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A Two-Stage Inventory Model for Imported Materials and Deteriorating Products with Expiration Date and Expired Rework

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Abstract

This research is based on the traditional Economic Production Quantity (EPQ) model, adding factors such as deteriorating items and expiration date, as well as the product's reworkability, to establish a realistic and effective two-stage production inventory model. Furthermore, it considers different currencies of material purchase and product sales to explore the impact of exchange rate changes on the model. The main purpose is to determine the optimal lengths of the production period and production cycle for finished products, and the optimal order quantity for materials, so that the total profit per unit time is maximum. Through mathematical analysis, the characteristics of the optimal solutions can be obtained. Further, we use several practical numerical examples to illustrate the solution procedure and perform sensitivity analyses with respect to the parameters of the proposed model. Finally, several management implications are found through the results, hoping to provide reference for decision makers to formulate production and purchase strategies.

Keywords: Production inventory model, imported materials, deteriorating items, expiration date, rework.

1. Introduction

Effective inventory management will help promote the stable development of the enterprise. Economic order quantity (EOQ) and economic production quantity (EPQ) model of optimal inventory quantity has been proposed and become an important reference for enterprises to carry out inventory management. However, with the diversification of product attributes and changes in business operations, some assumptions of the traditional EOQ/EPQ model are not sufficient to meet the actual situation, such as excluding deteriorating items. In real life, the qualities of some products decline with time, and those are not allowed to be sold beyond the expiration date. At the same time, with the improvement of production technology, some categories of deteriorating items can be reworked to restore their quality. Although there will be a certain amount of loss in rework, and it will also produce the cost, but compared with direct scrapping, the loss of rework to the enterprise's overall operation is relatively reduced. The above-mentioned actual product characteristics are not considered in the traditional EOQ or EPQ model.

Ghare and Schrader [6] first included the issue of the deterioration items and assumed that the deterioration of goods is subject to exponential decay. Covert and Philip [3] extended the theoretical basis proposed by Ghare and Schrader in 1963 to establish an EOQ model whose product deterioration rate follows a Weibull distribution subject to two parameters. Philip [14] extended that the product deterioration issue is an inventory model which follows a Weibull distribution with three parameters, making the study of the deterioration rate more complete. Fujiwara [4] discussed perishable products and put forward the EOQ mode of products related to freshness. Wu et al. [24] discussed an optimal replenishment policy for deteriorating items with stock-dependent demand and partial backlogging. Hsu et al. [7] considered the assumption that demand would decrease with the expiration date of the product, and established an inventory model that meets seasonal demand. Lee and Dye [11] proposed the use of preservation technology cost as a decision variable for deterioration items with a controllable deterioration rate, and combined with replacement strategies to establish an inventory-dependent inventory model. Maihami and Karimi [12] established the optimal selling price and inventory model for deteriorating products by adding the influence of variable demand and promotional factors. Yang et al. [25] discussed the optimal dynamic trade credit and preservation technology allocation for a deteriorating inventory model. Geetha and Udayakumar [5] set up an inventory model by considering the residual value of deteriorating items for deterioration items that depend on advertising needs. Khan et al. [10] jointly proposed the inventory model for goods with an expiration date and taking the price depend demand into consideration.

Teng et al. [22] found that the expiration date of a deteriorating item is an important factor in a buyer's purchase decision, and considered a situation where the deterioration rate increases as the expiration date approaches. Shah et al. [19] discussed an inventory model with deteriorating items and expiration date under preservation technology investment. Wu et al. [23] established an inventory model that uses a prepayment strategy for product sales with high deterioration rates to resist the risk of short product shelf life. Sebatjane and Adetunji [18] established a supply chain model, suggesting that retailers conduct clearance sales at the end of each replenishment cycle to clear out the ending on-hand inventory to prevent items from expiring. Priyan and Mala [15] designed a procedure for deciding an optimal strategy with the help of a game theory return matrix, to manage the problem of gradual deterioration of finished products as the expiration date approaches.

Schrady [17] carried out the earliest research on the process of rework of defective products, and determined the best quantity of purchase and repair. Chiu [2] considered the random defect rate under the traditional EPQ model, in which some defective products can be reworked, and some defective products will be scrapped and discarded. Jamal et al. [9] discussed the issue of rework batches of returned items, which were divided into return-and-rework and accumulation-rework strategies. Ojha et al. [13] discussed a production system with quality inspection, and defective products must be reworked. Chiu et al. [1] proposed an EPQ model in which some products will become waste products due to the failure rework. Sivashankari and Panayappan [20] discussed the impact of product defect rate and rework ratio to establish production inventory models. Taleizadeh et al. [21] determined the best selling price and replenishment batches based on the ratio of good products after rework, and established an EPQ model. Islam et al. [8] used a Markov process to explore an inventory model with rework problems with expired items. Sanjai and Periyasamy [16] distinguished the inventory into two states for allow shortage and no shortage allowed, and discussed the rework of defective products.

Moreover, the external market transactions often use different foreign currencies, while the domestic market uses the domestic currency. Therefore, exchange rate changes have a great impact on the purchase cost of materials and the sales revenue of products. In the previous inventory model, the transaction currencies of the purchase price and the sales price were assumed as the same, that is, the impact of exchange gains and losses will be ignored.

Based on the above issues, this study uses the traditional EPQ model as the basis, considering deteriorating items, expiration date and reworkability, and also considering the different currencies of material purchase and product sales, in order to establish a more realistic production inventory model. Firstly, the notations and assumptions required for the proposed model are established, and the relevant total profit function of the production and inventory is established accordingly. Then, the mathematical optimization method is used to find the optimal lengths of production period and production cycle, and the optimal material purchase quantity, so that the total profit is the maximum. Further, we use a few realistic numerical examples to illustrate the solution process, and perform sensitivity analyses on the main parameters. Finally, according to the results of numerical analysis, some meaningful management connotations are obtained, and it is expected to provide a reference for the manufacturer to make relevant inventory decisions.

2. Notations and Assumptions

For the establishment of the model, the notations used are explained as follows:

A Ordering cost for raw materials (domestic currency).

S Setup cost (domestic currency).

c Unit purchase price of materials (foreign currency).

P Production rate.

D Demand rate.

r Bank's selling rate.

v Unit production cost of finished products (domestic currency).

h_1 Holding cost of materials per unit per unit time (domestic currency).

h_2 Holding cost of finished products per unit per unit time (domestic currency).

s Unit selling price of finished products (domestic currency).

v_r Unit rework cost for expired goods (domestic currency).

τ Expiration date of finished goods.

δ Reprocessing yield.

θ_m Deterioration rate of materials.

$\theta(t)$ Deterioration rate of finished goods.

$I_{\max}$ Maximum inventory quantity of finished products.

t_1 Length of time period in which the finished product inventory reaches $I_{\max}$.

