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Omni-channel Pricing and Procurement Strategies with Consumer Returns and Order Cancellation under Reference Point Effects

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Abstract

This paper considers the online retailer's omni-channel retail operations under reference point effects in which consumers can cancel their orders before payment and return the products after payment if the products doesn't meet their expectation. The online retailer's optimal pricing and inventory decisions are derived under the omni-channel strategy by maximizing the total expected utility. The analysis reveals a threshold strategy on the retailer's optimal pricing and inventory decisions and the optimal total expected utility while considering the impact of reference point effects. Moreover, with the increase of the retailer's loss aversion or the optimism level, the order quantity and overall expected utility decrease, while the optimal price presents a threshold type. Finally, the comparison of the utility performance between with and without reference point effects is presented.

Keywords: Omni-channel, pricing-inventory management, consumer returns, consumer order cancellation, reference point effects.

1. Introduction

With the vigorous development of mobile Internet in recent years, the business model based on Internet is rapidly replacing the traditional business model with physical and offline channels. The way of communication between retailers and consumers has changed dramatically. In order to improve the probability of successful marketing and increase market share, many traditional retailers based on brick-and-mortar have developed a series of new retail models, such as online, mobile phone, email, QQ, etc. A new retail model that opens up various channels, "omni-channel retail" came into being. For instance, facing the competition from JD.com, both Suning and Gome, the two largest traditional physical home appliance retailers in China, launched their online businesses in 2009. Additionally, the world-famous e-commerce retailers, such as Amazon, Google and eBay etc., have also built their tech-enabled physical stores. Omni-channel retail focuses on "a truly integrated approach across the whole retail operation that delivers a seamless response to the consumer experience through all available shopping channels" (see Rigby [27] and Saghiri et al. [28]). Therefore, omni-channel business is becoming more and more pervasive as consumers tend to switch between online and offline channels and exhibit higher levels of satisfaction and loyalty (see Wallace et al. [34]). This not only brings unprecedented opportunities to retailers, but also brings new challenges to their pricing and inventory optimization, for instance, traditional pricing and inventory models optimize channel prices under the assumption that there is no inventory sharing between channels. But this assumption doesn't valid in omni-channel with which physical store inventory is used for fulfilling customer orders placed online. Thus, from academic and practical perspectives, it is necessary to investigate the coordination of pricing and inventory decisions in omni-channel retail environment.

Studies on omni-channel retailing are emerging, most of which are exploratory. At present, the omni-channel research mainly focuses on the following aspects, channel conflicts and synergies (see Kim and Chun [19], Lee et al. [20] and Wiener et al. [37]); construction and evaluation of omni-channel distribution and logistics system (see Hübner et al. [13, 14] and Murfield et al. [24]); channel choices of omni-channel consumers (see Gao et al. [6] and Park and Lee [25]); purchasing behavior of omni-channel consumers (see Blom et al. [2], Hosseini et al. [12], Shen et al. [30] and Yurova et al. [40]) and order fulfillment for omni-channel consumers (see Ishfaq and Raja [15] and Wollenburg et al. [38]). The studies that are closely related to ours are those of omni-channel pricing and inventory models. As omni-channel retailing is a relatively new area, there are few academic papers on optimization of omni-channel pricing and inventory operations, especially on joint pricing and inventory. For example, omni-channel pricing strategy (see Gao and Su [5], Halzack [7], Harsha et al. [9], Jin et al. [16] and Matthews [23]); omni-channel inventory strategy (see Gallino et al. [4] and Kembro et al. [18]); omni-channel joint pricing and inventory strategy (see Zhang et al. [41]). Among them, the most related work to ours is Zhang et al. [41], in which the authors consider the joint optimal pricing and inventory control problem for a omni-channel system with consumer returns and order cancellation. The difference between ours and Zhang et al. [41] is that we consider the retailer's behavior, i.e., reference point and loss aversion.

Recent studies on prospect theory and behavioral science have recognized that decision makers are subject to anchoring effects, i.e., the cognitive bias, which has an important impact on the behavior of decision makers to make judgement that biased towards an initially presented value (see Kahneman and Tversky [17]). In a seminal experimental work, Schweitzer and Cachon [29] showed that the realized order quantity of the decision maker often violates that of expected profit maximizing, and they introduced loss aversion (i.e., the losses result in larger disutility as compared to the utility gain of the same magnitude) in the prospect theory to explain such a deviation by relying on the risk attitudes toward gains and losses. However, they found that prospect theory cannot systematically explain the ordering behavior observed in experiments. The reason is that they ignored the non-zero reference point (i.e., non-zero reference payoff). The non-zero reference point acts as the decision maker's optimization objective towards realized profit, which depends on his/her decisions. Long and Nasiry [21] proposed an alternative based on newsvendor's non-zero reference point and showed that prospect theory can, in fact, account for the experimental results by considering such a non-zero reference point. In our daily lives, when one retailer makes decisions, the decision maker's valuation of a particular option actually relies on relative gain or loss from an expectation level. Hence, the reference point and loss aversion are reasonably included as another factors impacting retailers' pricing and ordering decisions.

