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A Multi-criteria Group Decision Making Method and Its Applications Based on Improved Intuitionistic Fuzzy Entropy and Information Integration Operator

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Abstract

In order to solve the problem of group decision making in which the criteria weight and the weight of decision makers are completely unknown in the intuitive fuzzy environment, a multi-criteria group decision method based on improved intuitionistic fuzzy entropy and information integration operator is proposed. Firstly, using the intuitive fuzzy numbers, language variable method and quantitative index value conversion formula, the initial evaluation criteria for decision makers are normalized. Secondly, an improved intuitive fuzzy entropy is introduced to determine the criteria weight and the weight of decision makers. Thirdly, using the intuitionistic fuzzy weight average operator and the intuitionistic fuzzy ordered weighted average operator, integrate the intuitive fuzzy decision matrix, and the improved score function is used to optimize the ranking of options. The feasibility of this method is proved theoretically, and the effectiveness of the method is verified by an example.

Keywords: Multi-criteria group decision making, intuitionistic fuzzy entropy, criteria weights, decision maker weights, score function.

1. Introduction

Multi-criteria group decision making refers to the process of choosing best scheme by integrating evaluation criteria and intuitionistic fuzzy information given by decision makers. In recently years, intuitionistic fuzzy multi-criteria group decision making theories and methods have been widely used in the fields of program evaluation, decision analysis, and pattern recognition (see Geng et al. [6]). In actual decision making process, decision makers are difficult to accurately express evaluation information due to time pressure and insufficient understanding of the problem, and are often more accustomed to use intuitionistic fuzzy sets instead of exact values or language variables (see Zhao et al. [34]). In multi-criteria group decision making, how to better determine attribute weights, decision maker weights, information aggregation, and decision making methods are the key to solve such problems. Scholars have conducted extensive researches and obtained rich results. The first is attribute weights determination method, such as Wan et al. [21] proposed two-objective linear programming model; Chen [2] constructed a linear programming model by consistency coefficient; Jin et al. [13] proposed interval intuitionistic fuzzy entropy method. The second is objective weights determination of decision makers, such as Wan et al. [22] proposed similarly weights determination method; Zhang et al. [32] proposed objective programming model method. The third is information aggregation, such as Hashemi et al. [8] proposed interval intuitionistic fuzzy set weighted average operator; Meng et al. [17] proposed arithmetic interval intuitionistic fuzzy number-Choquet aggregation operator; Further more, Zhou et al. [36] proposed continuous interval intuitionistic fuzzy number aggregation operator; Chen et al. [3] proposed decision information aggregation method based on evidence theory. The fourth is decision making method, such as Chen et al. [2] proposed interval intuitionistic fuzzy number QUALIFLEX group decision method; Wang et al. [25] and Wu et al. [27] proposed group decision method based on score function and accuracy function; Jiang [12] and Zhong [35] proposed group decision method based on evidence theory; Li et al. [15] proposed intuitionistic fuzzy group decision based on similarly; and Chen [4] proposed group decision method based on evidence reasoning and fuzzy preference relationship.

The above methods have a certain promotion effect on solving group decision making problems, but there are still problems in the implementation process: such as weights determination of decision makers (see Wan et al. [22] and Zhang et al. [32]), must be set same weights of decision makers, and in fact the weights of decision makers is generally different. Secondly, information aggregation methods(see Hashemi et al. [8], Meng et al. [17] and Zhou et al. [36]), When membership of evaluation criteria is zero, it may cause the loss of decision information; Chen et al. [3] is need to presuppose that evaluation criteria have the same weights and therefore have a limited scope of application. In addition, for decision making methods (see Chen [2], Jiang [12], Li et al. [15], Wang et al. [25], Wu et al. [27], Zhong [35]), None of these methods have taken into account risk preferences of decision makers, which is easily cause loss of information and bias decision making results. Therefore, these methods are not suitable for complex group decision making issues.

Based on the analysis discussed above, this paper introduce concepts of improved intuitionistic fuzzy entropy and score function, and utilize improved intuitionistic fuzzy entropy method to determine criteria weights and objective weights of decision maker, and construct a group decision method of hesitating fuzzy information by synthesizing decision evaluation information. Compared with the existing decision making methods, this method can fully preserve the completeness of decision evaluation information, overcome problems such as information omission in decision making and decision makers' risk preference, and improve the scientific and reliability of group decision making.

2. Basic Theory of Intuitionistic Fuzzy Set

2.1 Intuitionistic fuzzy set

Intuitionistic fuzzy set (IFS) is a kind of generalized fuzzy sets developed on the basis of Zadeh fuzzy sets. IFS can effective describe ambiguity and uncertainty in reality by comprehensively analysis the membership, non-membership, and hesitation information. Some basic concepts on IFSs are introduced below to facilitate future discussion.

Definition 1. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a non-empty sets, $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ is called an intuitionistic fuzzy sets (see Wang et al. [25] and Zhang et al. [33]). Among them, in which $\mu_A(x)$ is membership and $\nu_A(x)$ means non-membership, where $\mu_A(x) \in [0, 1]$ and $\nu_A(x) \in [0, 1]$, with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

For all $x \in X$, we call $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is for the degree of hesitation, it is clear that $0 \leq \pi_A(x) \leq 1$. Especially, when $\pi_A(x) = 0$, then A is degraded to a ordinary fuzzy set. Additionally, ordered pairs $(\mu_A(x), \nu_A(x))$ are called intuitionistic fuzzy numbers, where $A_1 = (1, 0, 0)$ and $A_2 = (0, 1, 0)$ are the largest and the smallest intuitionistic fuzzy numbers respectively.