T Length of the whole production cycle.

q_m The purchase quantity of raw materials in a production cycle.

Next, in order to establish the model, we need some assumptions, explained as follows:

(1) This inventory system considers a single finished product produced by a single material. Further, the currencies used for the material purchase and finished product sales are different. In order to facilitate the establishment of the model, it assumes that the bank's selling rate at the time of material purchase is fixed at r.

(2) The production rate of the finished product is fixed and greater than the demand rate, otherwise there will be no inventory issues.

(3) The material deterioration rate θ_m is fixed, $0 < \theta_m < 1$. The finished product deterioration rate $\theta(t)$ is an increasing function of time with an expiration date τ (That is $\theta(\tau) = 1$, and $t_1 \leq \tau$). Hence, the deterioration rate of the finished products is assumed as $\theta(t) = \dfrac{1}{1+\tau-t}$, where $0 \leq t \leq \tau$.

(4) Once the material deteriorates, it will not be repaired.

(5) If the finished goods have not been sold after the expiration date, rework is allowed. But the quantity of finished products after rework will be less than the original quantity, assuming the reprocessing yield is δ, where $0 \leq \delta \leq 1$. The finished products after rework are sold at one time.

(6) Shortages are not allowed for either materials or finished goods.

3. Model Formulation and Solution

Based on the previous notations and assumptions, we first briefly describe the two-stage inventory system. The manufacturer first purchases q_m quantity of materials and start the production at the time $t = 0$. Over time, the inventory level of materials decreases as the production process and the deterioration of raw material itself. Until t_1, all materials will be used up. The relationship between the inventory level of raw materials and time is shown in Figure 1

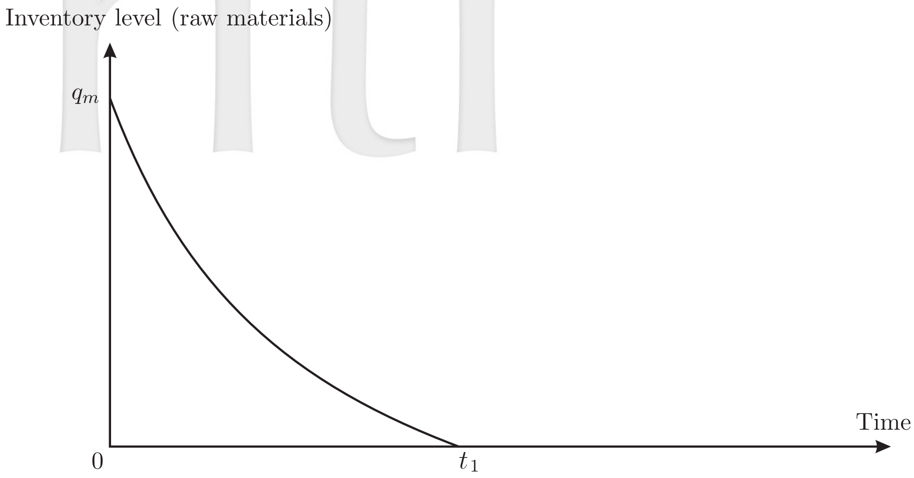

Figure 1: Inventory level of raw materials.

As seen in Figure 1, the change of the material inventory level during $[0, t_1]$ is related to the production input and deterioration. Therefore, the change of the material inventory level $I_m(t)$ at time t can be expressed by the following differential equation:

$$\frac{dI_m(t)}{dt} + \theta_m I_m(t) = P, \quad 0 \le t \le t_1. \tag{3.1}$$

According to the boundary condition $I_m(t_1) = 0$, solving equation (3.1) gives:

$$I_m(t) = \frac{[e^{\theta_m(t_1-t)} - 1]P}{\theta_m}, \quad 0 \le t \le t_1. \tag{3.2}$$

Therefore, the manufacturer's purchase quantity of materials per production cycle is as follows:

$$q_m = I_m(0) = \frac{(e^{\theta_m t_1} - 1)P}{\theta_m}. \tag{3.3}$$

For finished products, from $t = 0$, the manufacturer starts selling while producing. Since the production rate is greater than the demand rate, when reaching the time point t_1, the quantity of finished products reaches $I_{\max}$ and the production is stopped, after that, until $t = T$, the finished product is sold out.

Therefore, during $[0, t_1]$, the quantity of finished goods inventory is related to production, demand, and its own deterioration. That is, the change of the inventory level $I_1(t)$ at time t can be expressed by the following differential equation:

$$\frac{dI_1(t)}{dt} + \theta(t) I_1(t) = P - D, \quad 0 \le t \le t_1. \tag{3.4}$$

Since the boundary condition $I_1(0) = 0$, the inventory level in the interval $[0, t_1]$ can be obtained by solving (3.4) as follows:

$$I_1(t) = (1 + \tau - t) \cdot (P - D) \cdot \ln\left[\frac{1+\tau}{1+\tau-t}\right], \quad 0 \le t \le t_1. \tag{3.5}$$

Similarly, since the production stopped during the period $[t_1, T]$, the change of the inventory level is related to the demand rate and the deterioration of the finished product itself. The change of the inventory level $I_2(t)$ at time t can be expressed by the following differential equation:

$$\frac{dI_2(t)}{dt} + \theta(t)I_2(t) = -D, \quad t_1 < t < T. \tag{3.6}$$

Since the boundary condition $I_2(T) = 0$, the inventory level in the interval $[t_1, T]$ can be obtained by solving (3.6) as follows:

$$I_2(t) = (1 + \tau - t) \cdot D \cdot \ln\left[\frac{1 + \tau - t}{1 + \tau - T}\right], \quad t_1 < t < T. \tag{3.7}$$

Substituting t_1 into equations (3.5) and (3.7), since $I_1(t_1) = I_2(t_1)$, the length of the production period T is got as

$$T = 1 + \tau - \frac{(1 + \tau - t_1)^{\frac{P}{D}}}{(1 + \tau)^{\frac{P-D}{D}}}. \tag{3.8}$$

Since the expiration date of finished products is τ, which is different from the length of the production period T, it needs to consider two different situations: (1) $T \le \tau$ and (2) $T > \tau$ in this model, as described below.

Situation 1: $T \le \tau$

In this situation, finished products will be sold out within expiration date, so that the relationship between the finished product inventory level and time is shown in Figure 2.

