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Comparative Analysis of Fuzzy Critical Path Method in Agriculture Project Management

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Keywords

Project management
fuzzy networking problem
fuzzy number
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Abstract.

Agriculture is considered as a system that supplies valuable product and indefinite yields. It is essential to choose a suitable technique to maximize the yield and minimize losses in harvesting. Project management has techniques to assign activities that facilitate agriculture to standardize the quality, reduce expenses, develops the effectiveness and completes the project without delay. In this paper, an agriculture project has been constructed to minimize the cost of the project with the help of Fuzzy Critical Path Method (FCPM). Fuzzy numbers are more effective to deal with uncertainty which arises in the field of agriculture. Activities affecting the growth of saplings are identified, and a project network is drawn. The cost and duration of each activity is obtained from the observation. The expected duration for the completion of the project, total cost and the critical path is determined using a fuzzy network with generalized Trapezoidal fuzzy numbers, Heptagonal fuzzy number and Hendecagonal fuzzy number. The comparative analysis is done for a significant result of the fuzzy networking problem in agriculture project management.

1. Introduction

Project management can be used as a tool to construct a method to maximize resource utilization and minimize overall cost in agriculture project. The network technique called Critical Path Method (CPM) can be used for agriculture project analysis. In literature, CPM has been a valuable tool in planning and controlling complicated projects in management and engineering. The decision-maker can adopt a better strategy of optimizing the time or cost with the help of the critical path. This paper presents a new approach, which has not been proposed in the literature so far.

Fuzzy numbers can only be partially ordered, and hence cannot be compared. In decision-making, scheduling, market analysis with fuzzy uncertainties, the comparison of fuzzy numbers becomes essential. An approach has been developed to compare Trapezoidal Fuzzy Number (TrapFN), Heptagonal Fuzzy Number (HFN) and Hendecagonal Fuzzy Number (HDFN) to find the expected duration and total cost for completion of the agriculture project.

Zadeh [33] introduced an alternative way to deal with imprecise data to employ the idea of fuzziness in 1965, and Yager [31] developed the characterization of the extension principle. Different types of fuzzy sets are defined to clarify the vagueness of the existing problems. The fuzzy number is a fuzzy subset of the real line is defined by Dubois and Prade [8]. Detyniecki and Yagar [7] proposed the α-weighted valuations of fuzzy numbers. Thorani et al. [29] compared the various ranking methods for the fuzzy numbers with the possibility of ranking the crisp numbers. Huang and Huang [12] developed fuzzy aggregation evaluation based on some fuzzy and statistical techniques. Lu and Wang [15] improved the characteristic for the index of ranking fuzzy numbers.

Yao and Lin [32] presented various methods to calculate fuzzy completion time and Chanas and Zielinski [2] proposed the method for solving critical path analysis in the network with fuzzy activity times. Liang and Han [13] developed an algorithm that is presented to perform critical path analysis in a fuzzy environment, and Han et al. [11] applied the fuzzy critical path method to the airport's cargo ground operation systems. Also, Ravi Shankar et al. [21] used trapezoidal fuzzy numbers to rank the set of fuzzy numbers in fuzzy project work. Chanas and Kuchta [1] analyzed a generalized approach for multi-objective programming to optimize interval objective functions. Besides, Rathi and Balamohan ([19] and [20]) proposed a ranking of Heptagonal fuzzy number using value, and the ambiguity index also developed its arithmetic nature. Revathi and Valliathal ([23] and [25]) introduced hendecagonal fuzzy number and applied in fuzzy assignment problem to find the optimal solution using the penalty method. They [24] also developed hendecagonal fuzzy number with the similarity measure for pre-training analysis of student's placement training in a decision-making situation. Narayanamoorthy and Maheshwari [18] solved the fuzzy critical path method based on various ranking methods.

Chitra and Halder [6] developed project crashing time using a linear programming approach. Besides, Mazlum and Guneri [16] used business-oriented performance for CPM, PERT and Project Management in fuzzy nature. Zareei [34] used the critical path method in project scheduling to construct the biogas plants. The case study has also been carried out to solve vehicle route problems to minimize the total of routing costs and maximize customer services. Gul et al. [10] investigated the patient flow evaluation in an emergency room using fuzzy CPM and fuzzy PERT with the background of project management healthcare. This healthcare system provides the most quality service level, lower costs and limitless access. Takebira and Mohibullah [28] designed critical path method for making process layout of a T-Shirt approach within the earliest finish time.

A project is a series of tasks with an absolute beginning and ending, leading to a product. A project consists of numerous activities that must be carefully designed to achieve the desired stage and may take a long time to complete. Project management deals with preparation, decision, implementation, control, and finishing processes in the classes of a project. It consists of challenging demands for time, quality, scope, and cost. The main aim of CPM in project management is to reduce the overall manufacturing cost compared to the actual operating cost. An Agricultural process can be considered as a system that provides the most quality service level at a lower cost.

The paper is organized as: In Section 2, the purpose of studying fuzzy critical path method in agriculture project management has been discussed. Section 3 summarizes the notations, basic definitions, types of fuzzy numbers and their membership functions that are required for this study. In Section 4, arithmetic operations of hendecagonal fuzzy numbers have been proposed. In Section 5, the centroid method for ranking fuzzy numbers has been discussed. In Section 6, the fuzzy networking problem has been formulated. The proposed method has been applied to an agriculture project, and comparative analysis is done by considering fuzzy duration and fuzzy cost as three different fuzzy numbers. Section 7 presents the concluding remarks.

2. Need for FCPM in Agriculture Project

Apart from the natural calamity, the socio-economical changes also play a vital role in the agriculture sector. They directly or indirectly influence the yield of the agricultural product. The Agriculture sector is a major source of livelihood for 58output and productivity contribute to the overall economic development of the country. Therefore, it is crucial to analyze the agricultural process in a scientific approach.

Monjezi et al. [17] introduced the evaluation of a mechanized greenhouse construction project using CPM methods. In the several classifications of optimization, the information on the objective, constraints and impact of possible individual opinion is often imprecise. In the case of a simple system, the probabilistic approach may be used, which is inadequate in real-time situations. Therefore, it is necessary to provide a unique mathematical technique, which deals with uncertainty. An extension of the classical set theory called fuzzy set theory is an implementation to consider uncertainty in human judgment. Comparing two or more fuzzy numbers and ranking such numbers is one of the most fundamental techniques. In this situation, the ranking index proposed by Cheng [4] for the centroid point $(\tilde{x}_0, \tilde{y}_0)$ can be used.

Liu and Xu [14] discussed the delivery of fresh agricultural products to maximize the total cost and to minimize the customer services under random fuzzy environment. Smith [27] developed agricultural project management for monitoring and control of activities involved in the project. Revathi and Saravanan [22] solved the networking problem with the idea of fuzzy critical path method using the ranking approach. Chen and Hsueh [5] used simple approach for project networks to solve fuzzy CPM problems with fuzzy activity time. The goal of this approach is to convert fuzzy CPM problem into crisp one that can be solved by the conventional procedure. Further, the complicated projects in real-world applications are managed by this approach that is suitable for other types of fuzzy numbers.

Based on the above review, the present work aims to propose an approach to systemize the agricultural process to minimize the overall cost by identifying critical activities.

CPM and PERT plays a significant role in the literature to identify critical activity. It is tough to meet the outflow of the complex project by deterministic approaches. The parameters involved in the agricultural process are mostly imprecise and uncertain. A fuzzy set is one of the best tools to handle the uncertainty in parameters. Therefore, a fuzzy version of the CPM method is preferred for the present study. Cost is one of the key factors that is directly associated with the performance and effectiveness of the agriculture project. Therefore, a suitable method is used for improving the performance of agriculture projects. Activities related to this project are identified, and the project network is drawn. The FCPM is proposed for analyzing agriculture project. The comparison of Trapezoidal fuzzy number, Heptagonal fuzzy number and Hendecagonal fuzzy number have been made to produce efficient output. The expected duration, total cost for completion of the project and the fuzzy critical path are determined.