2.2. Intuitionistic fuzzy set algorithm

Definition 2. Let $A = (\mu_A(x), \nu_A(x))$ and $B = (\mu_B(x), \nu_B(x))$ be two intuitionistic fuzzy numbers, $\delta > 0$, then there are the following algorithms (see Wang et al. [25] and Zhang et al. [33]):

(1) The sum of the intuitionistic fuzzy numbers is

$$A + B = \{(x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)) \mid x \in X\}.$$

(2) The product of intuitionistic fuzzy numbers is

$$A \cdot B = \{(x, \mu_A(x) \cdot \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)) \mid x \in X\}.$$

(3) The multiply by number of intuitionistic fuzzy numbers is

$$\delta A = \{(x, 1 - (1 - \mu_A(x))^\delta, \nu_A(x)^\delta) \mid x \in X\}.$$

(4) The square of intuitionistic fuzzy numbers is

$$A^\delta = \{(x, \mu_A(x)^\delta, 1 - (1 - \nu_A(x))^\delta) \mid x \in X\}.$$

3. Score Function of Intuitionistic Fuzzy Number

Let $A = (\mu_A(x), \nu_A(x))$ and $B = (\mu_B(x), \nu_B(x))$ be two intuitionistic fuzzy numbers, then $S(A) = \mu_A(x) - \nu_A(x)$ and $S(B) = \mu_B(x) - \nu_B(x)$ are for score function; $H(A) = \mu_A(x) + \nu_A(x)$ and $H(B) = \mu_B(x) + \nu_B(x)$ are for accuracy function.

Obviously, $S(A) \in [-1, 1]$ and $H(A) \in [0, 1]$.

Property 1. *Let A and B be two intuitionistic fuzzy numbers, $S(A)$ and $S(B)$ are called score function, and $H(A)$ and $H(B)$ are called accuracy function, then there are the following properties:*

(1) *If $S(A) < S(B)$, then A is smaller than B, denoted by $A < B$;*

(2) *If $S(A) > S(B)$, then A is greater than B, denoted by $A > B$;*

(3) *If* $S(A) = S(B)$, *then:* (a) *if* $H(A) < H(B)$, *then* A *is smaller than* B, *denoted by* $A < B$; (b) *if* $H(A) > H(B)$, *then* A *is greater than* B, *denoted by* $A > B$; (c) *if* $H(A) = H(B)$, *then* $A = B$ *represent the same information, denoted by* $A = B$.

For score functions and accuracy functions, there have been more relevant researches ([11, 14, 24, 28]). Such as Xu et al. [28] and Lakshmana [14] defined score function and accuracy function based on membership and non-membership, but this method ignore the influence of hesitation, and sometimes it is impossible to distinguish intuitionistic fuzzy numbers, result in ranking failure. Jian [11] constructed score function and accuracy function from perspective of decision maker's attitude, and score function change with decision maker's attitude value . To some extent, the ranking order lack accuracy. (see Wang et al. [24]) assigned degree of hesitation to the membership and non- membership in a certain proportion, and a improved score function value is established, this score function have a strong subjectivity about how to allocate hesitation degree, so the score function is not completely consistent.

Based on the analysis discussed above, this section give an improved method for construct score function based on full consideration of membership, non-membership, hesitation, and decision maker's risk attitude.

Definition 3. Let $A = (\mu_A(x), \nu_A(x))$ be a intuitionistic fuzzy number, with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and $\mu_A(x) \in [0,1]$, $\nu_A(x) \in [0,1]$ for all $x \in X$, then the improved score function is defined as (see Wu et al. [27] and Zhang et al. [33]):

$$S(A) = \mu_A(x) + \frac{(\mu_A(x) - \nu_A(x))\pi_A(x)}{1 - (\mu_A(x) - \nu_A(x))\pi_A(x)}. \tag{3.1}$$

The improved score function (3.1) comprehensive consideration information such as $\mu_A(x), \nu_A(x), \pi_A(x)$ and decision makers' risk attitudes, so the decision results are relatively objective and reasonable.

Property 2. *Let* A' *and* B' *be two intuitionistic fuzzy numbers,* $S(A')$ *and* $S(B')$ *are for the improved score function, then there are the following properties:*

(1) *If* $S(A') < S(B')$, *then* A' *is smaller than* B', *denoted by* $A' < B'$;

(2) *If* $S(A') = S(B')$, *then* $A' = B'$ *represent the same information, denoted by* $A' = B'$;

(3) *If* $S(A') > S(B')$, *then* A' *is greater than* B', *denoted by* $A' > B'$.

Theorem 1. *Let* $A' = (\mu_{A'}(x), \nu_{A'}(x))$ *be intuitionistic fuzzy number, the improved score function* $S(A')$ *is about* u *strict monotonic increase, and about* v *strict monotonic decrease.*

Proof. According to concept of proved score function equation (3.1), we can get

$$S(A') = u + \frac{(u-v)(1-u-v)}{1-(u-v)(1-u-v)}.$$

Since $\dfrac{\partial S(A')}{\partial u} = 1 + \dfrac{1}{(1-u+u^2+v-v^2)^2} > 0.$

Then $S(A')$ is about u strict monotonic increase.