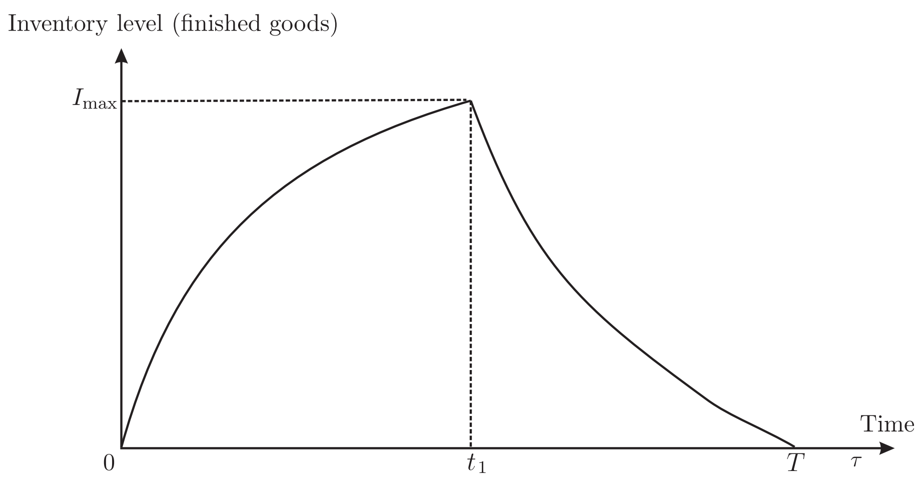

Figure 2: Relationship between finished product inventory level and time ($T \le \tau$).

In the situation of $T \le \tau$, the total profit of each production cycle of the inventory system includes setup cost, raw material ordering cost, raw material cost, production cost, raw material holding cost, finished product holding cost and sales revenue, which are explained as follows:

(1) Setup cost
The setup cost (expressed in the notation SC) of each production cycle is a fixed value, that is

$$SC = S. \tag{3.9}$$

(2) Raw material ordering cost
The material ordering cost (expressed in OC) consumed in purchase of each batch of materials is a fixed value, that is

$$OC = A. \tag{3.10}$$

(3) Raw material cost
Since the raw materials are imported from abroad, they are originally purchased in foreign currency and converted into domestic currency to get the cost of materials (expressed in MC) as

$$MC = crq_m = cr\frac{(e^{\theta_m t_1} - 1)P}{\theta_m}. \tag{3.11}$$

(4) Production cost
For every kilogram of finished products produced in a production cycle, the required production cost (expressed in PC) is

$$PC = vPt_1. \tag{3.12}$$

(5) Raw material holding cost
During the production period of $[0, t_1]$, the holding cost (expressed in HC_m) per kg of materials calculated in unit time is as

$$HC_m = h_1 \int_0^{t_1} I_m(t)dt = \frac{h_1 P(e^{\theta_m t_1} - \theta_m t_1 - 1)}{\theta_m^2}. \tag{3.13}$$

(6) Finished product holding cost
For the finished products accumulated during $[0, T]$, the holding cost calculated in unit time (expressed in HC_g) is

$$\begin{aligned} HC_g &= h_2\Big[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\Big] \\ &= h_2\Big[\int_0^{t_1}(1+\tau-t)\cdot(P-D)\cdot\ln\Big[\frac{1+\tau}{1+\tau-t_1}\Big]dt + \int_{t_1}^{T}(1+\tau-t)\cdot D\cdot\ln\Big[\frac{1+\tau-t}{1+\tau-T}\Big]dt\Big] \\ &= \frac{1}{2}h_2\Big\{\frac{1}{2}\{Pt_1(2-t_1+2\tau) + DT[T-2(1+\tau)]\} \\ &\quad + (1+t_1+\tau)^2\Big\{(D-P)\ln\Big[\frac{1+\tau}{1-t_1+\tau}\Big] + D\ln\Big[\frac{1-t_1+\tau}{1-T+\tau}\Big]\Big\}\Big\}. \end{aligned} \tag{3.14}$$

(7) Sales revenue
In a production cycle, the sales revenue (expressed in SR) of finished products is

$$SR = s \cdot D \cdot T. \tag{3.15}$$

Therefore, the total profit function (expressed in $TP_1(t_1, T)$) is

$$\begin{aligned} TP_1(t_1,T) &= (SR - SC - OC - MC - PC - HC_g - HC_m)/T \\ &= \Bigg\{ sDT - S - A - cr\frac{(-1+e^{\theta_m t_1})P}{\theta_m} - vPt_1 \\ &\quad - \frac{1}{2}h_2\Bigg\{\frac{1}{2}\{Pt_1(2-t_1+2\tau) + DT[T-2(1+\tau)]\} \\ &\quad + (1+t_1+\tau)^2\Big\{(D-P)\ln\left[\frac{1+\tau}{1-t_1+\tau}\right] + D\ln\left[\frac{1-t_1+\tau}{1-T+\tau}\right]\Big\}\Bigg\} \\ &\quad - \frac{h_1P(e^{\theta_m t_1} - \theta_m t_1 - 1)}{\theta_m^2}\Bigg\} \Bigg/ T. \end{aligned} \tag{3.16}$$

Situation 2: $T > \tau$

In this situation, because the finished product inventory has not been sold out within its expiration date, the unsold products are allowed to be reworked. But the quantity of finished products after rework will be less than the original quantity. The finished products after rework are sold at one time. The relationship between finished product inventory level and time is shown in Figure 3.

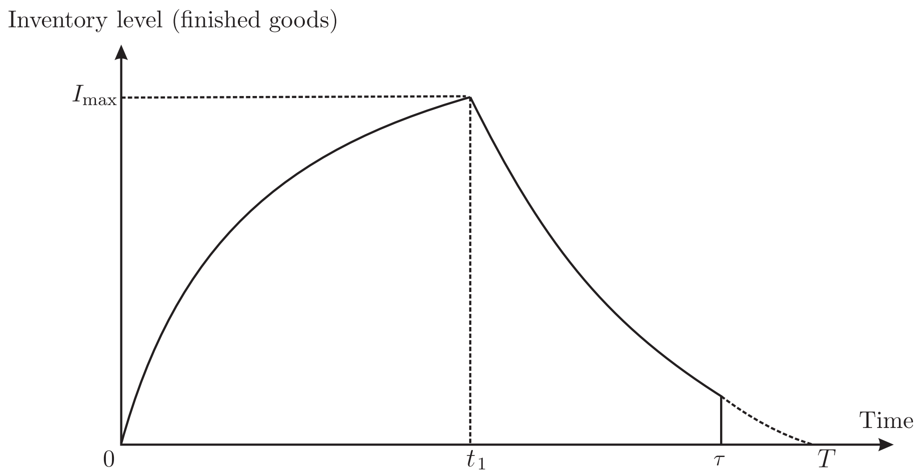

Figure 3: Relationship between finished goods inventory level and time ($T > \tau$).