3. Preliminaries

The mathematical notations are as follows:

$\oplus, \Theta, \otimes, \Phi$ - Addition and Subtraction of FNs.

$\tilde{A}$ - Fuzzy number.

$\tilde{T}$ - Trapezoidal fuzzy number.

$\tilde{H}$ - Heptagonal fuzzy number.

$\widetilde{HD}$ - Hendecagonal fuzzy number.

$|\tilde{A}^{\alpha}|$ - Length of the α-cut of $\tilde{A}$.

$[\tilde{T}^{\alpha}]$ -Alpha cut of the Trapezoidal fuzzy number.

$[\tilde{H}^{\alpha}]$ - Alpha cut of the Heptagonal fuzzy number.

$[\widetilde{HD}^{\alpha}]$ - Alpha cut of the Hendecagonal fuzzy number.

$[\widetilde{HD}]_{\text{Symm}}$ - Symmetric form of HDFN.

$\mu_{\tilde{A}}$ - Membership function of the fuzzy number $\tilde{A}$.

$\mu_{\tilde{T}}$ - Membership function of the fuzzy number $\tilde{T}$.

$\mu_{\tilde{H}}$ - Membership function of the fuzzy number $\tilde{H}$.

$\mu_{\widetilde{HD}}$ - Membership function of the fuzzy number $\widetilde{HD}$.

$(\tilde{x}_0, \tilde{y}_0)$ - Centroid of fuzzy numbers.

$f_A^L(x), f_A^R(x)$ - Left and right spread of the fuzzy number $\tilde{A}$.

$g_A^L(y), g_A^R(y)$ - Inverse functions of the fuzzy number $\tilde{A}$.

$R(\tilde{A})$ - Ranking value of fuzzy number $\tilde{A}$.

u, v, ω - Degree of confidence by the decision maker.

$\tilde{C}_{ij}$ -Fuzzy cost.

$\tilde{t}_{ij}$ -Fuzzy duration.

Definition 1. A fuzzy set is characterized by a membership function mapping the elements of a domain space or universe of discourse X to the unit interval $[0,1]$. (i.e.) $\mu_{\tilde{A}} : X \to [0,1]$.

Definition 2. An α-cut of a fuzzy set $\tilde{A}$ is a crispest $\tilde{A}^{\alpha}$ that contains all the elements of the universal set X that have a membership grade in $\tilde{A}$ greater or equal to specified value of α. Thus $\tilde{A}^{\alpha} = \{x \in X, \mu_{\tilde{A}}(x) \geq \alpha, 0 \leq \alpha \leq 1\}$.

Definition 3. A fuzzy set $\tilde{A}$ is defined on universal set of real numbers is said to be a generalized fuzzy number if its membership function has the following attributes, (i) For all $\alpha \in (0,1]$ α-sets $\tilde{A}^{\alpha}$ is a convex set, (ii) $\mu_{\tilde{A}}$ is an upper semi continuous function (iii) supp($\tilde{A}$) is a bounded set in R, (iv) The height of $\tilde{A} = \max_{x \in X} \mu_{\tilde{A}}(x) = \omega > 0$.

Definition 4. A fuzzy set $\tilde{A}$ is normal, if there exist at least $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$ otherwise, the fuzzy set is non-normal.

Definition 5. The membership function for the generalized (non-normal) Trapezoidal Fuzzy Number $\tilde{T} = (a, b, c, d; \omega)$ is defined as follows:

$$\mu_{\tilde{T}}(x) = \begin{cases} \omega\left(\frac{x-a}{b-a}\right), & a \leq x \leq b \\ \omega, & b \leq x \leq c \\ \omega\left(\frac{d-x}{d-c}\right), & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases} \qquad \text{where } 0 < \omega \leq 1.$$

Definition 6. The membership function for the generalized (non-normal) Heptagonal Fuzzy Number $\tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7; v, \omega)$ is defined as follows:

$$\mu_{\tilde{H}}(x) = \begin{cases} 0, & \text{for } x < h_1 \\ v\left(\frac{x-h_1}{h_2-h_1}\right), & \text{for } h_1 \leq x \leq h_2 \\ v, & \text{for } h_2 \leq x \leq h_3 \\ v + (\omega - v)\left(\frac{x-h_3}{h_4-h_3}\right), & \text{for } h_3 \leq x \leq h_4 \\ v + (\omega - v)\left(\frac{h_5-x}{h_5-h_4}\right), & \text{for } h_4 \leq x \leq h_5 \\ v, & \text{for } h_5 \leq x \leq h_6 \\ v\left(\frac{h_7-x}{h_7-h_6}\right), & \text{for } h_6 \leq x \leq h_7 \\ 0, & \text{for } x \geq h_7 \end{cases} \qquad \text{where } 0 < v \leq \omega \leq 1.$$

Definition 7. The membership function for the generalized (non-normal) Hendecagonal Fuzzy Number $\widetilde{HD} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}; u, v, \omega)$ is defined as follows:

$$\mu_{\widetilde{HD}}(x) = \begin{cases} u\left(\frac{x-h_1}{h_2-h_1}\right), & \text{for } h_1 \le x \le h_2 \\ u, & \text{for } h_2 \le x \le h_3 \\ u+(v-u)\left(\frac{x-h_3}{h_4-h_3}\right), & \text{for } h_3 \le x \le h_4 \\ v, & \text{for } h_4 \le x \le h_5 \\ v+(\omega-v)\left(\frac{x-h_5}{h_6-h_5}\right), & \text{for } h_5 \le x \le h_6 \\ v+(\omega-v)\left(\frac{h_7-x}{h_7-h_6}\right), & \text{for } h_6 \le x \le h_7 \\ v, & \text{for } h_7 \le x \le h_8 \\ u+(v-u)\left(\frac{h_9-x}{h_9-h_8}\right), & \text{for } h_8 \le x \le h_9 \\ u, & \text{for } h_9 \le x \le h_{10} \\ u\left(\frac{h_{11}-x}{h_{11}-h_{10}}\right), & \text{for } h_{10} \le x \le h_{11} \\ 0, & \text{otherwise} \end{cases} \quad \text{where } 0 < u < v \le \omega \le 1.$$

Graphical representation of Hendecagonal fuzzy number is as follows:

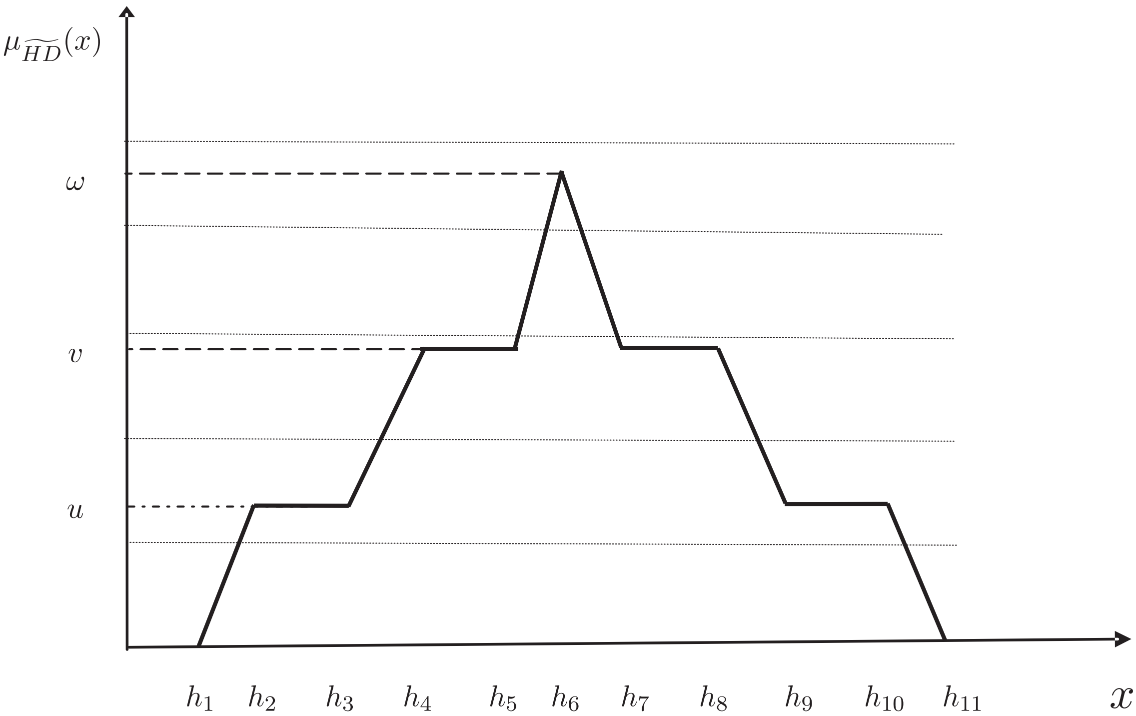

Figure 1: Graphical representation of Hendecagonal fuzzy number.