Similarly, there is $\dfrac{\partial S(A')}{\partial u} < 0$, so $S(A')$ is about v strict monotonic decrease. □

4. Multi-criteria Group Decision Making Method Based on Hesitating Fuzzy Information

4.1. Problem description

For a multi-criteria group decision making problem, $X = \{x_1, x_2, \ldots, x_n\}$ is the set of schemes, $O = \{o_1, o_2, \ldots, o_m\}$ is the set of criterias, $P = \{P_1, P_2, \ldots, P_k\}$ is the set of decision makers, $w = \{w_1, w_2, \ldots, w_m\}$ is the weights, where $0 \le w_i \le 1$ and $\sum_{i=1}^{m} w_i = 1$. Let $d_{ij}^k = (\mu_{ij}^k, \nu_{ij}^k)$ be the intuitionistic fuzzy decision matrix, where μ_{ij}^k indicates the degree that the alternative x_j satisfy the criteria o_i given by the decision maker P_k, v_{ij}^k indicates the degree that the alternative x_j dissatisfy the criteria o_i given by the decision maker P_k, $0 \le \mu_{ij}^k + \nu_{ij}^k \le 1$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, k$, then the intuitionistic fuzzy evaluation matrix $D^k = (d_{ij}^k)_{m \times n}$ can be expressed as:

$$D^k = \begin{bmatrix} (\mu_{11}^k, \nu_{11}^k) & (\mu_{12}^k, \nu_{12}^k) & \cdots & (\mu_{1n}^k, \nu_{1n}^k) \\ (\mu_{21}^k, \nu_{21}^k) & (\mu_{22}^k, \nu_{22}^k) & \cdots & (\mu_{2n}^k, \nu_{2n}^k) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{m1}^k, \nu_{m1}^k) & (\mu_{m2}^k, \nu_{m2}^k) & \cdots & (\mu_{mn}^k, \nu_{mn}^k) \end{bmatrix}. \tag{4.1}$$

4.2. Intuitionistic fuzzy number of value evaluation

Usually, different evaluation attribute's property, order of magnitude, dimension, or physical meaning may be inconsistent. So, in order to improve operability of evaluation method, it is necessary to establish a connection between various evaluation attributes, and express attribute evaluation as intuitionistic fuzzy numbers.

4.2.1. Qualitative indicators

Generally speaking, linguistic variables are more concise and accurate than other forms to describe complex or unclear semantics, and are often used to deal with qualitative indicators of uncertain decisions. Combined relevant researches (see Gumus et al. [7]), this section give the relationship between linguistic variables and intuitionistic fuzzy numbers, as shown in Table 1.

4.2.2. Quantitative indicators

In order to eliminate the impact of different indicators, normative processing should be done before decision making. This section use the conversion method of membership and non-membership of intuitionistic fuzzy numbers. The specific methods are as follows:

Table 1: Relationship between linguistic variables and intuitionistic fuzzy numbers.

Linguistic variables	Intuitionistic fuzzy number
Extreme Poor	(0.05,0.95,0.00)
Very Poor	(0.15,0.80,0.05)
Poor	(0.25,0.65,0.10)
Medium Poor	(0.35,0.55,0.10)
Medium	(0.50,0.40,0.10)
Medium Good	(0.65,0.25,0.10)
Good	(0.75,0.15,0.10)
Very Good	(0.85,0.10,0.05)
Extreme Good	(0.95,0.05,0.00)

For beneficial indicators, there are

$$\begin{cases} \mu_{ij} = \alpha \dfrac{r_{ij}}{\max\limits_{1\le j\le m}\{r_{ij}\}} \\ \nu_{ij} = \beta \dfrac{r_{ij}}{\max\limits_{1\le j\le m}\{r_{ij}\}} \end{cases} \tag{4.2}$$

For cost-type indicators, there are

$$\begin{cases} \mu_{ij} = \delta \dfrac{\min\limits_{1\le j\le m}\{r_{ij}\}}{r_{ij}} \\ \nu_{ij} = \varepsilon \dfrac{\min\limits_{1\le j\le m}\{r_{ij}\}}{r_{ij}} \end{cases} \tag{4.3}$$

Exceptionally, when $\min\limits_{1\le j\le m}\{r_{ij}\} = 0$, there are

$$\begin{cases} \mu_{ij} = \delta\left(1 - \dfrac{r_{ij}}{\max\limits_{1\le j\le m}\{r_{ij}\}}\right) \\ \nu_{ij} = \varepsilon\left(1 - \dfrac{r_{ij}}{\max\limits_{1\le j\le m}\{r_{ij}\}}\right) \end{cases} \tag{4.4}$$

Among them, r_{ij} is for attribute value, α and β are the probability that decision maker believe that indicator μ_j expected value is greater $\max\limits_{1\le j\le m}\{r_{ij}\}$, the value is approaching one; β and ε are the probability that decision maker believe that indicator μ_j expected value is less than $\min\limits_{1\le j\le m}\{r_{ij}\}$, and the value approaching zero.

4.3. Improved intuitionistic fuzzy entropy construction method

In recently years, intuitionistic fuzzy entropy theory have received extensive attention (see Jin et al. [13], Wang et al. [25], Wang et al. [26]). The main problem with these methods is that they do not reflect the influence of hesitation on intuitionistic fuzzy entropy. Therefore, it is impossible to accurately distinguish between intuitionistic fuzzy numbers.