When $T > \tau$, the total profit of each production cycle of the inventory system includes setup cost, raw material order cost, raw material cost, production cost, raw material holding cost, finished product holding cost, rework cost and sales revenue, among which, the setup cost, raw material ordering cost, raw material cost, production cost, and raw material holding cost are the same as in situation 1 with the formulas (3.9)∼(3.13), and the finished product holding cost, rework cost and sales revenue are explained as follows :

(1) Finished product holding cost
For the finished products accumulated during $[0,\tau]$, the holding cost calculated in unit time (expressed in HC_q) is

$$\begin{aligned}HC_q &= h_2\Big[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{\tau} I_2(t)dt\Big]\\ &= h_2\Big[\int_0^{t_1}(1+\tau-t)\cdot(P-D)\cdot\ln\Big[\frac{1+\tau}{1+\tau-t_1}\Big]dt\\ &\quad + \int_{t_1}^{\tau}(1+\tau-t)\cdot D\cdot\ln\Big[\frac{1+\tau-t}{1+\tau-T}\Big]dt\Big]\\ &= \frac{1}{2}h_2\Big\{-\frac{1}{2}D\tau(2+\tau)+P\Big(t_1+\frac{t_1^2}{2}+t_1\tau\Big)-D\ln\Big[\frac{1}{1-T+\tau}\Big]\\ &\quad +(1-t_1+\tau)^2\Big\{(D-P)\ln\Big[\frac{1+\tau}{1-t_1+\tau}\Big]+D\ln\Big[\frac{1-t_1+\tau}{1-T+\tau}\Big]\Big\}\Big\}. \qquad (3.17)\end{aligned}$$

(2) Rework cost
The cost of reworking the finished products beyond the expiration date (expressed in RC) is

$$\begin{aligned}RC &= v_r I_2(\tau)\\ &= v_r D\ln\Big[\frac{1}{1-T+\tau}\Big]. \qquad (3.18)\end{aligned}$$

(3) Sales revenue
In a production cycle, the sales revenue of finished products (expressed in SR) is

$$\begin{aligned}SR &= s[D\cdot\tau+\delta I_2(\tau)]\\ &= sD\Big[\tau+\delta\ln\Big[\frac{1}{1-T+\tau}\Big]\Big]. \qquad (3.19)\end{aligned}$$

Therefore, the total profit function (expressed in $TP_2\ (t_1,T)$) is

$$\begin{aligned}TP_2(t_1,T) &= (SR-SC-OC-MC-PC-HC_q-HC_m-RC)/\tau\\ &= \Big\{sD\Big[\tau+\delta\ln\Big[\frac{1}{1-T+\tau}\Big]\Big]-S-A-cr\frac{(e^{\theta_m t_1-1})P}{\theta_m}-vPt_1\\ &\quad -\frac{1}{2}h_2\Big\{-\frac{1}{2}D\tau(2+\tau)+P\Big(t_1+\frac{t_1^2}{2}+t_1\tau\Big)-D\ln\Big[\frac{1}{1-T+\tau}\Big]\\ &\quad +(1-t_1+\tau)^2\Big\{(D-P)\ln\Big[\frac{1+\tau}{1-t_1+\tau}\Big]+D\ln\Big[\frac{1-t_1+\tau}{1-T+\tau}\Big]\Big\}\Big\}\\ &\quad -\frac{h_1P(e^{\theta_m t_1}-\theta_m t_1-1)}{\theta_m^2}-v_rD\ln\Big[\frac{1}{1-T+\tau}\Big]\Big\}\Big/\tau. \qquad (3.20)\end{aligned}$$

The main purpose is to find a production strategy with the maximum total profit per unit time, that is, to find the length of the production period t_1 and the length of the whole production cycle T. First, from equation (3.8), it is known that $T = 1+\tau-\dfrac{(1+\tau-t_1)^{P/D}}{(1+\tau)^{(P-D)/D}}$, so $TP_i(t_1,T)$ can be simplified to $TP_i(t_1)$, where $i = 1, 2$. Then, for $TP_i(t_1)$, find its first-order differential function $dTP_i(t_1)/dt_1$ and second-order differential function $d^2TP_i(t_1)/dt_1^2$.

$$\begin{aligned}\frac{dTP_1(t_1,T)}{dt_1} &= \frac{1}{T^2}\frac{dT}{dt_1}\Big\{S+A+\frac{cr(e^{\theta_m t_1}-1)P}{\theta_m}+vPt_1\\ &\quad+\frac{h_2}{2}\Big\{Pt_1(1+\tau)-\frac{Pt_1^2}{2}+DT[T-2(1+\tau)]\Big\}+\frac{h_1P(e^{\theta_m t_1}-\theta_m t_1-1)}{\theta_m^2}\Big\}\\ &-\frac{1}{T}\Big\{cre^{\theta_m t_1}P+vP+\frac{h_2}{2}\Big\{P(1+\tau-t_1)+2D\frac{dT}{dt_1}[T-(1+\tau)]\Big\}+\frac{h_1P(e^{\theta_m t_1}-1)}{\theta_m}\Big\},\end{aligned} \tag{3.21}$$

$$\begin{aligned}\frac{d^2TP_1(t_1,T)}{dt_1^2} &= \frac{-2}{T^3}\frac{dT}{dt_1}\Big\{S+A+\frac{cr(e^{\theta_m t_1}-1)P}{\theta_m}+vPt_1\\ &\quad+\frac{h_2}{2}\Big\{Pt_1(1+\tau)-\frac{Pt_1^2}{2}+DT[T-2(1+\tau)]\Big\}+\frac{h_1P(e^{\theta_m t_1}-\theta_m t_1-1)}{\theta_m^2}\Big\}\\ &+\frac{1}{T^2}\frac{d^2T}{dt_1^2}\Big\{S+A+\frac{cr(e^{\theta_m t_1}-1)P}{\theta_m}+vPt_1\\ &+\frac{h_2}{2}\Big\{Pt_1(1+\tau)-\frac{Pt_1^2}{2}+DT[T-2(1+\tau)]\Big\}+\frac{h_1P(e^{\theta_m t_1}-\theta_m t_1-1)}{\theta_m^2}\Big\}\\ &+\frac{1}{T^2}\frac{dT}{dt_1}\Big\{cre^{\theta_m t_1}P+vP+\frac{h_2}{2}\Big\{P(1+\tau-t_1)+2D\frac{dT}{dt_1}[T-(1+\tau)]\Big\}+\frac{h_1P(e^{\theta_m t_1}-1)}{\theta_m}\Big\}\\ &+\frac{1}{T^2}\Big\{cre^{\theta_m t_1}P+vP+\frac{h_2}{2}\Big\{P(1+\tau-t_1)+2D\frac{dT}{dt_1}[T-(1+\tau)]\Big\}+\frac{h_1P(e^{\theta_m t_1}-1)}{\theta_m}\Big\}\\ &-\frac{1}{T}\Big\{cr\theta_m e^{\theta_m t_1}P+\frac{h_2}{2}\Big\{-P+2D\frac{d^2T}{dt_1^2}[T-(1+\tau)]+2D\Big(\frac{dT}{dt_1}\Big)^2\Big\}+h_1Pe^{\theta_m t_1}\Big\},\end{aligned} \tag{3.22}$$