Remark 1.

(i) If $u = v = 0$ then the Hendecagonal fuzzy number reduced to Triangular fuzzy number.

(ii) If $u = v = \omega$ then the Hendecagonal fuzzy number reduced to Trapezoidal fuzzy number.

(iii) If $u = v$ then the Hendecagonal fuzzy number reduced to Heptagonal fuzzy number.

Remark 2. In some cases, the membership function of the HDFN structure is complex it reflects the arithmetic operations to be complicated. The Symmetric HDFN (SymmHDFNs) decreases the complexity both in computation and representation

The SymmHDFN is defined as $\widetilde{HD}_{\text{symm}} = (\tilde{h}; u, v, \omega)_\beta$ where the ordinates are given by $h_i = h + (i-6)\beta$ for $i = 1$ to 11 and β denotes the equal space between the ordinates.

4. Arithmetic Operations of HDFNs

Chen SH's method [3] is used to develop arithmetic operations between HDFNs. Let us consider two HDFNs $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}; u_A, v_A, \omega_A)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}; u_B, v_B, \omega_B)$ then the arithmetic operations of HDFNs based on the function principle is a point wise operation and is defined as follows:

(i) Addition of two HDFNs $\tilde{A}$ and $\tilde{B}$:

$$\begin{aligned}\tilde{A} \oplus \tilde{B} = &(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8, a_9 + b_9,\\ &a_{10} + b_{10}, a_{11} + b_{11}; \min\{u_A, u_B\}, \min\{v_A, v_B\}, \min\{\omega_A, \omega_B\})\end{aligned}$$

(ii) Scalar Multiplication non-normal HDFN:

$$\lambda\tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5, \lambda a_6, \lambda a_7, \lambda a_8, \lambda a_9, \lambda a_{10}, \lambda a_{11}; u_A, v_A, \omega_A) & \text{if } \lambda > 0,\\ (\lambda a_{11}, \lambda a_{10}, \lambda a_9, \lambda a_8, \lambda a_7, \lambda a_6, \lambda a_5, \lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; u_A, v_A, \omega_A) & \text{if } \lambda < 0.\end{cases}$$

(iii) Subtraction of two HDFNs $\tilde{A}$ and $\tilde{B}$:

$$\begin{aligned}\tilde{A}\Theta\tilde{B} = \tilde{A} + (-\tilde{B}) = &(a_1 - b_{11}, a_2 - b_{10}, a_3 - b_9, a_4 - b_8, a_5 - b_7, a_6 - b_6, a_7 - b_5, a_8 - b_4,\\ &a_9 - b_3, a_{10} - b_2, a_{11} - b_1; \min\{u_A, u_B\}, \min\{v_A, v_B\}, \min\{\omega_A, \omega_B\}).\end{aligned}$$

Example 1. Let the HDFNs $\tilde{A} = (2, 5, 7, 9, 10, 13, 15, 17, 20, 23, 26; 0.3, 0.7, 0.9)$ and $\tilde{B} = (5, 8, 10, 12, 16, 19, 22, 25, 28, 30, 32; 0.4, 0.6, 0.8)$ is considered.

(i) The addition of the HDFNs and the graph of its membership function is shown in Figure 2

$$\begin{aligned}\tilde{A} \oplus \tilde{B} &= (7, 13, 17, 21, 26, 32, 37, 42, 48, 53, 58; \min\{0.3, 0.4\}, \min\{0.7, 0.6\}, \min\{0.9, 0.8\})\\ &= (7, 13, 17, 21, 26, 32, 37, 42, 48, 53, 58; 0.3, 0.6, 0.8).\end{aligned}$$

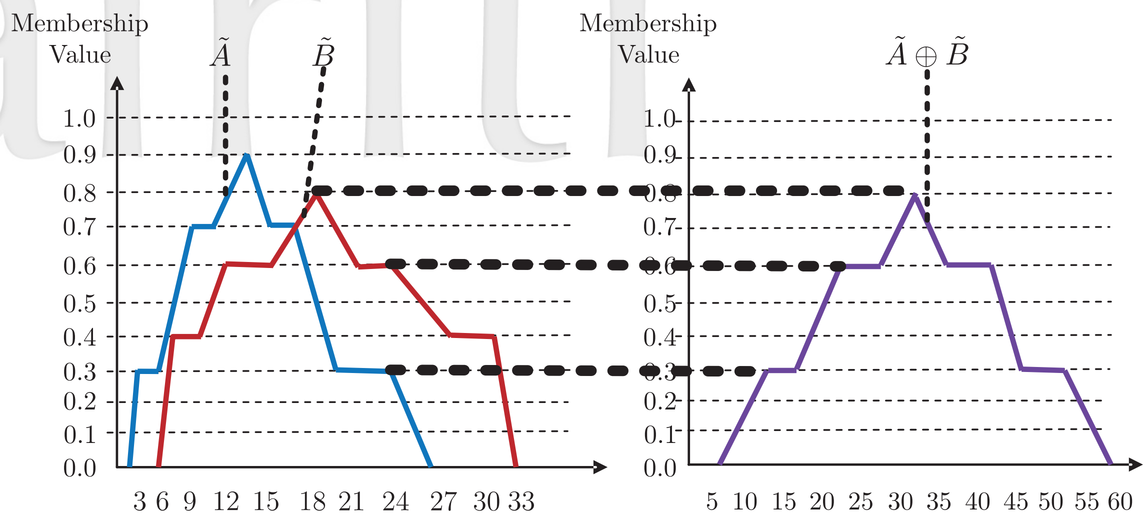

Figure 2: Addition of two HDFNs.

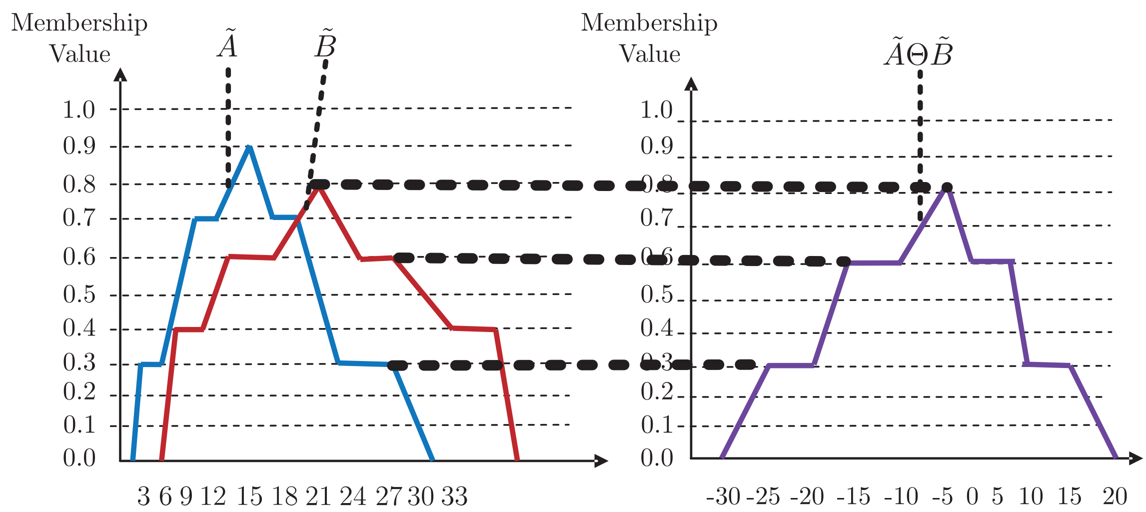

Figure 3: Subtraction of two HDFNs.