To our best knowledge, previous researchers, either through experimental work or through analytical modeling work, have studied the impact of reference point and loss aversion when the retailer is only making the pricing decisions (see Baron et al. [1] and Heidhues and Koszegi [10]) or the inventory decisions (see Baron et al. [1], Long and Nasiry [21], Shi et al. [31], Vipin and Amit [33], Wang et al. [36] and Xu et al. [39]). However, only a few papers have jointly investigated reference point and loss aversion in the coordination of pricing and inventory problem (see Mandal et al. [22]). Moreover, recent research on omni-channel pricing and inventory ignores the reference point and loss aversion (see Zhang et al. [41]). So the following technical problems will arise: (1) Does the optimal pricing and ordering strategies in omni-channel retail environment under reference point and loss aversion exist? If exist, are they unique? (2) How does the reference point and loss aversion influence the pricing and order quantity of a loss averse retailer with reference point? (3) How does the reference point and loss aversion affect the optimal pricing and inventory decisions?

To tackle these technical questions, we develop a online retailer's omni-channel retail operation model under reference point and loss aversion in which consumers can cancel their orders before payment and return the products after payment if the products does not meet their expectation. The online retailer's optimal pricing and inventory decisions are derived under the omni-channel strategy by maximizing the total expected utility. The analysis reveals a threshold strategy on the retailer's optimal pricing and inventory decisions as well as the optimal total expected utility while considering the impact of reference point effects. Moreover, our analysis also shows that the order quantity and overall expected utility decrease with the increase of the retailer's loss aversion or the level of optimism, while the optimal price presents a threshold type. Finally, we compare the performance between considering and not considering the reference point effects. We also identify the conditions under which the retailer benefits from the omni-channel retailing strategy under reference point effects. Thus, we contribute to the literature by studying the impact of human decision biases on the retailer's pricing and inventory strategies in omni-channel retail setting.

The rest of the paper is organized as follows. In section 2, we present a theoretical model to formulate the omni-channel under reference point effects as well as the consumer behavior under different channels. In Section 3, we explore the optimal decisions with reference point effects under the omni-channel strategy. Section 4 investigates the performance of adding reference point effects. Section 5 concludes our paper. We present all the proofs in Appendix.

2. Model description

2.1. Modeling the omni-channel under reference point effects

Consider a retailer, initially an online retailer, who previously operated a single online channel that only allowed consumers to shop online directly. Currently, the retailer has added a physical store and will implement a new sales strategy, i.e., the omni-channel strategy, which allows consumers to place orders online without paying immediately. With this allowance, a consumer who places an order online can choose to (i) pay online and wait for the package delivered, or (ii) reserve online in advance without payment, then visit the physical stores to touch and feel the product, and finally pay and pick up it (i.e., "ROPS" mode). The buying procedure of a consumer who orders online is depicted in Figure 1, where $1-\alpha$ and α be the fractions of consumers who choose to (i) and (ii), respectively.

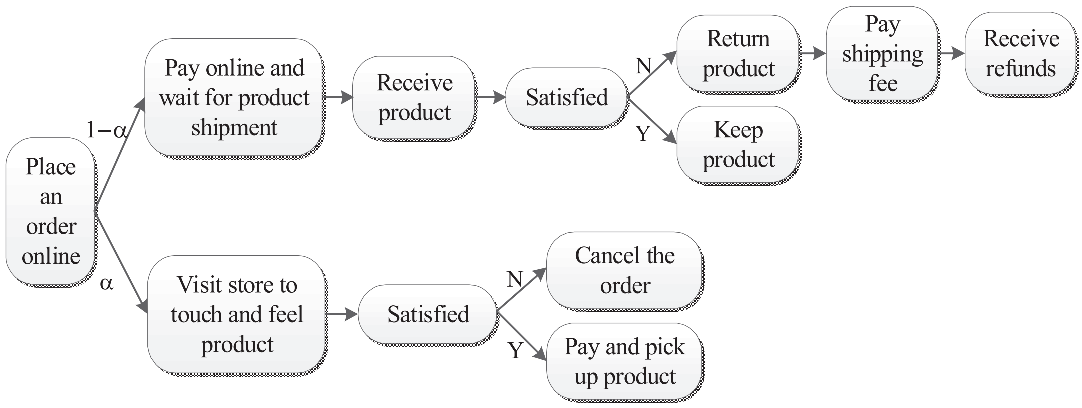

Figure 1: Buying procedure of a consumer who orders online under the omni-channel environment.