$$\begin{aligned}\frac{dTP_2(t_1,T)}{dt_1} &= \frac{1}{\tau}\Big\{\frac{dT}{dt_1}\frac{(\delta s-v_r)D}{(1-T+\tau)}-cre^{\theta_m t_1}P-vP\\ &\quad-\frac{h_2}{2}\Big\{P(1+\tau-t_1)+2D\frac{dT}{dt_1}[T-(1+\tau)]\Big\}-\frac{h_1P(e^{\theta_m t_1}-1)}{\theta_m}\Big\},\end{aligned} \tag{3.23}$$

$$\begin{aligned}\frac{d^2TP_2(t_1,T)}{dt_1^2} &= \frac{1}{\tau}\Big\{\frac{d^2T}{dt_1^2}\frac{(\delta s-v_r)D}{(1-T+\tau)}+\Big(\frac{dT}{dt_1}\Big)^2\frac{(\delta s-v_r)D}{(1-T+\tau)^2}-(cr\theta_m+h_1)e^{\theta_m t_1}P\\ &\quad-\frac{h_2}{2}\Big\{-P+2D\frac{d^2T}{dt_1^2}[T-(1+\tau)]+2D\Big(\frac{dT}{dt_1}\Big)^2\Big\}\Big\},\end{aligned} \tag{3.24}$$

where

$$\frac{dT}{dt_1}=\frac{P}{D}\Big(\frac{1+\tau-t_1}{1+\tau}\Big)^{(P-D)/D},$$

and

$$\frac{d^2T}{dt_1^2} = -\frac{P(P-D)}{(1+\tau)D^2}\Big(\frac{1+\tau-t_1}{1+\tau}\Big)^{(P-2D)/D}.$$

Let $\frac{dTP_i(t_1)}{dt_1} = 0$, the solutions of t_1 can be obtained separately and recorded as t^*_{11} and t^*_{12}, if $\left.\frac{d^2TP_i(t_1)}{dt_1^2}\right|_{t_1=t^*_{1i}} < 0$, then t^*_{1i} is the solution that makes $TP_i(t_1)$ the maximum value. Since it is not easy to find a definite solution for t^*_{1i} and directly check the concavity of profit function. Thus, to ensure that the obtained optimal t_1 value falls within the range and is determined as the optimal solution to the entire problem, we develop an algorithm to find the optimal solution t^*_1. Further, the concavity will be verified by numerical analysis in the next section.

Algorithm:

Step 1: Solve the equation $\frac{\partial TP_i(t_1)}{\partial t_1} = 0$ to get t^*_{1i}, where $i = 1, 2$.

Step 2: Substitute t^*_{1i} into (3.8) to find T^*_i, and determine whether T^*_i falls within the range, that is

1. If $T^*_1 \le \tau$, let $TP^*_1 = TP_1(t^*_{11}, T^*_1)$; otherwise, let $TP^*_1 = 0$.
2. If $T^*_2 > \tau$, let $TP^*_2 = TP_2(t^*_{12}, T^*_2)$; otherwise, let $TP^*_2 = 0$.

Step 3: Find $\max_{i=1,2} TP_i(t^*_{1i}, T^*_i)$, if $TP(t^*_1, T^*) = \max_{i=1,2} TP_i(t^*_{1i}, T^*_i)$, then (t^*_1, T^*) is the optimal solution.

The above algorithm can be solved using computer software Mathematica 12.0 under given parameters. Once the optimal solution (t^*_1, T^*) is obtained, we can use (3.3) to obtain the optimal order quantity of raw materials q^*_m, and to obtain the total annual profit $TP^* = TP(t^*_1, T^*)$ from (3.16) or (3.20).

4. Numerical Examples

4.1. Optimization results of numerical example

In order to verify this two-stage production inventory model, this paper takes plant essential oil products as an example, using the following parameters that meets the actual situation provided by the manufacturer's manager of production department.

Demand rate	$D =$ NT\$10,000 kg/year
Production rate	$P =$ NT\$20,000 kg/year
Ordering cost for materials	$A =$ NT\$2,000/order
Setup cost	$S =$ NT\$15,000/batch
Unit purchase price of materials	$c =$ \$6/kg
Unit production cost of finished products	$v =$ NT\$45/kg
Bank's selling rate (\$ to NT\$)	$r = 30$
Expiration date of finished products	$\tau = 2/12$ year
Unit selling price of finished products	$s =$ NT\$360/kg
Holding cost of materials per unit per unit time	$h_1 =$ NT\$5/kg
Holding cost of finished products per unit per unit time	$h_2 =$ NT\$15/kg
Deterioration rate of raw material	$\theta_m = 0.3$
Rework cost per unit	$v_r =$ NT\$60/kg
Reprocessing yield	$\delta = 0.8$

Using the above algorithm, we found that the optimal length of a production period is $t_1^* = 0.07727$ years, the optimal production cycle length is $T^* = 0.14943$ years, the optimal quantity of materials purchased in a production cycle is $q_m^* = 1,563.49$ kg, and the total annual profit is $TP^* =$ NT\$ 1,129,800. Because $\left.\frac{d^2TP_1(t_1)}{dt_1^2}\right|_{t_1=t_i^*} = -3.8043 \times 10^{-7} < 0$, it is shown that t_i^* is the solution that maximizes the total annual profit TP_1 (t_1). Figure 4 illustrates the graphical illustration of the total annual profit $TP_1(t_1)$ versus t_1. That is, the concavity of the total annual profit function can be verified, and the obtained solutions are optimal for maximizing the total annual profit function.

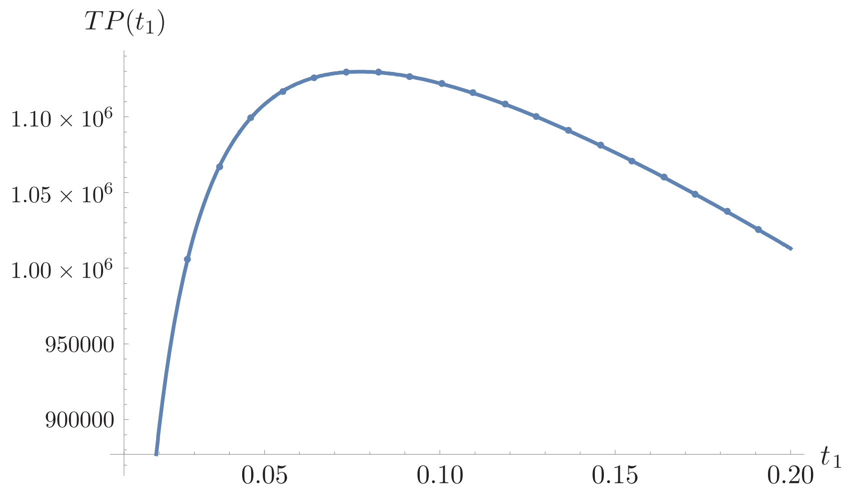

Figure 4: Graphical illustration of $TP_1(t_1)$ with respect to t_1.