(ii) The subtraction of the HDFNs and the graph of its membership function is shown in Figure 3

$$\tilde{A}\Theta\tilde{B} = (-30, -25, -21, -16, -12, -6, -1, 5, 10, 15, 21; 0.3, 0.6, 0.8).$$

5. Centroid Method for Fuzzy Numbers

A pair of centroid formulae $(\tilde{x}_0, \tilde{y}_0)$ suggested by Wang et al. [30] for TrapFN say $\tilde{T}$

satisfying two fundamental properties of exact centroid formulae, where

$$\tilde{x}_0(\tilde{T}) = \frac{\int_{-\infty}^{\infty} x\mu_{\tilde{T}}(x)dx}{\int_{-\infty}^{\infty} \mu_{\tilde{T}}(x)dx} = \frac{\int_a^b xf_A^L(x)dx + \int_b^c \omega x dx + \int_c^d xf_A^R(x)dx}{\int_a^b f_A^L(x)dx + \int_b^c \omega dx + \int_c^d f_A^R(x)dx}$$

and

$$\tilde{y}_0(\tilde{T}) = \frac{\int_0^{\omega} y(g_A^R(y) - g_A^L(y))dy}{\int_0^{\omega} (g_A^R(y) - g_A^L(y))dy}. \tag{5.1}$$

Where $\mu_{\tilde{T}}(x)$ is the membership function of the fuzzy number $\tilde{T}$, $f_A^L(x)$, $f_A^R(x)$ are left and right spreads of $\tilde{T}$, and $g_A^L(y), g_A^R(y)$ are their inverse functions. In 2007, Shieh [26] showed a considerable development on the centroid formulae and proposed the following formulae

$$\tilde{x}_0(\tilde{A}) = \frac{\int_{-\infty}^{\infty} x\mu_{\tilde{A}}(x)dx}{\int_{-\infty}^{\infty} \mu_{\tilde{A}}(x)dx} \quad \text{and} \quad \tilde{y}_0(\tilde{A}) = \frac{\int_0^{\omega} \alpha|\tilde{A}^{\alpha}|d\alpha}{\int_0^{\omega} |\tilde{A}^{\alpha}|d\alpha} \tag{5.2}$$

where $\tilde{A}$ is any fuzzy number with $\sup_{x\in R} \tilde{A}(x) = \omega$, $|\tilde{A}^{\alpha}|$ is the length of the α-cut of $\tilde{A}$, $0 < \alpha \leq 1$. The centroid formulae suggested by Shieh [26] is more significant, flexible and therefore the formulae in (5.2) are extended to present a pair of centroid formulae for TrapFN, HFN and HDFN.

Definition 8. The ranking function for generalized fuzzy number $\tilde{A}$ maps the set of all fuzzy number to a set of real numbers. The ranking index proposed by Cheng [4] for the centroid point $(\tilde{x}_0, \tilde{y}_0)$ is defined as $R(\tilde{A}) = \sqrt{\tilde{x}_0^2 + \tilde{y}_0^2}$, where the natural order exists, i.e.

$$\begin{aligned}
&\text{(i)} \quad \tilde{A} > \tilde{B} \quad \text{iff } R(\tilde{A}) > R(\tilde{B}),\\
&\text{(ii)} \quad \tilde{A} < \tilde{B} \quad \text{iff } R(\tilde{A}) < R(\tilde{B}),\\
&\text{(iii)} \quad \tilde{A} = \tilde{B} \quad \text{iff } R(\tilde{A}) = R(\tilde{B}).
\end{aligned}$$

Definition 9. Let $\tilde{T} = (a, b, c, d; \omega)$ be a TrapFN having the membership function as in Definition 5. The pair of centroid formula for the TrapFN $\tilde{T}$ is defined as follows:

$$\tilde{x}_0(\tilde{T}) = \frac{\int_{-\infty}^{\infty} x\mu_{\tilde{T}}(x)dx}{\int_{-\infty}^{\infty} \mu_{\tilde{T}}(x)dx} = \frac{\int_a^b xf(x)dx + \int_b^c \omega x dx + \int_c^d xg(x)dx}{\int_a^b f(x)dx + \int_b^c \omega dx + \int_c^d g(x)dx}$$

and

$$\tilde{y}_0(\tilde{T}) = \frac{\int_0^{\omega} \alpha|T^{\alpha}|d\alpha}{\int_0^{\omega} |T^{\alpha}|d\alpha} = \frac{\int_0^{\omega} \alpha[T_{\alpha}^R - T_{\alpha}^L]d\alpha}{\int_0^{\omega} [T_{\alpha}^R - T_{\alpha}^L]d\alpha}.$$

On simplification, the pair of centroid formulae $(\tilde{x}_0, \tilde{y}_0)$ for the TrapFN obtained as

$$\tilde{x}_0(\tilde{T}) = \frac{1}{3}\left[a + b + c + d - \frac{(cd - ab)}{(c+d) - (a+b)}\right] \text{ and } \tilde{y}_0(\tilde{T}) = \frac{\omega}{3}\left[1 + \frac{c - b}{(c+d) - (a+b)}\right].$$

Definition 10. Let $\tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7; v, \omega)$ be a HFN having the membership function as in Definition 6. The pair of centroid formula for the HFN $\tilde{H}$ is defined as follows:

$$\begin{aligned}
\tilde{x}_0(\tilde{H}) &= \frac{\int_{-\infty}^{\infty} x\mu_{\tilde{H}}(x)dx}{\int_{-\infty}^{\infty} \mu_{\tilde{H}}(x)dx} \\
&= \frac{\int_{h_1}^{h_2} xp_1(x)dx + \int_{h_2}^{h_3} vxdx + \int_{h_3}^{h_4} xq_1(x)dx + \int_{h_4}^{h_5} xq_2(x)dx + \int_{h_5}^{h_6} vxdx + \int_{h_6}^{h_7} xp_2(x)dx}{\int_{h_1}^{h_2} p_1(x)dx + \int_{h_2}^{h_3} vdx + \int_{h_3}^{h_4} q_1(x)dx + \int_{h_4}^{h_5} q_2(x)dx + \int_{h_5}^{h_6} vdx + \int_{h_6}^{h_7} p_2(x)dx}
\end{aligned}$$

and

$$\tilde{y}_0(\tilde{H}) = \frac{\int_0^{\omega} \alpha|\tilde{H}^{\alpha}|d\alpha}{\int_0^{\omega} |\tilde{H}^{\alpha}|d\alpha} = \frac{\int_0^{v} \alpha[H_{1\alpha}^R - H_{1\alpha}^L]d\alpha + \int_v^{\omega} \alpha[H_{2\alpha}^R - H_{2\alpha}^L]d\alpha}{\int_0^{v} [H_{1\alpha}^R - H_{1\alpha}^L]d\alpha + \int_v^{\omega} [H_{2\alpha}^R - H_{2\alpha}^L]d\alpha}.$$

