,N), TP_{2\text{-}2}(T_{2\text{-}2},N)\}$.*

Proof. It immediately follows from Lemma 2-(2) and Lemma 3-(1).

Theorem 4. *For a fixed value of N, if $M > N$ and $T_d = M-N$, then $TP_{2\text{-}1}(T_d,N) = TP_{2\text{-}2}(T_d,N) = TP_{2\text{-}1}(M-N,N) = TP_{2\text{-}2}(M-N,N)$, $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4$ and the following results are obtained.*
(1) *If $2A \geq \Delta_1$, then $TP^*(T^*,N) = TP_{2\text{-}1}(T_{2\text{-}1},N)$.*
(2) *If $2A < \Delta_1$, then $TP^*(T^*,N) = TP_{2\text{-}1}(T_d,N)$.*

Proof. It immediately follows from Lemma 2-(3) and Lemma 3-(2).

Theorem 5. *For a fixed value of N, if $M \leq N$, the following results are obtained.*

(1) *If* $2A \geq \Delta_0$, *then* $TP^*(T^*, N) = TP_{2\text{-}3}(T_{2\text{-}3}, N)$.
(2) *If* $2A < \Delta_0$, *then* $TP^*(T^*, N) = \max\{TP_{2\text{-}3}(T_d, N), TP_1(T_1, N)\}$.

Proof. It immediately follows from Lemma 1 and Lemma 4.

5. Algorithms

In order to determine the optimal values of $\mathcal{T}$ and N simultaneously, we develop the following algorithms based on the above theorems.

Algorithm 1

Step 0: Setup the parameters.

Step 1: Set $N = 1$.

Step 2: If $M > N$, then go to algorithm 1-1 to find $TP^*(T^*, N)$. Otherwise, go to Step 3.

Step 3: (1) IF $2A \geq \Delta_0$, then $TP^*(T^*, N) = TP_{2\text{-}3}(T_{2\text{-}3}, N)$ and go to Step 4.

(2) IF $2A < \Delta_0$, then $TP^*(T^*, N^*) = \max\{TP_{2\text{-}3}(T_d, N), TP_1(T_1, N)\}$ and go to Step 4.

Step 4: If $N < N^{\max}$ (using $D(N) \leq D^{\max}$, the maximum customer's trade credit period $N^{\max}$ can be obtained), then $N = N + 1$ and go to Step 2.

Step 5: The optimal solution $TP^*(T^*, N) = \max\{TP^*(T^*, N^*), N = 1, 2, \ldots, N^{\max}\}$.

Step 6: The optimal order quantity $Q^* = D(N^*)T^*$.

Step 7: Stop.

Algorithm 1-1

Step 0: Set $T_d = Q_d/D(N)$.

Step 1: If $T_d > M - N$, then go to Step 2. Otherwise, go to Step 5.

Step 2: If $cI_c > pI_e$, then go to Step 3. Otherwise go to Step 4.

Step 3: (1) If $2A \geq \Delta_0$, then $TP^*(T^*, N) = TP_{2\text{-}1}(T_{2\text{-}1}, N)$ and go to Step 8.

(2) If $\Delta_1 < 2A < \Delta_0$, then $TP^*(T^*, N) = \max\{TP_{2\text{-}1}(T_{2\text{-}1}, N), TP_1(T_1, N)\}$ and go to Step 8.

(3) If $2A \leq \Delta_2$, then $TP^*(T^*, N) = \max\{TP_{2\text{-}1}(T_d, N), TP_{2\text{-}1}(M - N, N), TP_1(T_1, N)\}$ and go to Step 8.

Step 4: (a) If $\Delta_2 < \Delta_0 < \Delta_1$, then go to Step 3.

(b) If $\Delta_0 < \Delta_2 < \Delta_1$, and

(1) If $2A \geq \Delta_2$, then $TP^*(T^*, N) = TP_{2\text{-}1}(T_{2\text{-}1}, N)$ and go to Step 8.