4.2. Effects of expiration date

Since different products may have different expiration dates depending on their characteristics, this section will discuss some different expiration dates with their optimal solution and the change in total profit. For convenience of analyses, we use the parameter value of 1 and consider different $\tau \in [1/12, 2/12, 3/12, \ldots, 1)]$, and calculate the corresponding optimal solutions and the total annual profits. The results are shown in Table 1.

Table 1: Optimal solutions for different values of τ.

τ	t_1^*	T^*	q_m^*	TP^*	Situation
1/12	0.01000	0.08333	200.30	1,212,120	2
2/12	0.07727	0.14943	1,563.49	1,129,800	1
3/12	0.07931	0.15359	1,605.28	1,135,480	1
4/12	0.08123	0.15751	1,644.56	1,140,560	1
5/12	0.08304	0.16121	1,681.58	1,145,120	1
6/12	0.08474	0.16470	1,716.55	1,149,250	1
7/12	0.08635	0.16800	1,749.65	1,153,020	1
8/12	0.08788	0.17113	1,781.04	1,156,450	1
9/12	0.08933	0.17411	1,810.85	1,159,610	1
10/12	0.09072	0.17694	1,839.23	1,162,520	1
11/12	0.09203	0.17964	1,866.26	1,165,210	1
12/12	0.09329	0.18222	1,892.07	1,167,771	1

The results in Table 1 show that when the expiration date is 1/12 (situation 2), the optimal length of time period is $t_1^* = 0.01$ years, the optimal production cycle is $T^* = 0.08333$ years, the optimal purchase quantity of raw materials is $q_m^* = 200.3$ kg, and the total annual profit is $TP^* =$ NT\$ 1,212,120; when the expiration date is 2/12 (situation 1), the result are $t_1^* = 0.07727$ years, $T^* = 0.14943$ years, $q_m^* = 1,563.49$ kg, $TP^* =$ NT\$ 1,129,800; when the expiration date is 12/12 (situation 1), the result are $t_1^* = 0.09329$ years, $T^* = 0.18222$ years, $q_m^* = 1,892.07$ kg, $TP^* =$ NT\$ 1,167,771.

From the numerical comparison results between situation 1 and situation 2, it is known that for products with a very short expiration date, due to the rapid circulation of commodities, their production period and production cycle are much shortened. The holding cost of materials and finished products per unit time is relatively reduced. And the case with the shortest expiration date have a higher total profit than the case with the second shortest expiration date. The numerical results from situation 1 show that the longer the expiration date of the product, the longer the production period and production cycle, and the longer the sales duration of the product. Although it will increase the holding cost of the finished product per unit time, it can be exempted from overdue rework, and also allows profits to be higher than products with short expiration date.

Therefore, under normal circumstances, that is $T^* < \tau$, it is recommended that manufacturers choose to sell out within the expiration date of the finished product. However, when the expiration date of the finished product is very short (as $\tau = 1/12$ in Table 1), it is recommended to let the production cycle exceed the expiration date and rework the expired products. When the expiration date is longer, the production cycle should be longer, more materials are purchased, and the total profit will increase. These results show that if you can find methods to extend the expiration date, it will help increase the total profit.

4.3. Sensitivity analysis

According to Table 2, when the demand rate D is 8,000 kg/year, the optimal length of production period is $t_1^* = 0.06554$ years, the optimal production cycle is $T^* = 0.15701$ years, the optimal material purchase quantity is $q_m^* = 1,323.79$ kg, and the total annual profit is TP^* =NT\$871,377; if D increases to 12,000 kg/year, t_1^* will be extended to 0.08982 years, T^* will be shortened to 0.14582 years, q_m^* will be increased to 1,820.81 kg, TP^* will be increased to NT\$ 1,393,530. It can be seen that when the market demand increases, manufacturers will first purchase a large amount of raw materials, and formulate a longer production period to increase production to meet the demand. The increase in the amount of single purchase of materials will reduce the number of orders. The total profit will increase due to the increase in revenue and the decrease in cost.

When the production rate P is 16,000 kg/year, the optimal length of production period is $t_1^* = 0.10375$ years, the optimal production cycle is $T^* = 0.16152$ years, the optimal material purchase quantity is $q_m^* = 1,686.1$ kg, and the total annual profit is TP^* =NT\$1,146,030; if P increases to 24,000 kg/year, t_1^* will be shortened to 0.06176 years, T^* will also be shortened to 0.14277 years, q_m^* will be reduced to 1,496.07 kg, and TP^* will be reduced to NT\$ 1,119,660. For manufacturers, if the demand cannot

Table 2: Impacts of demand rate and production rate changes on the optimal solution.

Parameters		t_1^*	T^*	q_m^*	TP^*
	8000	0.06554	0.15701	1,323.79	871,377
	9000	0.07134	0.15263	1,442.15	999,883
D	10000	0.07727	0.14943	1,563.49	1,129,800
	11000	0.08341	0.14720	1,689.21	1,261,040
	12000	0.08982	0.14582	1,820.81	1,393,530
	16000	0.10375	0.16152	1,686.10	1,146,030
	18000	0.08850	0.15444	1,614.31	1,136,850
P	20000	0.07727	0.14943	1,563.49	1,129,800
	22000	0.06863	0.14568	1,525.53	1,124,210
	24000	0.06176	0.14277	1,496.07	1,119,660

be expected to increase, the production rate should be maintained at the existing level. Excessive production and accumulation of finished products, in addition to causing inventory problems, may also lead to finished products expiring and requiring rework, and resulting in reduced profits.

Table 3: Impacts of changes in operating costs on the optimal solution.