On simplification, the pair of centroid formulae $(\tilde{x}_0, \tilde{y}_0)$ for the HFN $\tilde{H}$ is obtained as

$$\begin{aligned}
\tilde{x}_0(\tilde{H}) &= \frac{1}{3}\Big[\frac{v((h_7^2 + h_6^2 + h_6h_7) - (h_1^2 + h_2^2 + h_1h_2)) + (\omega - v)(h_5 - h_3)(h_3 + h_4 + h_5)}{v(h_7 - h_1 + h_6 - h_2) + (\omega - v)(h_5 - h_3)}\Big] \\
\tilde{y}_0(\tilde{H}) &= \frac{1}{3}\Big[\frac{v^2(h_7 - h_1) + 2v^2(h_6 - h_2) + (\omega - v)(\omega + 2v)(h_5 - h_3)}{v(h_7 - h_1 + h_6 - h_2) + (\omega - v)(h_5 - h_3)}\Big].
\end{aligned}$$

Definition 11. Let $\tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}; u, v, \omega)$ be a HDFN having the membership function as in Definition 7. The pair of centroid formula for the HDFN $\widetilde{HD}$ is

$$\begin{aligned}
\tilde{x}_0(\widetilde{HD}) &= \frac{\int_{-\infty}^{\infty} x\mu_{\widetilde{HD}}(x)dx}{\int_{-\infty}^{\infty} \mu_{\widetilde{HD}}(x)dx} \\
&= \Big[\int_{h_1}^{h_2} xp_1(x)dx + \int_{h_2}^{h_3} uxdx + \int_{h_3}^{h_4} xq_1(x)dx + \int_{h_4}^{h_5} vxdx + \int_{h_5}^{h_6} xr_1(x)dx \\
&\quad + \int_{h_6}^{h_7} xr_2(x)dx + \int_{h_7}^{h_8} vxdx + \int_{h_8}^{h_9} xq_2(x)dx + \int_{h_9}^{h_{10}} uxdx + \int_{h_{10}}^{h_{11}} xp_2(x)dx\Big] \\
&\quad \Big/ \Big[\int_{h_1}^{h_2} p_1(x)dx + \int_{h_2}^{h_3} udx + \int_{h_3}^{h_4} q_1(x)dx + \int_{h_4}^{h_5} vdx + \int_{h_5}^{h_6} r_1(x)dx \\
&\quad + \int_{h_6}^{h_7} r_2(x)dx + \int_{h_7}^{h_8} vdx + \int_{h_8}^{h_9} q_2(x)dx + \int_{h_9}^{h_{10}} udx + \int_{h_{10}}^{h_{11}} p_2(x)dx\Big]
\end{aligned}$$

and

$$\begin{aligned}
\tilde{y}_0(\widetilde{HD}) &= \frac{\int_0^{\omega} \alpha|\widetilde{HD}^{\alpha}|d\alpha}{\int_0^{\omega} |\widetilde{HD}^{\alpha}|d\alpha} \\
&= \frac{\int_0^{u} \alpha[HD_{1\alpha}^R - HD_{1\alpha}^L]d\alpha + \int_u^{v} \alpha[HD_{2\alpha}^R - HD_{2\alpha}^L]d\alpha + \int_v^{\omega} \alpha[HD_{3\alpha}^R - HD_{3\alpha}^L]d\alpha}{\int_0^{u} [HD_{1\alpha}^R - HD_{1\alpha}^L]d\alpha + \int_u^{v} [HD_{2\alpha}^R - HD_{2\alpha}^L]d\alpha + \int_v^{\omega} [HD_{3\alpha}^R - H_{3\alpha}^L]d\alpha}.
\end{aligned}$$

On simplification, the pair of centroid formulae $(\tilde{x}_0, \tilde{y}_0)$ for the HDFN $\widetilde{HD}$ is obtained as

$$\begin{aligned}\tilde{x}_0(\widetilde{HD}) &= \frac{1}{3}\Big\{\Big[u[(h_{11}^2+h_{10}^2+h_{10}h_{11})-(h_1^2+h_2^2+h_1h_2)]+(\omega-v)(h_5+h_6+h_7)(h_7-h_5)\\ &\quad +(v-u)[(h_9^2+h_8^2+h_9h_8)-(h_3^2+h_4^2+h_3h_4)]\Big]\\ &\quad \Big/\Big[u(h_{11}-h_1+h_{10}-h_2)+(v-u)(h_9-h_3+h_8-h_4)+(\omega-v)(h_7-h_5)\Big]\Big\}\end{aligned}$$

and

$$\begin{aligned}\tilde{y}_0(\widetilde{HD}) &= \frac{1}{3}\Big\{\Big[u^2(h_{11}-h_1+h_{10}-h_2)+(\omega-v)(2v+\omega)(h_7-h_5)\\ &\quad +(v-u)[(v+2u)(h_9-h_3)+(u+2v)(h_8-h_4)]\Big]\\ &\quad \Big/\Big[u(h_{11}-h_1+h_{10}-h_2)+(v-u)(h_9-h_3+h_8-h_4)+(\omega-v)(h_7-h_5)\Big]\Big\}\end{aligned}$$

6. Fuzzy Networking Problem

The fuzzy networking problem is used to find the longest path of unit flow entering at the start node and terminating at the finish node. A fuzzy activity is said to be critical if a delay in start, it will cause a further delay in the completion of the entire project. The fuzzy network can be constructed by using fuzzy activities. Here $\tilde{t}_{ij}$ is fuzzy duration of activity (i, j) for all defined i and j.

To find fuzzy critical path, it is necessary to compute the value of earliest start (ES) and latest start (LS) for all activities (i, j) using the following formulae

$$ES_i = \max_i\{ES_i + \tilde{t}_{ij}\} \quad \text{and} \quad LS_i = \min_j\{LS_j - \tilde{t}_{ij}\}. \tag{6.1}$$

The fuzzy critical activities have been identified using the following conditions for all activities (i, j)

$$\text{(i) } ES_i = LS_i, \quad \text{(ii) } ES_j = LS_j, \quad \text{and (iiii) } ES_j - ES_i = LS_j - LS_i = \tilde{t}_{ij}. \tag{6.2}$$

6.1. Numerical example

This section presents a hypothetical project problem to demonstrate the computational process of fuzzy critical path analysis proposed in equation (6.1) and (6.2). The objective is to find the fuzzy critical path, expected fuzzy duration and total fuzzy cost for the completion of the project. For analysis purpose, five acres of land has been considered. There are so many factors involved in the agriculture project before cultivation to the end of the yielding. Here we considered the main factors as activities that include preparing the land for cultivation, checking the water facility, planting the saplings, fencing the land, using fertilizers and natural pesticides for better yielding, and finally watering the sapling till the end of cultivation. Due to calamity, everything is not exact (uncertain). So, both the cost and time of the activities are taken as fuzzy cost and fuzzy duration. Consider the agriculture project activities with predecessor

Table 1: Activities in the agriculture project.

Name of the activity	Activity description	Predecessor	Approximate duration of the activity -t (in days)	Approximate cost - c (in thousands) rupees
$A(1,3)$	Land preparation and soil testing	–	$12-17$	$40-70$
$B(1,2)$	Digging bore well in the marked land	–	$5-10$	$25-40$
$C(3,4)$	Plantation of seedlings	A	$12-17$	$20-45$
$D(2,4)$	Arrangement of water facilities for irrigation	B	$12-17$	$32-45$
$E(4,7)$	Fencing around the agriculture field and necessary action to protect from animal attacks	D, C	$67-72$	$60-90$
$F(2,5)$	Process of irrigation over the land	B	$12-17$	$35-45$
$G(3,6)$	Applying manure for field	C	$5-10$	$42-60$
$H(6,7)$	Protection from weeds, applying fertilizers and secure from natural disasters	G	$47-52$	$15-26$
$I(5,7)$	Yield of Harvesting	F	$54-59$	$65-90$

restriction, approximate duration, and approximate cost in Table 1. Its corresponding fuzzy duration and fuzzy cost are given in Table 2 and Table 3, respectively. The extra cost per day for the agriculture project is considered as Rs. 20. The given comparison improves the result in many real-time application problems (Revathi and Valliathal [25]). Also, the degree of confidence is based on the expert's opinion. So, the given comparison includes the primary, secondary and tertiary (new HDFN) level of fuzzy numbers for improving the result. The uncertainty cost and duration is represented as three different fuzzy numbers, namely TrapFN, HFN and HDFN.