As shown in Figure 1, after placing an order online, if a consumer chooses to (i), the consumer knows the exact value of the product only after receiving it. If the actual value doesn't meet his/her expectation, he/she can return the product with a full refund (Choi and Guo [3]), but he/she needs to pay the return shipping fee for each unit of the product (Pei et al. [26]). Suppose the forward shipping fee is paid by the retailer while the return shipping fee is paid by the consumer and the express company charges the same shipping fee m for both of them. If an online consumer chooses to (ii), he/she needs to pay a travelling cost t ($t > 0$) to visit the store, and then decides whether to keep the product or cancel the order in store. The cancellation of the orders doesn't pay anything. Moreover, assume that there is an additional profit b from every consumer visiting the store (UPS [32]).

In omni-channel environment, the retailer charges the same price p over the physical and online channels. The inventory is shared across channels (Harsha and Subramanian

[8]), thus, there is a single decision Q to be made before demand is realized. The purchase cost is c per unit, s is the salvage value per unit satisfying $p > c > s$, and shortage cost is not considered. Let v be the consumer's valuation for the product which is random and follows the cdf $G(\cdot)$ (also the pdf $g(\cdot)$). Without loss of generality, we assume that $v > c$.

The aggregate market demand X under the omni-channel strategy is random which is continuously distributed over $[\underline{x}, \overline{x}] \subset R_+$ and follows the cdf $F(\cdot)$ (also the pdf $f(\cdot)$). $F(\cdot)$ and $f(\cdot)$ are differentiable over $[\underline{x}, \overline{x}]$. Let k $(0 < k < 1)$ be the fraction of the demand served under the single online channel, then the demand was kX when the retailer used to operate a single online channel. However, the omni-channel strategy will lead to an incremental demand $(1-k)X$ for the retailer from both the online and offline channels (Zhang et al. [41]). Let β and $1-\beta$ be the fractions of the incremental demand coming from the online and the offline market, respectively.

In this paper, the retailer is assumed to be loss-averse (more sensitive to losses than gains). Similar loss aversion has been modeled in the operations management literature by Herweg [11] and Long and Nasiry [21]. Following Mandal et al. [22], Schweitzei and Cachon [29] and Wang and Webster [35], to characterize the retailer's loss aversion, we introduce the piecewise-linear gain-loss utility function which is expressed by

$$\nu(y) = \begin{cases} \eta y, & y \geq 0, \\ \lambda \eta y, & y < 0, \end{cases} \tag{2.1}$$

where y is the difference between the realized payoff and reference payoff level (defined in (2.2) below). η captures the strength of the reference point effects. A higher value of η indicates a higher degree of sensitiveness to the deviations from the reference point, $\eta = 0$ represents having no reference point effects. $\lambda \geq 1$ is the loss aversion coefficient, i.e., the retailer is loss averse if $\lambda > 1$, and loss neutral if $\lambda = 1$.

This paper focuses on finding the optimal pricing and order quantity to maximize the retailer's total expected utility. Following the similar setting of the expected utility provided by Herweg [11] and Mandal et al. [22]. The total expected utility we consider consists of two components: intrinsic expected utility and gain-loss expected utility. The intrinsic expected utility component corresponds to the realized profit without considering the loss aversion, whereas, the gain-loss expected utility component corresponds to the psychological value of the profit, determined by (2.1), i.e., comparing the actual profit to a reference payoff level $r(p, Q)$, given that the retailer's price p and order quantity Q. Based on this, the retailer's total utility, denoted by $U_r(X, p, Q)$, is given by

$$\begin{aligned} U_r(X,p,Q) &= \pi(X,p,Q) + \nu(\pi(X,p,Q) - r(p,Q)) \\ &= \pi(X,p,Q) - \lambda\eta[r(p,Q) - \pi(X,p,Q)]^+ + \eta[\pi(X,p,Q) - r(p,Q)]^+ \end{aligned} \tag{2.2}$$

where $z^+ = \max\{z, 0\}$.

The first component of (2.2), $\pi(X, p, Q)$, i.e., the retailer's profit, is the intrinsic utility. The second term, $\lambda\eta[r(p, Q) - \pi(X, p, Q)]^+$, is the first sub-component of the gain-loss utility function, which emerges due to the loss aversion and the reference bias of the retailer. Finally, the third term, $\eta[\pi(X,p,Q) - r(p,Q)]^+$, is the second sub-component of the gain-loss utility function, which emerges only due to the reference bias of the retailer.