The intuitionistic fuzzy entropy proposed in equation (4.5) (see Burillo [1]) is

$$E(A)=\frac{1}{n}\sum_{i=1}^{n}\Big(1-\big(\mu_A(x_i)-\nu_A(x_i)\big)\Big)\cdot\sin\big(\frac{\pi}{2}\big)\big(u_A(x_i)+\nu_A(x_i)\big). \tag{4.5}$$

By analyzing intuitionistic fuzzy entropy equation (4.5), it can be seen that equation (4.5) only consider the change condition of the intuitionistic fuzzy entropy $\pi_A(x)$, without considering its degree of fuzziness, and therefore can not accurately distinguish intuitionistic fuzzy entropy.

In addition, when $\mu_A(x)\neq\nu_A(x)$ and $\mu_A(x)+\nu_A(x)$ in equation (4.5) are the same, then the same entropy value will be obtained.

For example, when $(\mu_A(x),\nu_A(x))$ is $(0.1,0.7)$, $(0.2,0.6)$, $(0.3,0.5)$ respectively, according to equation (4.5), the value of $E(A)$ are both 0.16, but the amount of information reflected is different.

The intuitionistic fuzzy entropy proposed in equation (4.6) (see Wang et al. [26]) is

$$E(A)=\frac{1}{n}\sum_{i=1}^{n}\frac{\min(u_A(x_i),v_A(x_i)+\pi_A(x_i))}{\max(u_A(x_i),v_A(x_i)+\pi_A(x_i))}. \tag{4.6}$$

By analyzing intuitionistic fuzzy entropy equation (4.6), and combining with the theorem "if and only if A is Fuzzy sets, the value of its entropy is zero", it can be known that the definition ignores the fuzziness of Fuzzy sets itself. Secondly, according to equation (4.6), when $\mu_A(x)\neq\nu_A(x)$ and $\mu_A(x)$ are the same, then the value of $E(A)$ are also equal. For example, when $(\mu_A(x),\nu_A(x))$ is $(0.1,0.7)$, $(0.1,0.6)$, $(0.1,0.5)$ respectively, the value of $E(A)$ are both 0.11, according to equation (4.6), but the information reflected by them is not exactly the same.

To make up for these deficiencies, this section combine reference (see Gao et al. [5]), and an improved intuitionistic fuzzy entropy construction method is proposed to measure the degree of uncertainty contained in intuitionistic fuzzy numbers.

$$E(A)=\frac{1-|\mu_A(x)-\nu_A(x)|^2+\pi_A(x)^2}{2}. \tag{4.7}$$

Where, $E(A)$ is the intuitionistic fuzzy entropy, the smaller value of $E(A)$, the less uncertainty of A; Conversely, the greater of value $E(A)$, the more uncertainty of A.

Property 3. *Let $A=(\mu_A(x),\nu_A(x))$ and $B=(\mu_B(x),\nu_B(x))$ be two intuitionistic fuzzy numbers, where $E(A)$ and $E(B)$ are improved intuitionistic fuzzy entropy, then there are the following properties:*

(1) $E(A) = E(B) = 0$, *if and only if* A *and* B *is non-fuzzy sets;*
(2) $E(A) = E(B) = 1$, *if and only if for all* $x \in X$, *there are* $\mu_A(x) = \nu_A(x)$ *and* $\mu_B(x) = \nu_B(x)$*;*
(3) *If for all* $x \in X$, $\pi_A(x)^2 - |\mu_A(x) - \nu_A(x)|^2 \leq \pi_B(x)^2 - |\mu_B(x) - \nu_B(x)|^2$, *then there are* $E(A) \leq E(B)$.

Proof. Because $\mu_A(x) + \nu_A(x) + \pi_A(x) = 1$, $0 \leq \mu_A(x)$ and $\nu_A(x)$ and $\pi_A(x) \leq 1$.

(1) $E(A) = 0 \Leftrightarrow \dfrac{1 - |\mu_A(x) - \nu_A(x)|^2 + \pi_A(x)^2}{2} = 0$

$$\begin{aligned}
&\Leftrightarrow \pi_A(x) = 0,\ \mu_A(x) - \nu_A(x) = 1 \text{ or } \mu_A(x) - \nu_A(x) = -1 \\
&\Leftrightarrow \pi_A(x) = 0,\ \mu_A(x) = 1,\ \nu_A(x) = 0 \text{ or } \pi_A(x) = 0,\ \mu_A(x) = 0,\ \nu_A(x) = 1 \\
&\Leftrightarrow A \text{ is non-fuzzy sets.}
\end{aligned}$$

Similarly, $E(B) = 0 \Leftrightarrow B$ is non-fuzzy sets.

(2) $E(A) = 1 \Leftrightarrow \dfrac{1 - |\mu_A(x) - \nu_A(x)|^2 + \pi_A(x)^2}{2} = 1$

$$\begin{aligned}
&\Leftrightarrow 1 - |\mu_A(x) - \nu_A(x)|^2 + \pi_A(x)^2 = 2 \\
&\Leftrightarrow 1 - \pi_A(x)^2 + |\mu_A(x) - \nu_A(x)|^2 = 0 \\
&\Leftrightarrow 1 - \pi_A(x)^2 = 0, \text{ and } |\mu_A(x) - \nu_A(x)|^2 = 0 \\
&\Leftrightarrow \pi_A(x) = 1 \text{ and } \mu_A(x) = \nu_A(x) \\
&\Leftrightarrow \mu_A(x) = \nu_A(x).
\end{aligned}$$

Similarly, $E(B) = 1 \Leftrightarrow \mu_B(x) = \nu_B(x)$.