Based on the ranking value of fuzzy duration, ES and LS are computed using equation (6.1), and the fuzzy critical path is identified using equation (6.2) as shown in Figure 4.

The listed in the agriculture project (Table 1 and Table 2) has been converted as project network and its critical path using duration as shown in Figure 4.

Table 2. The activities and their fuzzy duration (TrapFN, HFN and HDFN) with ranking value.

Activities	Approximate duration-t (in days)	Trapezoidal fuzzy duration	Heptagonal fuzzy duration	Hendecagonal fuzzy duration	Ranking value
$1-2$	$5-10$	$(5, 5.5, 9.5, 10; 0.9)$	$(5, 5.5, 7, 7.5, 8, 9.5, 10; 0.6, 0.9)$	$(5, 5.5, 6, 6.5, 7, 7.5, 8, 8.5, 9, 9.5, 10; 0.3, 0.6, 0.9)$	7.5
$1-3$	$12-17$	$(12, 12.5, 16.5, 17; 0.8)$	$(12, 12.5, 14, 14.5, 15, 16.5, 17; 0.4, 0.8)$	$(12, 12.5, 13, 13.5, 14, 14.5, 15, 15.5, 16, 16.5, 17; 0.1, 0.4, 0.8)$	14.5
$2-4$	$12-17$	$(12, 12.5, 16.5, 17; 0.7)$	$(12, 12.5, 14, 14.5, 15, 16.5, 17; 0.5, 0.7)$	$(12, 12.5, 13, 13.5, 14, 14.5, 15, 15.5, 16, 16.5, 17; 0.2, 0.5, 0.7)$	14.5
$2-5$	$12-17$	$(12, 12.5, 16.5, 17; 0.8)$	$(12, 12.5, 14, 14.5, 15, 16.5, 17; 0.6, 0.8)$	$(12, 12.5, 13, 13.5, 14, 14.5, 15, 15.5, 16, 16.5, 17; 0.3, 0.6, 0.8)$	14.5
$3-4$	$12-17$	$(12, 12.5, 16.5, 17; 0.8)$	$(12, 12.5, 14, 14.5, 15, 16.5, 17; 0.6, 0.8)$	$(12, 12.5, 13, 13.5, 14, 14.5, 15, 15.5, 16, 16.5, 17; 0.2, 0.6, 0.8)$	14.5
$3-6$	$5-10$	$(5, 5.5, 9.5, 10; 0.7)$	$(5, 5.5, 7, 7.5, 8, 9.5, 10; 0.5, 0.7)$	$(5, 5.5, 6, 6.5, 7, 7.5, 8, 8.5, 9, 9.5, 10; 0.3, 0.5, 0.7)$	7.5
$4-7$	$67-72$	$(67, 67.5, 71.5, 72; 0.9)$	$(67, 67.5, 69, 69.5, 70, 71.5, 72; 0.4, 0.9)$	$(67, 67.5, 68, 68.5, 69, 69.5, 70, 70.5, 71, 71.5, 72; 0.1, 0.4, 0.9)$	69.5
$5-7$	$54-59$	$(54, 54.5, 58.5, 59; 1)$	$(54, 54.5, 56, 56.5, 57, 58.5, 59; 0.3, 0.6, 1)$	$(54, 54.5, 55, 55.5, 56, 56.5, 57, 57.5, 58, 58.5, 59; 0.3, 0.6, 1)$	56.5
$6-7$	$47-52$	$(47, 47.5, 51.5, 52; 0.9)$	$(47, 47.5, 49, 49.5, 50, 51.5, 52; 0.6, 0.9)$	$(47, 47.5, 48, 48.5, 49, 49.5, 50, 50.5, 51, 51.5, 52; 0.3, 0.6, 0.9)$	49.5

Table 3: The activities and their fuzzy costs (TrapFN, HFN and HDFN).

Activities	Approximate cost (in thousands) rupees	Trapezoidal fuzzy cost	Heptagonal fuzzy cost	Hendecagonal fuzzy cost
$1-2$	$25-40$	$(25, 26, 38, 40; 0.9)$	$(25, 26, 29, 30, 31, 38, 40; 0.6, 0.9)$	$(25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 40; 0.3, 0.6, 0.9)$
$1-3$	$40-70$	$(40, 42, 69, 70; 0.8)$	$(40, 42, 49, 53, 56, 69, 70; 0.4, 0.8)$	$(40, 42, 44, 47, 49, 53, 56, 59, 63, 69, 70; 0.1, 0.4, 0.8)$
$2-4$	$32-45$	$(32, 34, 43, 45; 0.7)$	$(32, 34, 37, 38, 39, 43, 45; 0.5, 0.7)$	$(32, 34, 35, 36, 37, 38, 39, 41, 42, 43, 45; 0.2, 0.5, 0.7)$
$2-5$	$35-45$	$(35, 36, 44, 45; 0.8)$	$(35, 36, 39, 40, 41, 44, 45; 0.6, 0.8)$	$(35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45; 0.3, 0.6, 0.8)$
$3-4$	$20-45$	$(20, 21, 43, 45; 0.8)$	$(20, 21, 29, 30, 32, 43, 45; 0.6, 0.8)$	$(20, 21, 24, 26, 29, 30, 32, 36, 39, 43, 45; 0.2, 0.6, 0.8)$
$3-6$	$42-60$	$(42, 44, 58, 60; 0.7)$	$(42, 44, 50, 52, 54, 58, 60; 0.5, 0.7)$	$(42, 44, 46, 48, 50, 52, 54, 56, 57, 58, 60; 0.3, 0.5, 0.7)$
$4-7$	$60-90$	$(60, 61, 89, 90; 0.9)$	$(60, 61, 70, 75, 81, 89, 90; 0.4, 0.9)$	$(60, 61, 63, 66, 70, 75, 81, 86, 87, 89, 90; 0.1, 0.4, 0.9)$
$5-7$	$65-90$	$(65, 67, 88, 90; 1)$	$(65, 67, 75, 78, 81, 88, 90; 0.6, 1)$	$(65, 67, 69, 72, 75, 78, 81, 83, 86, 88, 90; 0.3, 0.6, 1)$
$6-7$	$15-26$	$(15, 16, 25, 26; 0.9)$	$(15, 16, 20, 21, 22, 25, 26; 0.6, 0.9)$	$(15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26; 0.3, 0.6, 0.9)$

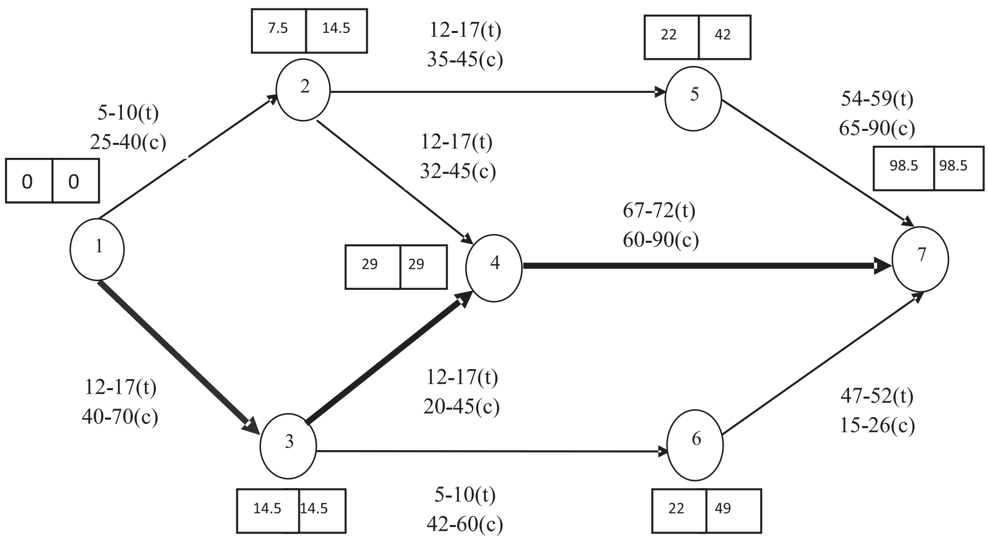

Figure 4: Network diagram with critical path using fuzzy duration.