Parameters		t_1^*	T^*	q_m^*	TP^*
	1600	0.07639	0.14777	1,545.37	1,132,500
	1800	0.07683	0.14860	1,554.46	1,131,150
A	2000	0.07727	0.14943	1,563.49	1,129,800
	2200	0.07771	0.15024	1,572.46	1,128,470
	2400	0.07815	0.15106	1,581.38	1,127,140
	12000	0.07033	0.13643	1,421.64	1,150,790
	13500	0.07389	0.14311	1,494.36	1,140,060
S	15000	0.07727	0.14943	1,563.49	1,129,800
	16500	0.08050	0.15544	1,629.50	1,119,960
	18000	0.08358	0.16118	1,692.77	1,110,490
	36.0	0.07838	0.15149	1,586.17	1,222,910
	40.5	0.07782	0.15045	1,574.71	1,176,350
v	45.0	0.07727	0.14943	1,563.49	1,129,800
	49.5	0.07674	0.14842	1,552.50	1,083,270
	54.0	0.07621	0.14744	1,541.75	1,036,740
	4.0	0.07741	0.14969	1,566.42	1,130,210
	4.5	0.07734	0.14956	1,564.95	1,130,010
h_1	5.0	0.07727	0.14943	1,563.49	1,129,800
	5.5	0.07720	0.14929	1,562.03	1,129,600
	6.0	0.07713	0.14916	1,560.57	1,129,400
	12.0	0.07764	0.15012	1,571.07	1,130,930
	13.5	0.07746	0.14977	1,567.27	1,130,370
h_2	15.0	0.07727	0.14943	1,563.49	1,129,800
	16.5	0.07709	0.14908	1,559.73	1,129,240
	18.0	0.07691	0.14874	1,556.00	1,128,690

According to Table 3, when the ordering cost for materials A is NT\$1,600 , the optimal length of production period is $t_1^* = 0.07639$ years, the optimal production cycle is $T^* = 0.14777$ years, the optimal material purchase quantity is $q_m^* = 1,545.37$ kg, and the total annual profit is TP^* =NT\$1,132,500; if A increases to NT\$2,400, t_1^* will be extended to 0.07815 years, T^* will be extended to 0.15106 years, q_m^* will be increased to 1,581.38 kg, TP^* will be reduced to NT\$ 1,127,140 due to the increase in cost. It can be seen that when the order cost of each batch increases, the manufacturer will increase purchase quantity of materials to reduce purchase frequency. The increase in purchase quantity also prolongs the length of production period and the production cycle, and the amount of the finished products also increases accordingly. Instead, it causes an increase in holding costs and makes the total profit decline.

When the setup cost S is NT$12,000, the optimal length of production period is $t_1^* = 0.07033$ years, the optimal production cycle is $T^* = 0.13643$ years, the optimal material purchase quantity is $q_m^* = 1,421.64$ kg, and the total annual profit is TP^* =NT$1,150,790; if S increases to NT$18,000, t_1^* will be extended to 0.08358 years, T^* will be extended to 0.16118 years, q_m^* will be increased to 1,692.77 kg, TP^* will be reduced to NT$ 1,110,490 due to the increase in cost. It can be seen that when the setup cost of each batch increases, the manufacturer will choose to reduce the frequency of equipment replacement in order to save costs, and increase the purchase quantity of materials to extend the length of production period and the production cycle. At the same time, the output of finished products will also increase, but it causes an increase in holding costs and reduces the total profits.

When the production cost v is NT$36/kg, the optimal length of production period is $t_1^* = 0.07838$ years, the optimal production cycle is $T^* = 0.15149$ years, the optimal material purchase quantity is $q_m^* = 1,586.17$ kg, and the total annual profit is TP^* =NT$1,222,910; if v increases to NT$54/kg, t_1^* will be shortened to 0.07621 years, T^* will be shortened to 0.14744 years, q_m^* will be reduced to 1,541.75 kg, TP^* will be reduced to NT$ 1,036,740 due to the increase in cost. It can be seen that when the production cost increase, the manufacturer will take measures to improve production efficiency, reduce material purchases, and shorten the length of production period and production cycles in order to reduce the production cost, but the increase in the cost also reduces profits.

When the holding cost of raw material per unit per unit time h_1 is NT$4.0/kg, the optimal length of production period is $t_1^* = 0.07741$ years, the optimal production cycle is $T^* = 0.14969$ years, the optimal material purchase quantity is $q_m^* = 1,566.42$ kg, and the total annual profit is TP^* =NT$1,130,210; if h_1 increases to NT$6.0/kg, t_1^* will be shortened to 0.07713 years, T^* will also be shortened to 0.14916 years, q_m^* will be reduced to 1,560.57 kg, TP^* will be reduced to NT$ 1,129,400 due to the increase in cost. It can be seen that with the increase in the holding cost of materials, manufacturers will hope to reduce the holding period of materials and request to shorten the length of production period and production cycle, or reduce the amount of materials purchased, because the increase in the cost also reduces the profits.

When the holding cost of finished product per unit per unit time h_2 is NT$12.0/kg, the optimal length of production period is $t_1^* = 0.07764$ years, the optimal production cycle is $T^* = 0.15012$ years, the optimal material purchase quantity is $q_m^* = 1,571.07$ kg, and the total annual profit is TP^* =NT$1,130,930; if h_2 increases to NT$18.0/kg, t_1^* will be shortened to 0.07691 years, T^* will also be shortened to 0.14874 years, q_m^* will be reduced to 1,556 kg, TP^* will be reduced to NT$ 1,128,690 due to the increase in cost. It can be seen that due to the increase in the holding cost of finished products, the manufacturer will hope to shorten the length of production period and production cycle in order to save the production cost, or to reduce the amount of materials purchased to reduce the output of finished products. But the increase in the cost also caused a decrease in the profit.

Table 4: Impacts of prices and exchange rate changes on the optimal solution.

Parameters		t_1^*	T^*	q_m^*	TP^*
	4.8	0.08394	0.16183	1,700.03	1,507,200
	5.4	0.08039	0.15525	1,627.42	1,318,320
c	6.0	0.07727	0.14943	1,563.49	1,129,800
	6.6	0.07449	0.14423	1,506.61	941,623
	7.2	0.07200	0.13955	1,455.58	753,735
	28	0.07931	0.15323	1,605.24	1,255,440
	29	0.07827	0.15129	1,583.94	1,192,600
r	30	0.07727	0.14943	1,563.49	1,129,800
	31	0.07631	0.14763	1,543.81	1,067,040
	32	0.07538	0.14590	1,524.87	1,004,320
	288	0.07727	0.14943	1,563.49	409,804
	324	0.07727	0.14943	1,563.49	769,804
s	360	0.07727	0.14943	1,563.49	1,129,800
	396	0.07727	0.14943	1,563.49	1,489,800
	432	0.07727	0.14943	1,563.49	1,849,800

According to Table 4, when the material purchase price c is \$4.8/kg, the optimal length of production period is $t_1^* = 0.08394$ years, the optimal production cycle is $T^* = 0.16183$ years, the optimal material purchase quantity is $q_m^* = 1,700.03$ kg, and the total annual profit is TP^* =NT\$1,507,200; if c increases to \$7.2/kg, t_1^* will be shortened to 0.072 years, T^* will be shortened to 0.13955 years, q_m^* will be reduced to 1,455.58 kg, TP^* will be reduced to NT\$ 753,735. It can be seen that when the purchase price increases, the manufacturer often adopt a strategy of reducing the purchase quantity, and wait for the purchase price to decrease. As the amount of materials purchased is reduced, the production period and production cycle are also shortened, the output of finished products is relatively reduced, and the total profit is reduced.