(2) If $\Delta_0 < 2A < \Delta_2$, then $TP^*(T^*, N) = \max\{TP_{2\text{-}1}(T_d, N), TP_{2\text{-}1}(M - N, N)\}$, and go to Step 8.

(3) If $2A \le \Delta_0$, then $TP^*(T^*, N) = \max\{TP_{2\text{-}1}(T_d, N), TP_{2\text{-}1}(M-N, N), TP_1(T_1, N)\}$ and go to Step 8.

Step 5: If $T_d < M - N$, then go to Step 6. Otherwise, go to Step 7.

Step 6: (1) If $2A \le \Delta_1$, then $TP^*(T^*, N) = \max\{TP_{2\text{-}1}(T_{2\text{-}1}, N), TP_{2\text{-}1}(T_{2\text{-}2}, N)\}$ and go to Step 8.

(2) If $2A < \Delta_1$, then $TP^*(T^*, N) = \max\{TP_{2\text{-}1}(T_d, N), TP_{2\text{-}1}(M-N, N), TP_{2\text{-}2}(T_{2\text{-}2}, N)\}$ and go to Step 8.

Step 7: (1) If $2A \le \Delta_1$, then $TP^*(T^*, N) = TP_{2\text{-}1}(T_{2\text{-}1}, N)$ and go to Step 8.

(2) If $2A < \Delta_1$, then $TP^*(T^*, N) = TP_{2\text{-}1}(T_d, N)$ and go to Step 8.

Step 8: Go to Step 4 of Algorithm 1.

6. Numerical Examples

We consider two demand functions $D(N)$ to demonstrate the theoretical results and illustrate the proposed algorithm. The sensitivity analysis of parameters on the optimal solution is performed and some managerial insights are drawn.

Example 1. exam1 Let $D = D(N) = \alpha + \beta N^r$, where $\alpha = D(0)$ is the initial demand per unit time, β and r are the customer's trade credit period (N) sensitive parameters of demand. Consider $\alpha = 80$ units/day, $\beta = 30$, $r = 0.12$, $A = \$1000$/order, $M = 30$ days, $h = \$4.5$/unit/year, $c = \$28$/unit, $p = \$45$/unit, $I_e = 0.10$ per year, $I_c = 0.15$ per year, $Q_d = 2000$ units/order, and $D^{\max} = 150$ units/day. We implemented the proposed algorithm with a form interface. The decision-maker can use the developed visualization system, shown in Figure 1, to obtain the optimal solutions and sensitivity

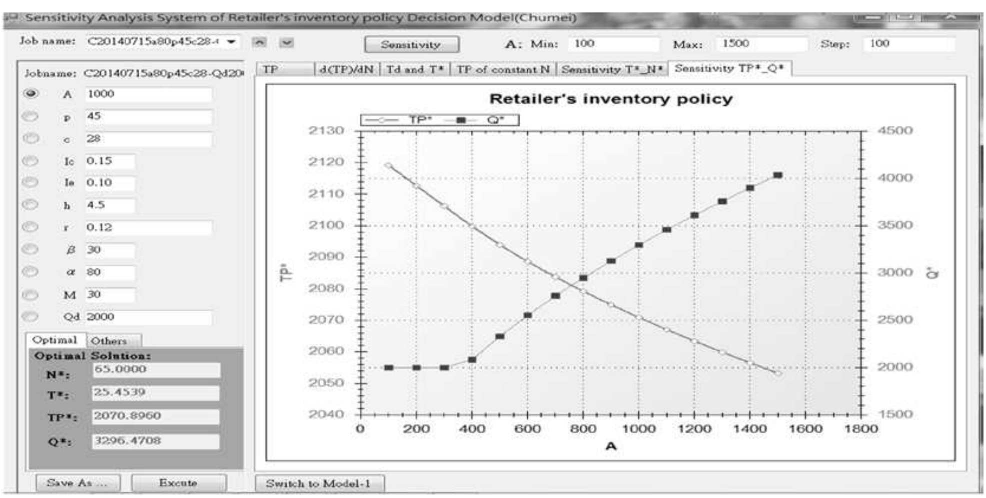


Figure 1: Snapshot of application for retailer's inventory policy.