2.2. Consumer behavior under different channels

In omni-channel retail environment, the exact inventory information is available to the consumers when they place orders online, and they will not order the product for those out of stock. Hence, a consumer who places an order online doesn't suffer a utility loss from the stock-out risk. In addition, the inventory is shared across channels. Thus, on one side, consumers can choose any channel to buy the product according to their preference. On the other, the retailer has the ability to fulfill a transaction from both channels. Table 1 shows the utility of a consumer at different trading times for both online and offline payment under the omni-channel strategy.

Table 1: Consumer utility under two different channels.

Demand source	Purchase channels	At the time of placing the order	At the time of receiving the product	
		Place order	Keep (Buy) product	Return product/ Cancel order/ Leave store
Online consumers	Buy online directly	$-p$	$v-p$	$-m$
	ROPS	0	$v-p-t$	$-t$
Offline consumers	Buy offline directly	0	$v-p-t$	$-t$

According to Table 1, the consumer's surplus corresponding to buy online directly and ROPS are $\max\{v-p,-m\}$ and $\max\{v-p-t,-t\}$, respectively. Follows from the principle of consumer utility maximization, if a consumer chooses the buy online directly channel, the probabilities that the consumer keeps and returns the product are $\overline{G}(p-m)$ and $G(p-m)$, respectively. If a consumer chooses the ROPS channel, the probabilities that the consumer chooses to buy or not buy the product are $\overline{G}(p)$ and $G(p)$, respectively. The consumer's surplus corresponding to buy offline directly is $\max\{v-p-t,-t\}$, thus, the probabilities that an offline consumer chooses to buy or not to buy the product are $\overline{G}(p)$ and $G(p)$, respectively.

2.3. Notations and assumptions

The related parameters and variables used in this paper are summarized in Table 2; other notations will be defined as needed.

To facilitate the analysis, we define the following functions.

$$L_1(p) = \overline{G}(p-m) - (p-s)g(p-m), \tag{2.3}$$

$$L_3(p) = \overline{G}(p) - (p-s)g(p), \tag{2.4}$$

Table 2: Summary of notations.

Notation	Description
Subscriptions	
O	The omni-channel strategy.
Decision	
p	The online retailer's retail price.
Q	The online retailer's inventory (ordering) quantity.
Parameters	
m	The unit shipping fee.
t	The unit travelling cost of consumers to visit the store.
v	The valuation of the product by the consumer, a random value that follows distribution $G(\cdot)$ and density $g(\cdot)$, $\overline{G}(\cdot) = 1 - G(\cdot)$.
c	The unit inventory and procurement cost.
s	The salvage price for a leftover unit.
b	The cross-selling benefit.
α	The fraction of online consumers choosing to ROPS and $1-\alpha$ is the fraction of online consumers choosing to buy online directly.
β	The fraction of the incremental demand (brought by the omni-channel strategy) coming from the online market $1-\beta$ and is the fraction of the incremental demand coming from offline market.
k	The fraction of the market demand served under the single online channel strategy.
η	Reference point effects parameter ($\eta > 0$).
λ	Coefficient of loss aversion ($\lambda \geq 1$).
ρ	The online retailer's level of optimism ($0 \leq \rho \leq 1$).
X	The aggregate market demand under the omni-channel strategy, a random value that follows distribution $F(\cdot)$ and density $f(\cdot)$, $\overline{F}(\cdot) = 1 - F(\cdot)$.
$H(z)$	Partial expectation of the random variable X, i.e., $H(z) = \int_0^z xf(x)dx$.

$$L_2(p) = \lambda_1 L_1(p) + \lambda_2 L_3(p), \tag{2.5}$$

$$A(p) = (p-s)\overline{G}(p-m) - m, \text{ and } B(p) = (p-s)\overline{G}(p) + b, \tag{2.6}$$

where $\lambda_1 = (1-\alpha)[\beta(1-k)+k]$, $\lambda_2 = \alpha[\beta(1-k)+k] + (1-\beta)(1-k)$.

The properties of functions (2.3)-(2.6) can be referred to Lemma 1-3 of Zhang et al. [41]. Furthermore, we need the following assumptions.

Assumption 1. *Both $F(\cdot)$ and $G(\cdot)$ have increasing failure rates, i.e., $f(x)/\overline{F}(x)$ is increasing in x and $g(v)/\overline{G}(v)$ is increasing in v.*

Assumption 2. *Suppose $-g(p-m)-(p-s)g'(p-m) < 0$ and $-g(p)-(p-s)g'(p) < 0$ throughout the paper.*

It is worth mentioning that Assumption 2 ensures the retailer's expected utility function to be concave with respect to p and Q and thus the maximum p and Q are unique. In addition, the following lemma can be obtained (see Zhang et al. [41]).