When the bank selling exchange rate r is 28 (\$ to NT\$), the optimal length of production period is $t_1^* = 0.07931$ years, the optimal production cycle is $T^* = 0.15323$ years, the optimal material purchase quantity is $q_m^* = 1,605.24$ kg, and the total annual profit is TP^* =NT\$1,255,440; if r increases to 32, t_1^* will be shortened to 0.07538 years, T^* will be shortened to 0.1459 years, q_m^* will be reduced to 1,524.87 kg, TP^* will be reduced to NT\$ 1,004,320 due to the loss of exchange rate and the increase of costs. If the manufacturer cannot reflect the increased raw material cost due to exchange rate fluctuations to the selling price, it will adopt a conservative purchase strategy to reduce the material purchase amount of each batch, and the production period and production cycle are also relatively shortened, and the total profit will also reduce.

When the unit selling price s is NT$288/kg, the optimal length of production period is $t_1^* = 0.07727$ years, the optimal production cycle is $T^* = 0.14943$ years, the optimal material purchase quantity is $q_m^* = 1,563.49$ kg, and the total annual profit is TP^* =NT$409,804; if s increases to NT$432/kg, t_1^* remains at 0.07727 years, T^* remains at 0.14943 years, q_m^* remains at 1,563.49 kg, TP^* will increase to NT$ 1,849,800 due to the increase in the selling price. It can be seen that the increase in the selling price can directly increase the total profit of the manufacturer.

Table 5: Effects of deterioration and rework parameters on the optimal solution.

Parameters		t_1^*	T^*	q_m^*	TP^*
	0.24	0.07889	0.15244	1,592.73	1,134,230
	0.27	0.07807	0.15091	1,577.90	1,132,010
θ_m	0.30	0.07727	0.14943	1,563.49	1,129,800
	0.33	0.07650	0.14798	1,549.46	1,127,620
	0.36	0.07575	0.14658	1,535.80	1,125,460
	48	0.07727	0.14943	1,563.49	1,129,800
	54	0.07727	0.14943	1,563.49	1,129,800
v_r	60	0.07727	0.14943	1,563.49	1,129,800
	66	0.07727	0.14943	1,563.49	1,129,800
	72	0.07727	0.14943	1,563.49	1,129,800
	0.5	0.07727	0.14943	1,563.49	1,129,800
	0.6	0.07727	0.14943	1,563.49	1,129,800
δ	0.7	0.07727	0.14943	1,563.49	1,129,800
	0.8	0.07727	0.14943	1,563.49	1,129,800
	0.9	0.07727	0.14943	1,563.49	1,129,800

According to Table 5, when the deterioration rate of raw material θ_m is 0.24/year, the optimal length of production period is $t_1^* = 0.07889$ years, the optimal production cycle is $T^* = 0.15244$ years, the optimal material purchase quantity is $q_m^* = 1,592.73$ kg, and the total annual profit is $TP^* =$ NT$1,134,230; if θ_m increases to 0.36/year, t_1^* will be shortened to 0.07575 years, T^* will be shortened to 0.14658 years, q_m^* will be reduced to 1,535.8 kg, TP^* will be reduced to NT$ 1,125,460. It can be seen that when the deterioration rate increases, the manufacturer will hope to reduce the holding time of materials or reduce the amount of materials purchased by shortening the length of production period and production cycle. However, the increase in holding costs due to material deterioration also causes a decrease in profits.

The optimal solution (the optimal length of production period $t_1^* = 0.07727$, the optimal production cycle $T^* = 0.14943$, the optimal material purchase quantity $q_m^* = 1,563.49$ kg, the total annual profit TP^* =NT$1,129,800) does not change with the changes in rework cost v_r. The reason is that the optimal solution falls in situation 1, the products will be sold out within the expiration date, and there is no need for rework. So v_r has no effects on t_1^*, T^*, q_m^* and TP^*.

The change of processing yield δ has no effect on the optimal solution (the optimal length of production period $t_1^* = 0.07727$, the optimal production cycle $T^* = 0.14943$, the optimal material purchase quantity $q_m^* = 1,563.49$ kg, the total annual profit TP^* =NT\$1,129,800). The reason is that the optimal solution falls in situation 1, and there is no need for rework.

Through sensitivity analyses, we sort out the impact trends of parameter changes in the model on the relevant optimal solutions, and summarize in Table 6.

Table 6: The impact trends of parameters on the optimal solution.

	t_1^*	T^*	q_m^*	TP^*
D	+	−	+	+
P	−	−	−	−
A	+	+	+	−
S	+	+	+	−
c	−	−	−	−
v	−	−	−	−
r	−	−	−	−
s	×	×	×	+
h_1	−	−	−	−
h_2	−	−	−	−
θ_m	−	−	−	−
v_r	×	×	×	×
δ	×	×	×	×
τ	+	+	+	+

Note: + means positive impact, − means negative impact, × means no impact.

5. Conclusions

This study established a two-stage production inventory model based on the traditional EPQ model, considering imported materials, and the deterioration, expiration date, rework of the finished products. In addition, we considered different currencies to explore the impact of exchange rate changes on the model. The main purpose is to determine the optimal length of the production period, the length of the production cycle and the amount of materials purchased, to determine the maximum total profit of the manufacturer.

Integrating the analysis results of the numerical example, we conclude that:

(1) The increase in demand rate will increase the length of the production period, increase the amount of materials purchased, and increase the profit.

(2) The higher the production rate, the shorter the production period, production cycle and the order quantity of materials. Further, it is not benefit for the total profit when the production rate increases.

(3) The increases in material ordering cost and setting cost will extend the length of production period and production cycle, and increase the material purchase quantity, but will reduce the total profit.

(4) The increases of material purchase price, unit production cost, exchange rate, holding costs of materials and finished products, and deterioration rate of materials will shorten the length of production period and production cycle, and reduce the amount of materials purchased, while reducing total profits.

(5) The selling price of the finished product has no effect on the production period, production cycle and the amount of materials purchased, but it has a positive effect on the total profit.

(6) Longer expiration date of finished products will extend the length of production period and production cycle, increase the material purchase quantity and total profit.

From the perspective of operation and management, when the market demand increases, manufacturers should increase the purchase of raw materials, and formulate a longer production period to increase production to meet demand. For manufacturers, in order to increase the total profit, in addition to internal throttling, open source is more important. In the highly competitive environment, the possibility of raising selling price is greatly reduced. If the demand can be increased, the increase in profits is more direct. Under normal circumstances, enterprises should complete production, storage and sales within the expiration date of finished products as much as possible. At the same time, if it can effectively extend the expiration date of the finished product, it will be very conducive to the increase in total profit. Special attention should be paid to changes in exchange rates that will have many effects on companies that use imported materials for production, and have a more direct impact on total profit.

This study can be seen as a basis and reference for related topics in management decisions and future academic research. Subsequent research can be based on the model of this study, considering the production rate changes with the demand rate, the deterioration rate is not fixed, and shortage, etc..

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