(3) $\pi_A(x)^2 - |\mu_A(x) - \nu_A(x)|^2 \leq \pi_B(x)^2 - |\mu_B(x) - \nu_B(x)|^2$

$$\begin{aligned}
&\Leftrightarrow \frac{1 - |\mu_A(x) - \nu_A(x)|^2 + \pi_A(x)^2}{2} \leq \frac{1 - |\mu_B(x) - \nu_B(x)|^2 + \pi_B(x)^2}{2} \\
&\Leftrightarrow E(A) \leq E(B).
\end{aligned}$$

□

4.4. Criteria weights determination based on improved intuitionistic fuzzy entropy

How to determine reasonable criteria weights is the key to solve group decision making problem. For evaluation criteria o_i, the intuitionistic fuzzy entropy E_i^k of decision makers P_k for criteria o_i can be expressed as

$$E_i^k = \sum_{j=1}^{n} \lambda_j e_{ij}^k. \tag{4.8}$$

Among them, e_{ij}^k is the intuitionistic fuzzy entropy of decision maker P_k for scheme x_j under criteria o_i, λ_j is the weights of scheme x_j. As each scheme has equal status, here we take scheme weights

$$\lambda_j = 1/n \quad (j = 1, 2, \ldots, n), \text{ and } \sum_{j=1}^{n} \lambda_j = 1.$$

Where E_i^k reflect uncertainty of the intuitionistic fuzzy number, and uncertainty affect the criteria weights. It can be seen that the greater E_i^k, the greater criteria uncertainty, and it should be assigned a smaller weight; Conversely, the smaller E_i^k, the smaller criteria uncertainty, and it should be assigned a greater weight.

Base on the above analysis, the intuitionistic fuzzy entropy E_i^k can be further modified into

$$E_i^k = \frac{1}{n}\sum_{j=1}^{n} e_{ij}^k = \frac{1}{n}\sum_{j=1}^{n} \frac{1 - |\mu_{ij}^k - \nu_{ij}^k|^2 + (\pi_{ij}^k)^2}{2}. \tag{4.9}$$

For example, $E_i^k = \frac{1}{n}\sum_{j=1}^{n} \frac{1 - |\mu_{1j}^1 - \nu_{1j}^1|^2 + (\pi_{1j}^1)^2}{2}$.

According to equation (4.9), decision makers' weights on evaluation criteria can be expressed as

$$w_i^k = \frac{1 - E_i^k}{\sum_{i=1}^{m}(1 - E_i^k)}, \text{ where } i = 1, 2, \ldots, m. \tag{4.10}$$

According to equation (4.10), decision makers' weights matrix W^k on criteria sets (denoted as $\{o_i \mid i = 1, 2, \ldots, m\}$) can be expressed as

$$W^k = [w_1^k, w_2^k, \ldots, w_m^k], \text{ where } k = 1, 2, \ldots, k. \tag{4.11}$$

Comprehensive analysis of intuitionistic fuzzy evaluation matrix of P_k, we can get criteria weights o_i is

$$w_i = \frac{1}{k}\sum_{i=1}^{k} w_i^l. \tag{4.12}$$

4.5. Decision maker weights determination based on improved intuitionistic fuzzy entropy

During the group decision making process, due to the complexity of the issue and differences in decision makers' knowledge, experience etc, decision maker often have different attitudes towards the same evaluation criteria. So, we need to determine decision maker weights. Let decision makers' weights be λ_k ($k = 1, 2, \ldots, k$), evaluation criteria be o_i ($i = 1, 2, \ldots, m$), and evaluation object x_j ($j = 1, 2, \ldots, n$). The basic idea of determining decision maker weights is: If the decision evaluation information uncertainty is greater, it shows that decision maker is less aware of evaluation object, and the smaller weights should be given; Conversely, the greater weights should be given. According to the improved intuitionistic fuzzy entropy analysis, the objective weights of decision maker can be expressed as

$$\lambda_k = \frac{1 - H_k}{K - \sum_{k=1}^{k} H_k}. \tag{4.13}$$

Among them, λ_k is for the objective weights of decision maker P_k, where $0 \leq \lambda_k \leq 1$ and $\sum\limits_{i=1}^{k} \lambda_k = 1$. $H_k = \sum\limits_{i=1}^{m} w_i E_i^k$ is for weighted intuitionistic fuzzy entropy, and $0 \leq H_k \leq 1$. Here, k is the number of decision maker, m is the number of evaluation criteria, w_i is the weights, where $i = 1, 2, \ldots, m$, and E_i^k is the intuitionistic fuzzy entropy of decision maker P_k for criteria o_i.

Obviously, equation (4.13) reflect the inverse relationship between the objective weights of decision maker and the uncertainty of the evaluation information. The greater H_k, the smaller λ_k, Conversely, the smaller H_k, the greater λ_k.

4.6. Intuitionistic fuzzy cluster aggregation operator

Information aggregation is an important step to solve multi-criteria group decision making problem. This section first gives the concept of intuitionistic fuzzy set weighted average operator and intuitionistic fuzzy number ordered weighted average operator.