Lemma 1. $L_i(p)$ $(i = 1, 2, 3)$ *is decreasing in* p.

In the following sections, we first present the optimal joint pricing and ordering policies for the omni-channel strategy. Second, the impacts of reference point effects, loss degree and retailer's optimistic level on the optimal pricing and ordering strategy as well as the optimal expected utility are emphatically analyzed.

3. The Omni-channel Strategy

In this section, we analyze the optimal omni-channel pricing and inventory strategy for the online retailer. The profit of the online retailer is

$$\begin{aligned}
&\pi_O(X,p,Q)\\
&= pK_1(1-\alpha)\overline{G}(p-m)(X\wedge Q) + sK_1(1-\alpha)G(p-m)(X\wedge Q) - mK_1(1-\alpha)(X\wedge Q)\\
&\quad + pK_1\alpha\overline{G}(p)(X\wedge Q) + sK_1\alpha G(p)(X\wedge Q) + bK_1\alpha(X\wedge Q)\\
&\quad + pK_2\overline{G}(p)(X\wedge Q) + sK_2G(p)(X\wedge Q) + bK_2(X\wedge Q) - cQ + s[Q-(X\wedge Q)]\\
&= \begin{cases} [\lambda_1 A(p) + \lambda_2 B(p)]\,X - (c-s)Q, & X < Q,\\ [\lambda_1 A(p) + \lambda_2 B(p)]\,Q - (c-s)Q, & X \ge Q, \end{cases}
\end{aligned} \tag{3.1}$$

where $\wedge$ is the minimum operator. $K_1 = \beta(1-k)+k$, $K_2 = (1-\beta)(1-k)$ and $K_1+K_2 = 1$. $\lambda_1 = (1-\alpha)K_1$, $\lambda_2 = \alpha K_1+K_2$ and $\lambda_1+\lambda_2 = 1$. In (3.1), the first three terms denotes the revenue from online consumers who buy online directly, while the fourth to sixth terms represent the revenue from ROPS consumers. The seventh to ninth terms represent the revenue from offline consumers. cQ and $s[Q-(X\wedge Q)]$ present the procurement cost and the salvage revenue of leftover units, respectively. Moreover, since the retailer's maximum possible payoff and the minimum possible payoff are $(1-\alpha)(p-c-m)Q + \alpha(p-c)Q$ and $[\lambda_1 A(p) + \lambda_2 B(p)]\underline{x} - (c-s)Q$, respectively. Thus, the retailer's reference point is

$$r_O(p,Q) = \rho[(1-\alpha)(p-c-m) + \alpha(p-c)]Q + (1-\rho)\{[\lambda_1 A(p) + \lambda_2 B(p)]\underline{x} - (c-s)Q\},$$

where $\rho \in [0,1]$ represents the retailer's optimism level. For a given reference level $r_O(p,Q)$, the total expected utility for the retailer is

$$\begin{aligned}
&E[\widetilde{U}_r(X,p,Q)]\\
&= E[\pi_O(X,p,Q)] - \lambda\eta E[r_O(p,Q)-\pi_O(X,p,Q)]^+ + \eta E[\pi_O(X,p,Q)-r_O(p,Q)]^+\\
&= E[\pi_O(X,p,Q)] - \lambda\eta\int_{\underline{x}}^{\sigma}\{r_O(p,Q) - [\lambda_1 A(p)+\lambda_2 B(p)]x + (c-s)Q\}f(x)dx\\
&\quad + \eta\int_{\sigma}^{Q}\{[\lambda_1 A(p)+\lambda_2 B(p)]x - (c-s)Q - r_O(p,Q)\}f(x)dx
\end{aligned}$$

$$+\eta\{[\lambda_1 A(p)+\lambda_2 B(p)]Q-(c-s)Q-r_O(p,Q)\}\overline{F}(Q), \tag{3.2}$$

where $\sigma=\sigma(p,Q)=[r_O(p,Q)+(c-s)Q]/[\lambda_1 A(p)+\lambda_2 B(p)]$. The retailer's task is to find out the optimal pricing and order quantity to maximize his overall expected utility.

In what follows, we first analyze the ordering strategy of the retailer under reference point effects when the price is exogenous. Secondly, we further analyze the joint pricing and ordering strategies of the retailer under reference point effects when the price is endogenous.