analysis. The optimal solution obtained is: optimal replenishment cycle $T^* = T_{2\text{-}3} = 25.45$ days, optimal order quantity $Q^* = 3296.47$ units, optimal customer's trade credit period $N^* = 65$days, and optimal total profit per day $TP^*(T^*, N^*) = \$2070.90$.

This implies that the retailer orders the optimal quantity $Q^* = 3296.47$ units, which is larger than the minimum order quantity $Q_d = 2000$ units with trade credit $M = 30$ days being offered by the supplier. The effects of various values of Q_d on the optimal solution are shown in Table 1.

Table 1: Optimal solutions for different values of Q_d.

Q_d (units)	T^* (days)	N^* (days)	Q^* (units)	$TP^*(T^*, N^*)(\$)$
0	$T_{2\text{-}3} = 25.45$	65	3296.47	2070.90
2000	$T_{2\text{-}3} = 25.45$	65	3296.47	2070.90
3296	$T_{2\text{-}3} = 25.45$	65	3296.47	2070.90
3297	$T_d = 25.46$	65	3297	2070.90
4000	$T_d = 30.89$	65	4000	2069.42
5847	$T_d = 45.15$	65	5847	2057.64
5848	$T_d = 45.12$	66	5848	2057.63
6000	$T_d = 46.30$	66	6000	2056.38
6752	$T_d = 52.10$	66	6752	2049.82
6753	$T_1 = 25.90$	30	3240.16	2049.82
8000	$T_1 = 25.90$	30	3240.16	2049.82
10000	$T_1 = 25.90$	30	3240.16	2049.82

From Table 1, the following results are obtained.

(1) If the value of Q_d increases, the value of $TP^*(T^*, N^*)$ decreases. This means that if the minimum order quantity increases, the optimal total profit per day will decrease.

(2) A retailer should order the minimum order quantity Q_d to get the trade credit $M = 30$ days to maximize the total profit per day, when the minimum order quantity is from 3297 to 6752 units. However, the retailer should abandon the trade credit and keep the customer's trade credit period at 30 days when the minimum order quantity is very large (i.e., $Q_d \geq 6753$). (3) The value of Q_d significantly influences the values of T^*, N^* and Q^*.

Example 2. exam2 To analyze the effects of parameters on optimal solutions, we use the same data given in Example 1. The sensitivity analysis is performed by varying the parameters and the results are shown graphically in Figure 2.∼Figure 6. Based on these figures, we have the following results:

(1) When ordering cost A increases, it is seen that the optimal replenishment cycle T^* and the optimal order quantity Q^* increase whereas the optimal total profit per day $TP^*(T^*, N^*)$ decreases (see Figure 2.). As shown in Figure 2, N^* will maintain the

same value if the ordering cost is over \$400. That is, the optimal customer's trade credit period is not affected by the ordering cost when the ordering cost is large (i.e., $A \geq 400$).

(2) A higher retailer's trade credit period value M results in a higher value of $TP^*(T^*, N^*)$, but a lower value of T^* (see Figure 3.). In addition, the customer's trade credit period N^* and order quantity Q^* are not decreasing functions of the retailer's trade credit period M if M is less than 65 days, while N^* and Q^* are decreasing functions of M if M is over 65 days.

(3) A higher value of r (or β) results in higher values of Q^*, N^*, and $TP^*(T^*, N^*)$, but a lower value of T^*. This implies that the retailer should increase the order quantity and the customer's trade credit period, but shorten the replenishment cycle (see Figure 4. and Figure 5.).