Definition 4. Let $A_j = (\mu_{A_j}(x), \nu_{A_j}(x))$ $j = 1, 2, \ldots, n)$ be a group of intuitionistic fuzzy numbers (see Xu [28]), then

$$IFWA(A_1, A_2, \ldots, A_n) = \Big(1 - \prod_{j=1}^{n}(1 - \mu_{A_j})^{\lambda_j}, \prod_{j=1}^{n}(\nu_{A_j})^{\lambda_j}\Big). \tag{4.14}$$

is called intuitionistic fuzzy set weighted average operator, where $\lambda_j = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ is the decision maker weights vector, where $\lambda_j \in [0, 1]$, $j = 1, 2, \ldots, n$ and $\sum\limits_{j=1}^{n} \lambda_j = 1$.

Definition 5. Let $A_j = (\mu_{A_j}(x), \nu_{A_j}(x))$ $j = 1, 2, \ldots, n)$ be a intuitionistic fuzzy numbers, then

$$IFOWA(A_1, A_2, \ldots, A_n) = \Big(1 - \prod_{j=1}^{n}(1 - \mu_{A_j})^{w_j}, \prod_{j=1}^{n}(\nu_{A_j})^{w_j}\Big). \tag{4.15}$$

is called intuitionistic fuzzy ordered weighted average operator (see Xu [28]), where $w = (w_1, w_2, \ldots, w_n)^T$ is the criteria weights vector, $w_j \in [0, 1]$, $j = 1, 2, \ldots, n$ and $\sum\limits_{j=1}^{n} \lambda_j = 1$.

4.7. Decision making steps

Based on the analysis discussed above, when criteria weights and decision maker weights are completely unknown, this paper proposes a multi-criteria group decision method based on hesitating fuzzy information. Firstly, calculate criteria weights and decision maker objective weights based on the intuitionistic fuzzy entropy. The improved score function is used to obtain the comprehensive score value, and realize the optimal ranking of the scheme. The specific method involves the following steps:

Step 1. For group decision making issues, determine scheme sets $X = \{x_1, x_2, \ldots, x_n\}$, evaluation criteria sets $O=\{0_1, o_2, \ldots, o_m\}$, and decision maker sets $P=\{P_1, P_2, \ldots, P_k\}$;

Step 2. According to equation (4.2), (4.3), (4.4) and Table 1, normalization initial evaluation information, construct intuitionistic fuzzy evaluation matrix (denote as $D^k = (d^k_{ij})_{m\times n} = \big((\mu^k_{ij}, \nu^k_{ij})_{m\times n}\big)$ of decision maker P_k for scheme x_j $(k = 1, 2, \ldots, k)$ under criteria o_i;

Step 3. According to equation from (4.8) to (4.12), determine criteria weights vector $w = (w_1, w_2, \ldots, w_m)^T$ by using the improved intuitionistic fuzzy entropy;

Step 4. According to equation (4.6), (4.8), (4.9) and (4.13), determine the decision maker objective weights vector $\lambda_k = (\lambda_1, \lambda_2, \ldots, \lambda_k)^T$ by using the improved intuitionistic fuzzy entropy;

Step 5. According to equation (4.14), determine intuitionistic fuzzy evaluation matrix of decision maker P_k for scheme x_j by using IFWA operator;

Step 6. According to equation (4.15), determine scheme comprehensive attribute value by using IFOWA aggregation operator;

Step 7. According to improved score function (3.1), determine the comprehensive score value, and get the most desirable scheme.

5. Application Example

5.1. Example analysis

Now suppose that there are four experts P_1, P_2, P_3, P_4 to form a decision making group. Three test options x_1, x_2 and x_3 for a new type of equipment were evaluated and optimized. To ensure the objectivity of the options determined, now select four attributes to evaluate the scheme, o_1 is for feasibility, o_2 is for economy, o_3 is for risk tolerance, and o_4 is for intended implementation. Using the expert consultation method, and now evaluation information of decision makers is presented, as shown in Table 2. Using group decision making method proposed in this paper to select the most desirable scheme.

Step 1. According to experts specific evaluation information, based on the improved intuitionistic fuzzy entropy method, we can calculate separately intuitionistic fuzzy entropy E^k_i and criteria weights w_i of decision maker P_k for criteria o_i;

Table 2: Criteria evaluation information of 3 alternatives.

Decision maker	Criteria	x_1	x_2	x_3
P_1	o_1	(0.85,0.11)	(0.75,0.06)	(0.60,0.31)
	o_2	(0.74,0.14)	(0.70,0.15)	(0.55,0.20)
	o_3	(0.49,0.30)	(0.64,0.15)	(0.74,0.16)
	o_4	(0.62,0.35)	(0.56,0.07)	(0.52,0.21)
P_2	o_1	(0.72,0.16)	(0.58,0.34)	(0.84,0.06)
	o_2	(0.82,0.13)	(0.58,0.32)	(0.61,0.32)
	o_3	(0.31,0.48)	(0.81,0.16)	(0.65,0.21)
	o_4	(0.40,0.36)	(0.65,0.13)	(0.74,0.21)
P_3	o_1	(0.77,0.10)	(0.64,0.25)	(0.81,0.08)
	o_2	(0.81,0.14)	(0.68,0.21)	(0.45,0.51)
	o_3	(0.40,0.46)	(0.75,0.06)	(0.60,0.32)
	o_4	(0.62,0.19)	(0.49,0.07)	(0.76,0.10)
P_4	o_1	(0.76,0.10)	(0.71,0.15)	(0.57,0.20)
	o_2	(0.82,0.12)	(0.58,0.31)	(0.61,0.32)
	o_3	(0.40,0.47)	(0.75,0.09)	(0.60,0.31)
	o_4	(0.71,0.18)	(0.49,0.29)	(0.45,0.37)