3.1. The case when pricing decision is exogenous

When pricing decision is exogenous, denote the exogenous price as p_O^e. Then (3.1) and (3.2) becomes

$$\pi_O(X,p_O^e,Q)=\begin{cases}[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\,X-(c-s)Q, & X<Q,\\ [\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\,Q-(c-s)Q, & X\geq Q,\end{cases} \tag{3.3}$$

and

$$\begin{aligned}&E[\widetilde{U}_r(X,p_O^e,Q)]\\ &=E[\pi_O(X,p_O^e,Q)]-\lambda\eta\int_{\underline{x}}^{\sigma}\{r_O(p_O^e,Q)-[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]x+(c-s)Q\}f(x)dx\\ &\quad+\eta\int_{\sigma}^{Q}\{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]x-(c-s)Q-r_O(p_O^e,Q)\}f(x)dx\\ &\quad+\eta\{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]Q-(c-s)Q-r_O(p_O^e,Q)\}\overline{F}(Q),\end{aligned} \tag{3.4}$$

where $\sigma=\sigma(p_O^e,Q)=[r_O(p_O^e,Q)+(c-s)Q]/[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]$, $r_O(p_O^e,Q)=\rho[(1-\alpha)(p_O^e-c-m)+\alpha(p_O^e-c)]Q+(1-\rho)\{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\underline{x}-(c-s)Q\}$. The existence and uniqueness of the optimal order quantity is characterized via the following result.

Proposition 1.

(i) *The expected utility function $E[\widetilde{U}_r(X,p_O^e,Q)]$ is concave for all Q if*

$$\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)>\frac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{1+\eta}$$

and thus there exists a unique optimal order quantity Q_O^* that maximizes $E[\widetilde{U}_r(X,p_O^e,Q)]$.

(ii) *With the omni-channel strategy, the optimal order quantity Q_O^* of the online retailer is characterized by the following equation*

$$\overline{F}(Q_O^*)=$$

$$\begin{cases}1, & \lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)\leq\dfrac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{1+\eta},\\ \dfrac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{(1+\eta)[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]}, & \\ & \lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)>\dfrac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{1+\eta},\end{cases} \tag{3.5}$$

where $\sigma^* = \sigma(p_O^e, Q_O^*) = \dfrac{r_O(p_O^e, Q_O^*) + (c-s)Q_O^*}{\lambda_1 A(p_O^e) + \lambda_2 B(p_O^e)}$.

The following result characterizes the impact of the strength of reference point effects (η), loss aversion coefficient (λ) and optimism factor (ρ) on the optimal order quantity Q_O^*.

Proposition 2.

(i) *If* $\lambda \leq \left\{\dfrac{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q_O^*)}{\rho[p_O^e - (1-\alpha)m - s]} - 1\right\}\dfrac{1}{F(\sigma^*)} + 1$, *then the optimal order quantity* Q_O^* *increases with* η*; furthermore, if* $\lambda > \left\{\dfrac{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q_O^*)}{\rho[p_O^e - (1-\alpha)m - s]} - 1\right\}\dfrac{1}{F(\sigma^*)} + 1$, *then* Q_O^* *decreases with* η*, where* $\sigma^* = \sigma(p_O^e, Q_O^*) = \dfrac{r_O(p_O^e, Q_O^*) + (c-s)Q_O^*}{\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)}$.

(ii) *The optimal order quantity* Q_O^* *decreases with* λ.

(iii) *The optimal order quantity* Q_O^* *decreases with* ρ.

From Proposition 2 (i), we find that the reference point effect factor (η) follows a threshold policy. That is, there exists a threshold $\lambda_O = \left\{\dfrac{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q_O^*)}{\rho[p_O^e - (1-\alpha)m - s]} - 1\right\}\dfrac{1}{F(\sigma^*)} + 1$ such that when $\lambda \geq \lambda_O$ (i.e., the retailer is highly loss averse), the optimal inventory level decreases as the reference point effect (η) increases. Whereas, when λ is low, the retailer's optimal inventory level increases as η increases. Hence, a high loss aversion bias will lead to lower inventory level with increasing reference effects, while a lower loss aversion bias leads to the opposite. The intuition is that if the retailer is highly loss averse, as η increases (i.e., the retailer is more sensitive to the difference between the actual profit and the reference payoff level), he will be more conservative and will avoid losses due to the excessive ordering. This thus affects he to stock less. Similarly, if the loss aversion is low, although the retailer is sensitive to the difference between the actual profit and the reference payoff level, but because he is not pessimistic about the available profit, which will make him to increase the inventory level.

Proposition 2 (ii) shows that the retailer's optimal stocking decision decreases as the loss aversion coefficient λ increases. The intuition behind this is that the retailer's high loss aversion behavior will cause he to worry about the loss of sales caused by excess orders, thus reducing the order quantity. From Proposition 2 (iii), we find that the ordering quantity decreases as the retailer's level of optimism increases. This is because the retailer's reference payoff increases as ρ increases, so the disutility due to loss aversion increases, which leads to a lower inventory level. The optimistic level also reflects the retailer's degree of risk aversion. The bigger the optimistic level is, the more risk averse the retailer becomes.