(4) It is observed that as I_c increases, T^*, Q^*, and $TP^*(T^*, N^*)$ decrease. This indicates that the retailer should decrease the replenishment cycle and order quantity (see Figure 6.). In addition, the customer's trade credit period N^* is a decreasing function of the interest charged I_c if I_c is over 0.13, but N^* maintains its value if I_c is less than 0.13.

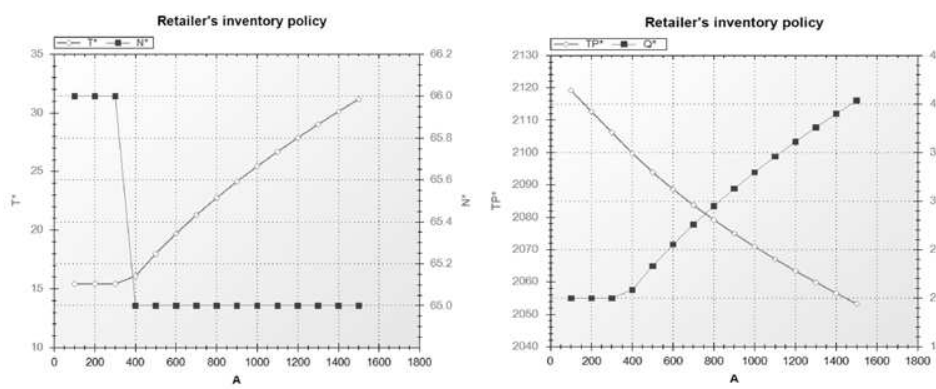


Figure 2: Effect of changing ordering cost A.

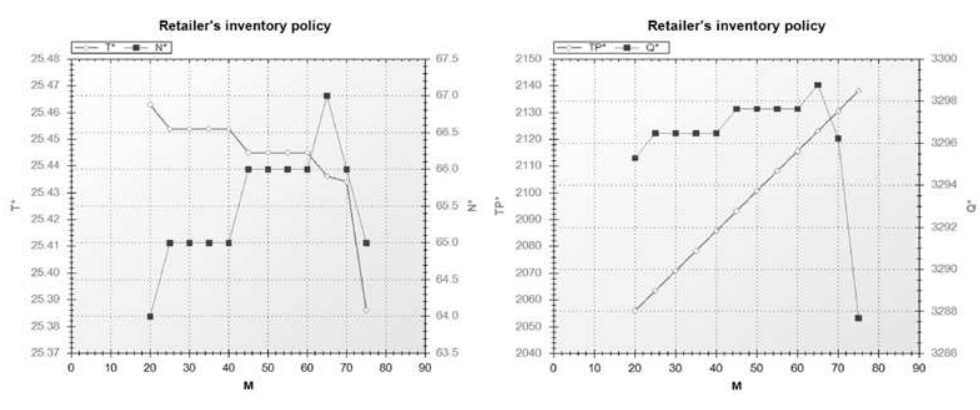


Figure 3: Effect of changing supplier's credit period M.

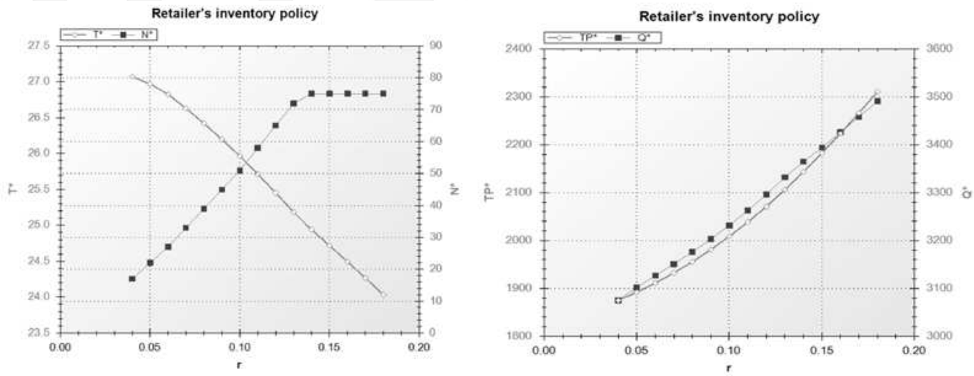


Figure 4: Effect of changing r.