Take intuitionistic fuzzy entropy E_i^k of decision maker P_1 for criteria o_1 as an example:

According to equation (4.9), we can get

$$\begin{aligned}
E_1^1 &= \frac{1}{3}\sum_{j=1}^{3} e_{1j}^1 = \frac{1}{3}(e_{11}^1 + e_{12}^1 + e_{13}^1) \\
&= \frac{1}{3}\Big(\frac{1-(0.85-0.11)^2+0.04^2}{2} + \frac{1-(0.75-0.06)^2+0.19^2}{2} + \frac{1-(0.60-0.31)^2+0.09^2}{2}\Big) \\
&= 0.3230.
\end{aligned}$$

Similarly, intuitionistic fuzzy entropy E_i^k of all decision maker P_k for criteria o_i can be determined, where i and $k = 1, 2, 3, 4$.

According to equation (4.10) and (4.11), determine the decision maker P_k weights vector for criteria o_i are:

$$\begin{aligned}
W^1 &= [w_1^1, w_2^1, w_3^1, w_4^1] = [0.2809, 0.2549, 0.2430, 0.2212], \\
W^2 &= [w_1^2, w_2^2, w_3^2, w_4^2] = [0.2747, 0.2511, 0.2459, 0.2284], \\
W^3 &= [w_1^3, w_2^3, w_3^3, w_4^3] = [0.2704, 0.2589, 0.2342, 0.2365], \\
W^4 &= [w_1^4, w_2^4, w_3^4, w_4^4] = [0.2685, 0.2567, 0.2459, 0.2289].
\end{aligned}$$

According to equation (4.12), we can get evaluation criteria weights $w_1 = 0.2736$, $w_2 = 0.2554$, $w_3 = 0.2423$, and $w_4 = 0.2287$.

Step 2. Determine decision maker objective weights λ_k based on improved intuitionistic fuzzy entropy, where $k = 1, 2, 3, 4$;

Calculate H_k according to formula $H_k = \sum_{i=1}^{4} w_i E_i^k$, where $k = 1, 2, 3, 4$.

Take H_1 as an example:

$$\begin{aligned} H_1 &= w_1 E_1^1 + w_2 E_2^1 + w_3 E_3^1 + w_4 E_4^1 \\ &= 0.2736 \times 0.3230 + 0.2554 \times 0.3857 + 0.2423 \times 0.4143 + 0.2287 \times 0.4669 \\ &= 0.3940 \end{aligned}$$

Similarly, we can get $H_2 = 0.3922$, $H_3 = 0.3725$ and $H_4 = 0.4091$.

According to equation (4.13), calculate decision maker objective weights $\lambda_1 = 0.2492$, $\lambda_2 = 0.2499$, $\lambda_3 = 0.2580$ and $\lambda_4 = 0.2429$.

Step 3. Calculate intuitionistic fuzzy decision matrix based on objective weights and group evaluation information of decision makers;

Firstly, according to equation (4.14), aggregate information of decision maker P_k for scheme x_j. For example, the intuitionistic fuzzy number of scheme x_1 for criteria o_1 is

$$\begin{aligned} \prod_{j=1}^{4} A_{11} &= \Big(1 - \prod_{j=1}^{4}(1-\mu_j)^{\lambda_j}, \prod_{j=1}^{4}(\nu_j)^{\lambda_j}\Big) \\ &= \Big(1 - (1-0.85)^{\lambda_1} \times (1-0.72)^{\lambda_2} \times (1-0.77)^{\lambda_3} \times (1-0.76)^{\lambda_4}, \\ &\qquad 0.11^{\lambda_1} \times 0.16^{\lambda_2} \times 0.10^{\lambda_3} \times 0.10^{\lambda_4}\Big) \\ &= \Big(1 - (1-0.85)^{0.2492} \times (1-0.72)^{0.2499} \times (1-0.77)^{0.2580} \times (1-0.76)^{0.2429}, \\ &\qquad 0.11^{0.2492} \times 0.16^{0.2499} \times 0.10^{0.2580} \times 0.10^{0.2429}\Big) \\ &= (0.7806, 0.1152). \end{aligned}$$

Similarly, we can get all intuitionistic fuzzy number of scheme x_j for criteria o_i, where $i = 1, 2, 3, 4$ and $j = 1, 2, 3$, as shown in Table 3.

Step 4. According to equation (4.15) and Table 2, we can get the comprehensive decision fuzzy number of each scheme;

Table 3: Alternative fuzzy decision matrix.