The next proposition characterizes the impact of the strength of reference point effects (η), loss aversion coefficient (λ) and optimism factor (ρ) on the optimal total expected utility $E[\widetilde{U}_r, (X, p_O^*, Q)]$.

Proposition 3.

(i) *If* $\lambda \leq \left\{\frac{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]Q_O^*\overline{F}(Q_O^*)+\Omega}{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\sigma^*}-1\right\}\frac{1}{F(\sigma^*)}+1$, *then the total expected utility* $E[\widetilde{U}_r,(X,p_O^*,Q)]$ *under the optimal order quantity* Q_O^* *increases with* η*; further, if* $\lambda > \left\{\frac{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]Q_O^*\overline{F}(Q_O^*)+\Omega}{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\sigma^*}-1\right\}\frac{1}{F(\sigma^*)}+1$, *then* $E[\widetilde{U}_r,(X,p_O^*,Q)]$ *decreases with* η*, where* $\Omega = [\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)][(\lambda-1)H(\sigma^*)+H(\sigma_O^*)]$, $\sigma^* = \sigma(p_O^e,Q_O^*) = \frac{r_O(p_O^e,Q_O^*)+(c-s)Q_O^*}{\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)}$.

(ii) *The total expected utility* $E[\widetilde{U}_r,(X,p_O^*,Q)]$ *under the optimal order quantity* Q_O^* *decreases with* λ.

(iii) *The total expected utility* $E[\widetilde{U}_r,(X,p_O^*,Q)]$ *under the optimal order quantity* Q_O^* *decreases with* ρ.

Proposition 3 indicates that the optimal total expected utility $E[\widetilde{U}_r,(X,p_O^*,Q)]$ has a similar analytic properties with the optimal order quantity Q_O^*. This also confirms the fact that high risk implies high return and low risk comes with low return.

3.2. The case when pricing decision is endogenous

When pricing decision is endogenous, the optimal values of p and Q can be characterized via the following Proposition 4.

Proposition 4. *When pricing is endogenous, with the omni-channel channel strategy, the following results hold true:*

(i) *There exists a unique optimal price* p_O^* *of the online retailer which is given by*

$$L_2(p_O^*) = \frac{\eta\rho Q_O^*[(\lambda-1)F(\hat{\sigma}^*)+1]}{(1+\eta)[Q_O^*\overline{F}(Q_O^*)+H(Q_O^*)]+(\lambda-1)\eta H(\hat{\sigma}^*)-\eta(1-\rho)\underline{x}[(\lambda-1)F(\hat{\sigma}^*)+1]}. \tag{3.6}$$

(ii) *There exists a unique optimal order quantity* Q_O^* *of the online retailer which is given by*

$$\overline{F}(Q_O^*) = \begin{cases} 1, & \lambda_1 A(p_O^*)+\lambda_2 B(p_O^*) \leq \dfrac{\eta\rho[p_O^*-(1-\alpha)m-s][(\lambda-1)F(\hat{\sigma}^*)+1]+(c-s)}{1+\eta}, \\ \dfrac{\eta\rho[p_O^*-(1-\alpha)m-s][(\lambda-1)F(\hat{\sigma}^*)+1]+(c-s)}{(1+\eta)[\lambda_1 A(p_O^*)+\lambda_2 B(p_O^*)]}, & \lambda_1 A(p_O^*)+\lambda_2 B(p_O^*) > \dfrac{\eta\rho[p_O^*-(1-\alpha)m-s][(\lambda-1)F(\hat{\sigma}^*)+1]+(c-s)}{1+\eta}, \end{cases} \tag{3.7}$$

where $\hat{\sigma}^* = \hat{\sigma}(p_O^*,Q_O^*) = \frac{r_O(p_O^*,Q_O^*)+(c-s)Q_O^*}{\lambda_1 A(p_O^*)+\lambda_2 B(p_O^*)}$.