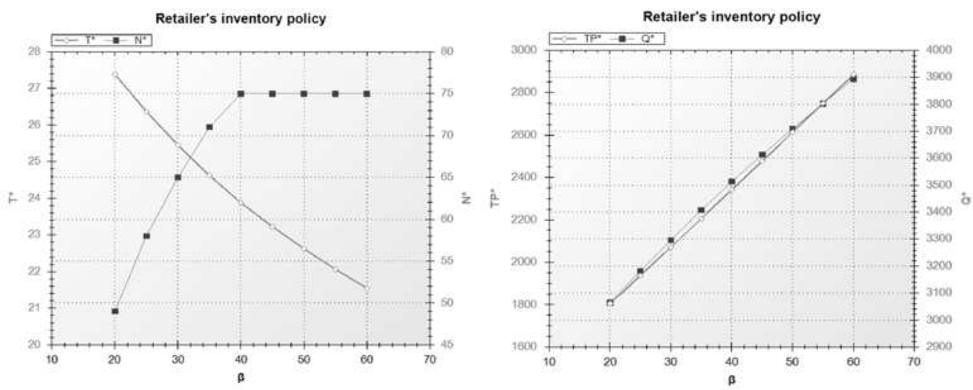


Figure 5: Effect of changing β.

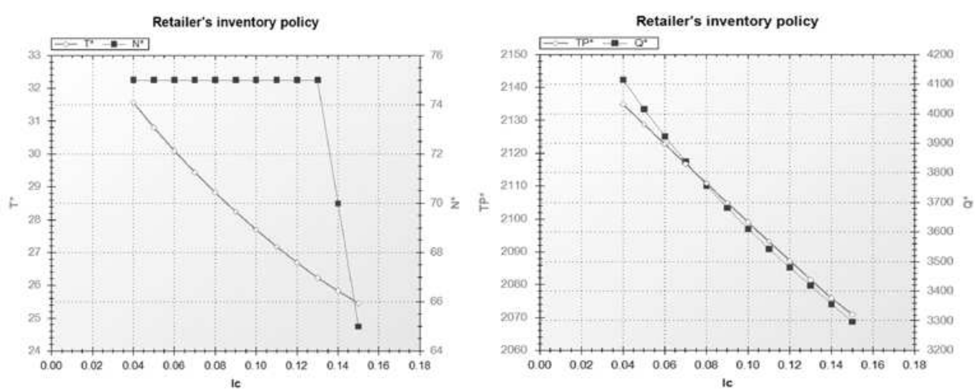
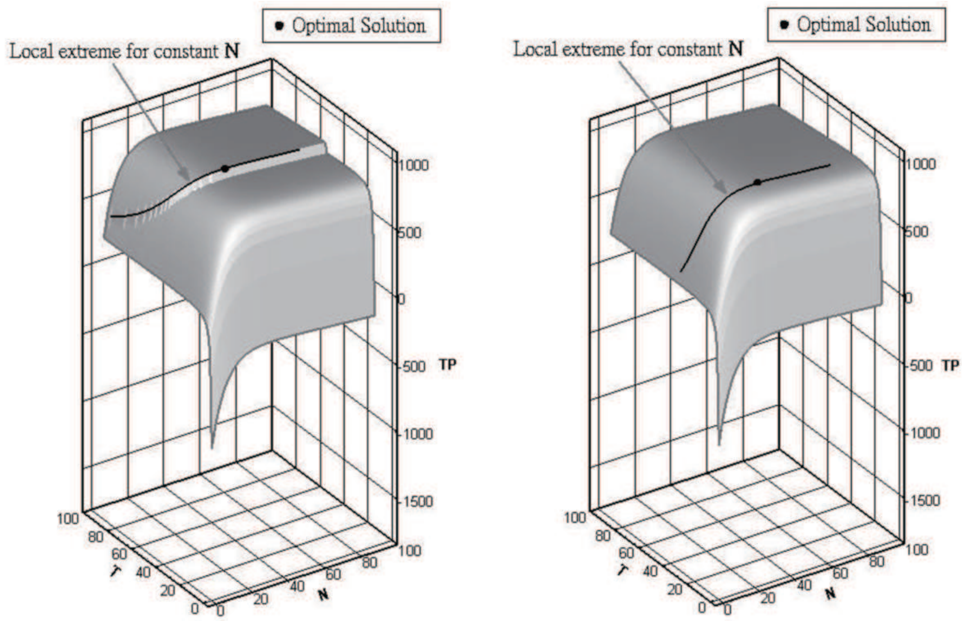