Table 4: The centroid value and ranking value of Trapezoidal fuzzy number.

Activity	Fuzzy cost	$\tilde{x}_0$	$\tilde{y}_0$	$R(\tilde{T})$
(1,2)	(25,26,38,40;0.9)	32.259	0.433	32.5
(1,3)	(40,42,69,70;0.8)	55.246	0.393	55.4
(2,4)	(32,34,43,45;0.7)	38.5	0.329	38.6
(2,5)	(35,36,44,45;0.8)	40	0.385	40.2
(3,4)	(20,21,43,45;0.8)	32.255	0.392	32.4
(3,6)	(42,44,58,60;0.7)	51	0.383	51.2
(4,7)	(60,61,89,90;0.9)	75	0.445	75.2
(5,7)	(65,67,88,90;1)	77.5	0.485	77.7
(6,7)	(15,16,25,26;0.9)	20.5	0.435	20.7

The centroid value and ranking values for the cost of TrapFN, HFN and HDFN are computed and listed in Table 4, Table 5 and Table 6, respectively.

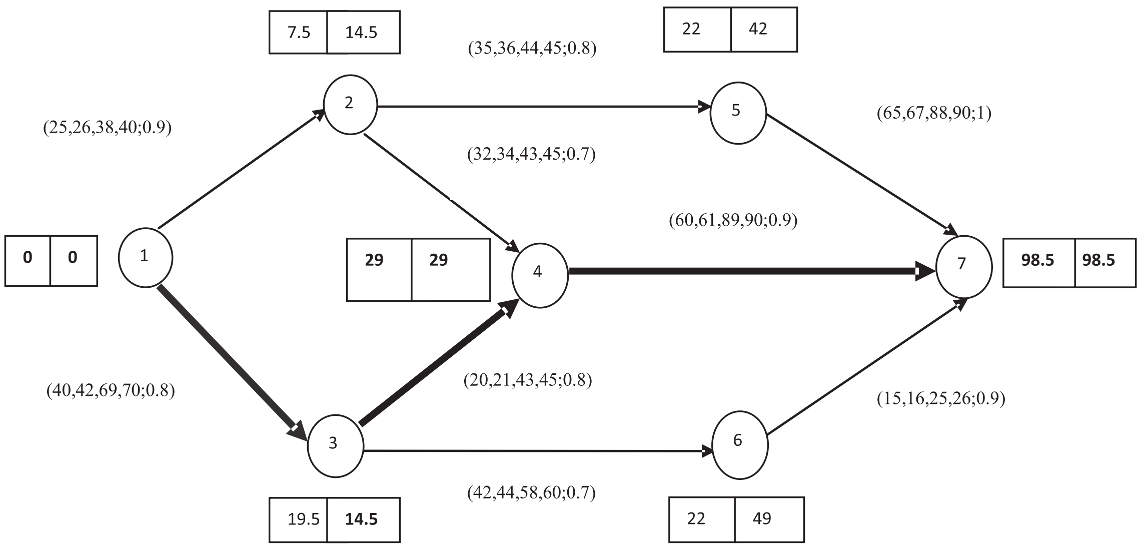

Figure 5: Network diagram for Trapezoidal fuzzy cost with critical path.

The critical path is $1 \to 3 \to 4 \to 7$ and project duration is 98.5 days with indirect cost is Rs. 1,970. The cost associated to critical activities for Trapezoidal fuzzy cost is (120,124,201,205; 0.8), and its ranking value is 163 (Figure 5 and Table 4).

Direct cost is = 423.9 = Rs. 4,23,900.

The associated total cost for the agriculture project using trapezoidal fuzzy cost is $=$ Rs. $4,23,900+$ Rs. $1,970 =$ Rs. 4,25,870.

Table 5: The centroid value and ranking value of Heptagonal fuzzy number.

Activity	Fuzzy cost	$\tilde{x}_0$	$\tilde{y}_0$	$R(\tilde{H})$
(1,2)	(25,26,29,30,31,38,40;0.6,0.9)	32.179	0.304	32.3
(1,3)	(40,42,49,53,56,69,70;0.4,0.8)	54.964	0.233	55
(2,4)	(32,34,37,38,39,43,45;0.5,0.7)	38.482	0.246	38.5
(2,5)	(35,36,39,40,41,44,45;0.6,0.8)	40	0.302	40.1
(3,4)	(20,21,29,30,32,43,45;0.6,0.8)	32.215	0.301	32.3
(3,6)	(42,44,50,52,54,58,60;0.5,0.7)	51.048	0.255	51.1
(4,7)	(60,61,70,75,81,89,90;0.4,0.9)	75.064	0.268	75.1
(5,7)	(65,67,75,78,81,88,90;0.6,1)	77.54	0.327	77.7
(6,7)	(15,16,20,21,22,25,26;0.6,0.9)	20.524	0.31	20.6

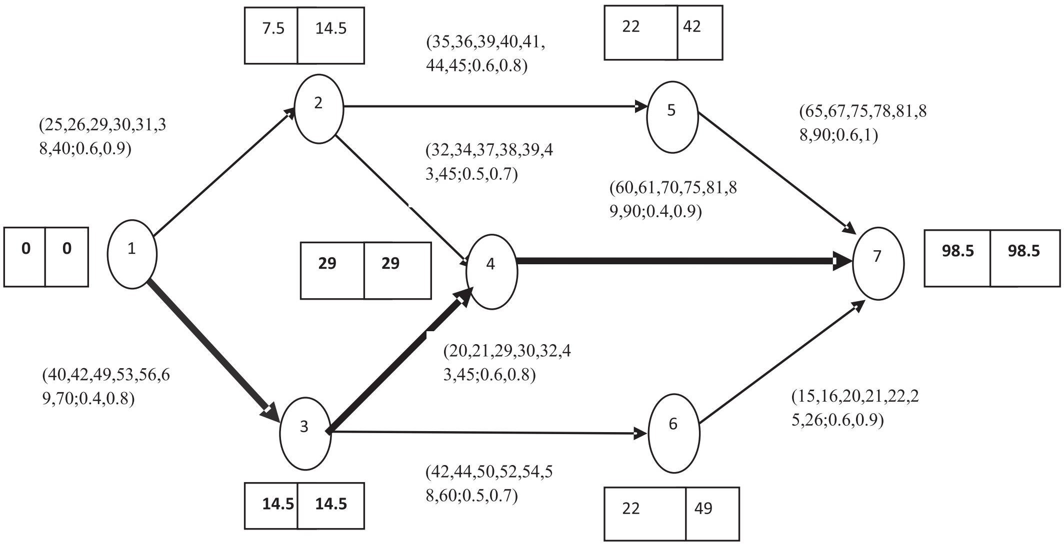

Figure 6: Network diagram with Heptagonal fuzzy cost.

The critical path is $1 \to 3 \to 4 \to 7$ and project duration is 98.5 days with indirect cost is Rs. 1,970. The cost associated to critical activities for Heptagonal fuzzy cost is (120,124,148,158,169,201,205;0.4,0.8), and its ranking value is 162.4 (Figure 6 and Table 5).

Direct cost $= 422.7 \times 1000 =$ Rs. 4,22,700

The associated total cost for the agriculture project using heptagonal fuzzy cost is $=$ Rs. $4,22,700+$ Rs. $1,970 =$ Rs. 4,24,670.