Scheme	Decision matrix			
x_1	(0.7806,0.1152)	(0.8000,0.1324)	(0.4033,0.4201)	(0.6011,0.2562)
x_2	(0.6758,0.1671)	(0.6400,0.2358)	(0.7444,0.1063)	(0.5526,0.1154)
x_3	(0.7328,0.1304)	(0.5584,0.3210)	(0.6525,0.2405)	(0.6440,0.1990)

Take decision intuition fuzzy numbers of scheme x_1 as an example:

$$IFOWA(A_1) = \Big(1 - \prod_{j=1}^{n}(1-\mu_{1j})^{w_j}, \prod_{j=1}^{n}(\nu_{1j})^{w_j}\Big)$$
$$= \Big(1-(1-0.7806)^{w_1}\times(1-0.8000)^{w_2}\times(1-0.4033)^{w_3}\times(1-0.6011)^{w_4},$$
$$0.1152^{w_1}\times 0.1324^{w_2}\times 0.4201^{w_3}\times 0.2562^{w_4}\Big)$$
$$= \Big(1-(1-0.7806)^{0.2736}\times(1-0.8000)^{0.2554}\times(1-0.4033)^{0.2423}\times(1-0.6011)^{0.2287},$$
$$0.1152^{0.2736}\times 0.1324^{0.2554}\times 0.4201^{0.2423}\times 0.2562^{0.2287}\Big)$$
$$= (0.6869, 0.1961).$$

Similarly, we can get $IFOWA(A_1) = (0.6616, 0.1603)$ and $IFOWA(A_1) = (0.6543, 0.2097)$.

Step 5. According to equation (3.1), calculate the comprehensive score value of scheme x_j (denote as $S(A_j)$), where $j = 1, 2, 3$;

According to $S(A) = \mu_a + \dfrac{(\mu_a - \nu_a)\pi_a}{1-(\mu_a-\nu_a)\pi_a}$, we can get $S(A_j)$, where $j = 1, 2, 3$;

Such as $S(A_1) = 0.6869 + \dfrac{(0.6869-0.1961)(1-0.6869-0.1961)}{1-(0.6869-0.1961)(1-0.6869-0.1961)} = 0.74782.$

Similarly, we can get $S(A_2) = 0.75963$ and $S(A_3) = 0.7187$.

Therefore, the ranking order of the options is $x_2 > x_1 > x_3$, and the most desirable scheme is x_2.

5.2. Comparative analysis

To verify the feasibility and effectiveness of the method proposed in this paper, now we compare this method to intuitionistic fuzzy similarity group decision method (see Li [15]), intuitionistic fuzzy TOPSIS group decision method (see Yue [30]), and VIKOR group decision method (see Yang [29]), the ranking order are shown in Table 4.

Table 4: Comparison of three decision methods.

method	Ranking order
Similarity group decision method [15]	$x_2 > x_1 > x_3$
TOPSIS group decision method [30]	$x_2 > x_3 > x_1$
VIKOR group decision method [29]	$x_2 > x_1 > x_3$
This decision method	$x_2 > x_1 > x_3$

Compare intuitionistic fuzzy similarity group decision method (see Li [15]) with hesitating fuzzy information group decision method in this paper, we know that the ranking order of two methods are exactly the same, and the most desirable scheme is x2, this verifies the feasibility of the method proposed in this paper. This paper face the problem of a multi-criteria group decision making in which the weights of criteria and decision makers are completely unknown, comprehensive evaluation of criteria and decision maker weights for each scheme, thus the evaluation is more persuasive. In the similarly group decision method (see Li [15]), need to evaluate the fuzzy similarity between schemes and assigned criteria weights. By adjusting the reliability value to evaluate the scheme, thus the evaluation results have certain subjectivity.

Compare with the TOPSIS decision making method (see Yue [30]), this decision method have the following advantages: This method use improved intuitionistic fuzzy entropy decision method to determine criteria weights and comprehensive weights of decision makers, all based on initial data analysis. So, the decision results is relatively objective and reasonable; the criteria weights of TOPSIS decision making method is presupposed, and assuming that all decision makers have the same comprehensive weights. And in actual decision making, due to the complexity of the issues, knowledge and experience of decision makers, etc, the comprehensive weights of decision makers should be different. Furthermore, from the perspective of scheme ranking order, this method determines the comprehensive score value based on the improved score function, the TOPSIS decision method is obtain a comprehensive decision matrix throw a weighted assembly single decision matrix, so the ranking order will be slightly different. But the most desirable scheme is exactly the same.

Compare with the VIKOR group decision making method (see Yang [29]), it can be seen that the decision ranking results obtained by the two methods are exactly the same. The VIKOR group decision making method use the IFWA weighted aggregation operator to obtain a comprehensive intuitive fuzzy decision matrix. Then, the group utility value and individual regret value determine the comprehensive compromise evaluation value of each scheme, which can accurately distinguish the advantages and disadvantages of each scheme. This paper use improved intuitive fuzzy entropy to determine the weight of criteria and decision makers, and use IFWA operator and IFOWA operator to integrate information, and use new score function to achieve the selection of options, so the decision result is more realistic.

In summary, compare with the similarity decision method (see Li [15]), the TOPSIS decision making method (see Yue [30]), and the VIKOR group decision method (see Yang [29]), this method is superior and has a wide range of applications.

6. Conclusion

For multi-criteria group decision problem, this paper introduce concepts of improved intuitionistic fuzzy entropy and score function, and based on improved intuitionistic fuzzy entropy to determine criteria weights and decision maker objective weights. By aggregating decision information, a group decision method of hesitating fuzzy information is proposed. Compare with the existing decision methods, this method gives full consideration to the hesitating fuzzy information of individual and group of decision makers, improve the scientific and reliability of group decision making, which provides a new way to solve the multi-criteria group decision making based on fuzzy hesitation information.

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