Figure 6: Effect of changing interest charged I_c.

Example 3. exam3 We consider the same demand function in Jaggi et al. [16]. Given $D = D(N) = S - (S - s)(1 - r)^N$, where S is the maximum demand per unit time, $s = D(0)$ is the initial demand per unit time, and r is the rate of saturation of demand. Let $S = D^{\max} = 100$ units/day, $s = 30$ units/day, $r = 0.12$, $A = \$500$/order, $M = 60$

days, $h = \$4.5$/unit/year, $c = \$30$/unit, $p = \$40$/unit, $I_e = 0.10$ per year, $I_c = 0.15$ per year and $Q_d = 4000$ units/order. A three-dimensional visualization of decision model for retailer's inventory policy are shown in Figure 7. Figure 7(a) demonstrates the curve showing local extreme under various values of N. The optimal solution is obtained as follows: optimal replenishment cycle $T^* = T_d = 40.37$ days, optimal order quantity $Q^* = Q_d = 4000$ units, optimal customer's trade credit period $N^* = 34$ days, and optimal total profit per day $TP^*(T^*, N^*) = \$959.86$. It implies that the retailer orders the minimum order quantity $Q_d = 4000$ unit to get the trade credit $M = 60$ days in order to maximize the total profit per day. Figure 7(b) shows the three-dimensional graph of $Q_d = 0$ [16].



(a) $Q_d = 4000$ (b) $Q_d = 0$

Figure 7: Three-dimensional graph of decision model for retailer's inventory policy.

The effects of the different Q_d on the optimal solution are shown in Table 2. From Table 2, a higher value of Q_d results in lower values of N^* and $TP^*(T^*, N^*)$. It means that the retailer should reduce the customer's trade credit period when the predetermined quantity is large. Figure 8 presents the optimal curve for the various values of Q_d. Furthermore, as the minimum order quantity Q_d is 4000, 6000, or 8000 units, the retailer should order the minimum order quantity Q_d to get the trade credit $M = 60$ days to maximize the total profit per day. However, the retailer should abandon the trade credit when the minimum order quantity is very large (i.e., $Q_d \geq 10000$).

Table 2: Optimal solutions for different values of Q_d.

Q_d (units)	T^* (days)	N^* (days)	Q^* (units)	$TP^*(T^*, N^*)(\$)$
0	$T_{2\text{-}2} = 20.81$	35	2063.9408	971.13
2000	$T_{2\text{-}2} = 20.81$	35	2063.9408	971.13
4000	$T_d = 40.37$	34	4000	959.86
6000	$T_d = 60.55$	34	6000	939.71
8000	$T_d = 80.73$	34	8000	917.30
10000	$T_1 = 20.24$	33	2003.4383	900.03
12000	$T_1 = 20.24$	33	2003.4383	900.03