The critical path is $1 \to 3 \to 4 \to 7$ and project duration is 98.5 days with indirect cost is Rs. 1,970. The cost associated to critical activities for Hendecagonal fuzzy cost is (120, 124,131,139,148,158,169,181,189,201,205; 0.1, 0.4, 0.8), and its ranking value is 161.1 (Figure 7 and Table 6).

Direct cost $= 420.8 \times 1000 =$ Rs. 4,20,800.

Table 6: The centroid value and ranking value of Hendecagonal fuzzy number.

Activity	Fuzzy cost	$\tilde{x}_0$	$\tilde{y}_0$	$R(\widetilde{HD})$
(1,2)	(25,26,27,28,29,30,31,32,33,38,40;0.3,0.6,0.9)	31.564	0.223	31.6
(1,3)	(40,42,44,47,49,53,56,59,63,69,70;0.1,0.4,0.8)	53.807	0.231	53.9
(2,4)	(32,34,35,36,37,38,39,41,42,43,45;0.2,0.5,0.7)	38.476	0.215	38.5
(2,5)	(35,36,37,38,39,40,41,42,43,44,45;0.3,0.6,0.8)	40	0.248	40.1
(3,4)	(20,21,24,26,29,30,32,36,39,43,45;0.2,0.6,0.8)	31.703	0.258	31.8
(3,6)	(42,44,46,48,50,52,54,56,57,58,60;0.3,0.5,0.7)	51.254	0.208	51.3
(4,7)	(60,61,63,66,70,75,81,86,87,89,90;0.1,0.4,0.9)	75.336	0.272	75.4
(5,7)	(65,67,69,72,75,78,81,83,86,88,90;0.3,0.6,1)	77.549	0.285	77.6
(6,7)	(15,16,17,18,20,21,22,23,24,25,26;0.3,0.6,0.9)	20.529	0.262	20.6

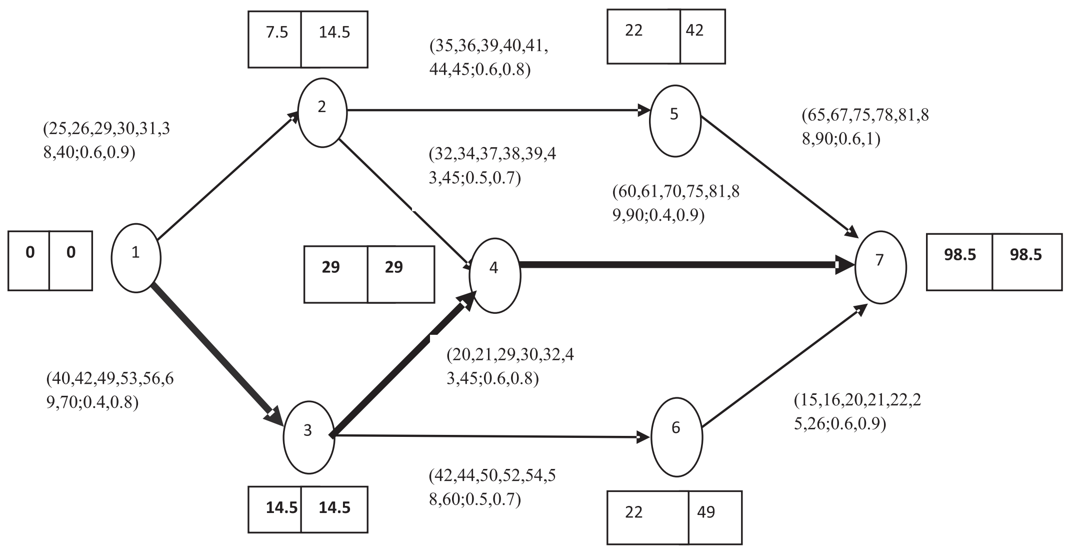

Figure 7: Network diagram for Hendecagonal fuzzy cost with critical path.

The associated total cost for the agriculture project using hendecagonal fuzzy cost is = Rs. $4,20,800+$ Rs. $1,970 =$ Rs. 4,22,770.

The comparison of the result is given in the following table and graph (Figure 8).

The critical path obtained is same in all the three cases; however, the total cost for the agriculture project varies for all the cases. Based on this the fuzzy cost ranking value, HDFN is minimum. So, HDFN is suitable for handling the agriculture project.

7. Conclusion

In this study, a fuzzy version of CPM is applied, and an agriculture project is developed. The profitable strategy has become unavoidable in the agriculture division. Therefore, the main aim is to minimize the cost of this agriculture project. The activ-

Table 7: Agriculture project fuzzy cost analysis.

Fuzzy Number	Critical Path	Expected Duration	Critical Activity Ranking Value (Cost)	Total Cost
TrapFN	$1 \to 3 \to 4 \to 7$	98.5	163	Rs. 4,25,870
HFN	$1 \to 3 \to 4 \to 7$	98.5	162.4	Rs. 4,24,670
HDFN	$1 \to 3 \to 4 \to 7$	98.5	161.1	Rs.4,22,770

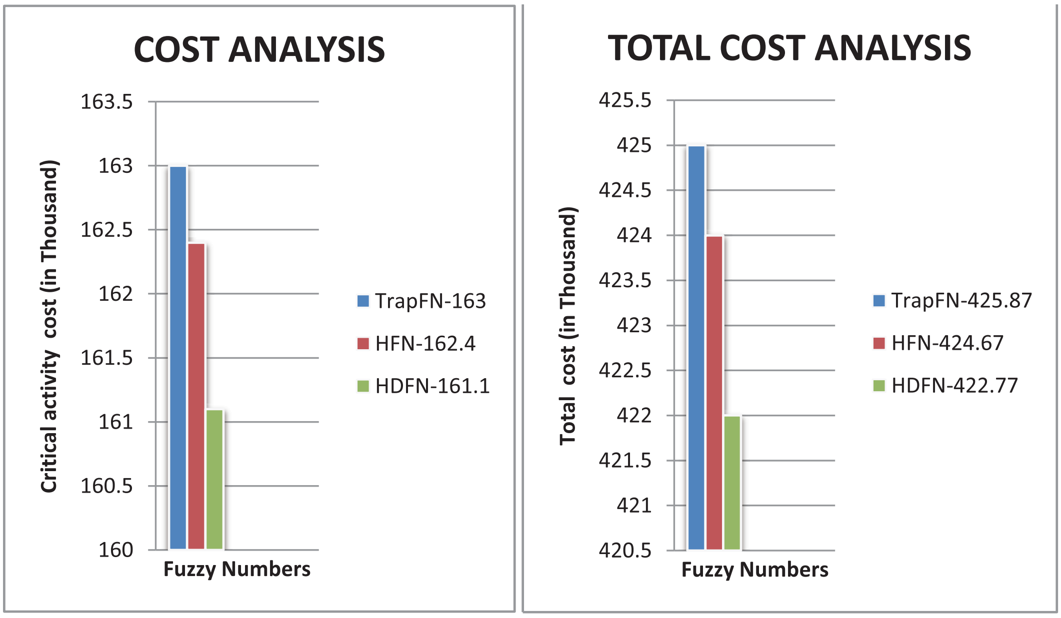

Figure 8: Comparison of cost for various fuzzy numbers.

ities of the proposed method are taken as a fuzzy duration and fuzzy cost. Using the activities, the project network is drawn. In this analysis, the cost of each activity is taken using various fuzzy numbers. The expected fuzzy duration for the completion of the project, total cost and the fuzzy critical path is computed for all the three cases. Based on this analysis, the centroid ranking value of critical activity cost and total cost for HDFN is minimum. Therefore, the proposed method is suitable and more effective for handling the agriculture project using HDFN. In future, the Symmetric form of fuzzy numbers can be developed to reduce the complexity of representation and computation for HFN and HDFN.

Declaration of interest

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of this article.

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