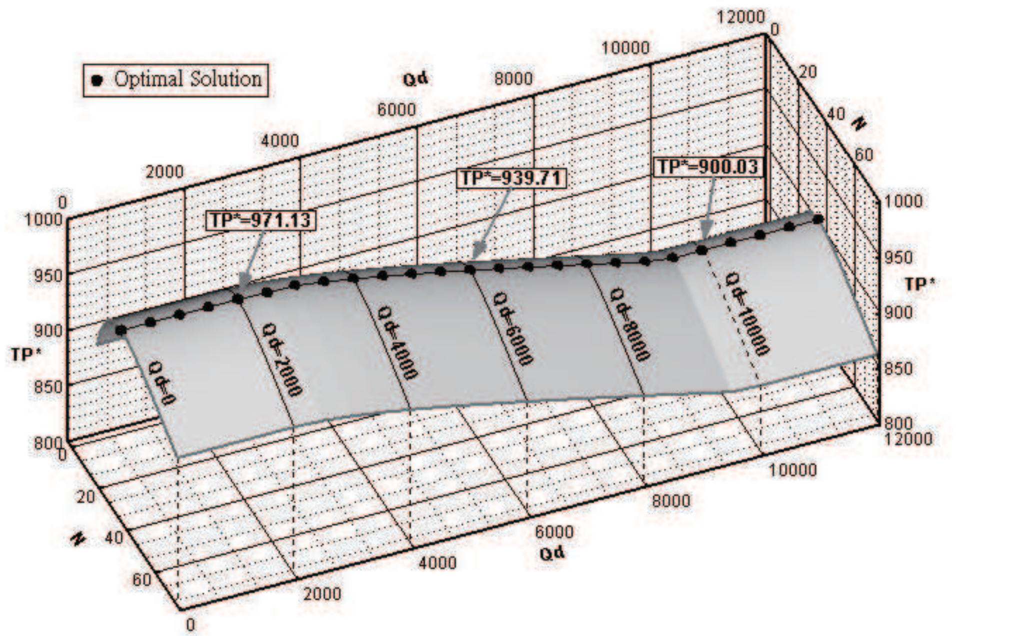


Figure 8: Optimal curve for different values of Q_d.

7. Conclusions

In this study, we developed an inventory model with credit-dependent demand to determine the optimal ordering policy and customer's trade credit period when the supplier provides a permissible delay in payments linked to order quantity. We established Theorems 1-5 to characterize the optimal solutions. Further, an easy-to-use algorithm was proposed and an application with a graphical user interface was created to find the optimal customer's trade credit period, order quantity, and replenishment for the retailer. Finally, we provided several numerical examples to illustrate the theoretical results, and we obtained the following managerial phenomena. (1) A higher value of minimum order quantity Q_d results in a lower value of the optimal total profit per unit time. In addition, the retailer should abandon the trade credit to maximize the total profit per unit time when the minimum order quantity is very large. (2) A higher value of ordering cost A causes higher values of optimal replenishment cycle and order quantity, and it causes a lower value of the optimal total profit per unit time. However, the optimal customer's trade credit period is not affected by the ordering cost when the ordering cost is large. (3) A higher value of retailer's trade credit period M results in a higher value of optimal total profit per unit time, but a lower value of optimal replenishment cycle. That is, when the retailer's trade credit period is longer, the retailer should shorten the replenishment cycle to maximize the total profit per unit time. (4) A higher value of r (or β) results in higher values of optimal order quantity, customer's trade credit period and total profit per unit time, and it results in a lower value of optimal replenishment cycle. (5) A higher value of interest charged I_c causes lower values of optimal replenishment cycle, order quantity, and total profit per unit time. This means that if the interest charge is large, the retailer should shorten the replenishment cycle and reduce the order quantity to maximize the total profit per unit time.

The proposed model could be extended in several ways in further research. For instance, we may extend the proposed model to include deteriorating items. We could also consider the demand as a function of the selling price, product quality, and other characteristics. Finally, we could generalize the model to allow for the shortages, quantity discounts, and inflation rates.

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(Received July 2020; accepted January 2022)