Remark 1. (i) When $\eta = 0$, i.e., there is no reference point effects, the problem reduces to the model of omni-channel strategy proposed by Zhang et al. [41]. Then the optimal price $p_O^{\text{no-rp}^*}$ and order quantity $Q_O^{\text{no-rp}^*}$ of the online retailer are given by $L_2(p_O^{\text{no-rp}^*}) = 0$ and

$$\overline{F}(Q_O^{\text{no-rp}^*}) = \begin{cases} 1, & \lambda_1 A(p_O^{\text{no-rp}^*}) + \lambda_2 B(p_O^{\text{no-rp}^*}) \leq c - s, \\ \dfrac{(c-s)}{\lambda_1 A(p_O^{\text{no-rp}^*}) + \lambda_2 B(p_O^{\text{no-rp}^*})}, & \lambda_1 A(p_O^{\text{no-rp}^*}) + \lambda_2 B(p_O^{\text{no-rp}^*}) > c - s, \end{cases}$$

respectively. Moreover, we can see from (3.6) that $L_2(p_O^*) > 0$, thus, it follows from Lemma 1 that $p_O^* \leq p_O^{\text{no-rp}^*}$. This indicates that the optimal price with reference point effects is smaller than that without reference point effects.

(ii) When $\lambda = 1$, i.e., the online retailer is loss-neutral, then the optimal price $p_O^{\text{lo-neu}^*}$ and order quantity $Q_O^{\text{lo-neu}^*}$ of the online retailer are given by

$$L_2(p_O^{\text{lo-neu}^*}) = \frac{\eta\rho Q_O^{\text{lo-neu}^*}}{(1+\eta)E(X \wedge Q_O^{\text{lo-neu}^*}) - \eta(1-\rho)\underline{x}},$$

and

$$\overline{F}(Q_O^{\text{lo-neu}^*}) = \begin{cases} 1, \quad \lambda_1 A(p_O^{\text{lo-neu}^*}) + \lambda_2 B(p_O^{\text{lo-neu}^*}) \leq \dfrac{\eta\rho[p_O^{\text{lo-neu}^*} - (1-\alpha)m - s] + (c-s)}{1+\eta}, \\ \dfrac{\eta\rho[p_O^{\text{lo-neu}^*} - (1-\alpha)m - s] + (c-s)}{(1+\eta)[\lambda_1 A(p_O^{\text{lo-neu}^*}) + \lambda_2 B(p_O^{\text{lo-neu}^*})]}, \\ \qquad \lambda_1 A(p_O^{\text{lo-neu}^*}) + \lambda_2 B(p_O^{\text{lo-neu}^*}) > \dfrac{\eta\rho[p_O^{\text{lo-neu}^*} - (1-\alpha)m - s] + (c-s)}{1+\eta}. \end{cases}$$

Next in Proposition 5, we study the impact of the strength of reference point effects (η), loss aversion factor (λ) and optimism factor (ρ) on the optimal price p_O^*.

Proposition 5. *For the optimal price p_O^*, the following results holds.*
(i) *For the optimal price p_O^* satisfying*

$$L_2(p_O^*) > \frac{\rho Q_O^*[(\lambda-1)F(\hat{\sigma}^*) + 1]}{E(X \wedge Q_O^*) - (1-\rho)\underline{x} + (\lambda-1)[H(\hat{\sigma}^*) - (1-\rho)\underline{x}F(\hat{\sigma}^*)]},$$

p_O^* is increasing in η. Otherwise, p_O^* is decreasing in η.
(ii) *For the optimal price p_O^* satisfying*

$$L_2(p_O^*) > \frac{\eta\rho Q_O^* F(\hat{\sigma}^*)}{H(\hat{\sigma}^*) - (1-\rho)\underline{x}F(\hat{\sigma}^*)},$$

p_O^* is increasing in λ. Otherwise, p_O^* is decreasing in λ.

(iii) *For the optimal price p_O^* satisfying*

$$L_2(p_O^*) > \frac{\eta Q_O^*[(\lambda-1)F(\hat{\sigma}^*)+1] + \eta\rho(\lambda-1)Q_O^* f(\hat{\sigma}^*)\dfrac{\partial\hat{\sigma}^*}{\partial\rho}}{\eta(\lambda-1)f(\hat{\sigma}^*)[\hat{\sigma}^* - (1-\rho)\underline{x}]\dfrac{\partial\hat{\sigma}^*}{\partial\rho} + \eta\underline{x}[(\lambda-1)F(\hat{\sigma}^*)+1]}$$

where
$$\frac{\partial\hat{\sigma}^*}{\partial\rho} = \frac{[p_O^* - (1-\alpha)m - s]Q_O^* - [\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\underline{x}}{\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)},$$

p_O^* is increasing in ρ. Otherwise, p_O^* is decreasing in ρ.

Follows from Lemma 2 that $L_2(p)$ is decreasing in p. Thus, from Proposition 5 (i), there exists a threshold

$$p_O^* = L_2^{-1}\Big(\frac{\rho Q_O^*[(\lambda-1)F(\sigma^*)+1]}{E(X\wedge Q_O^*) - (1-\rho)\underline{x} + (\lambda-1)[H(\hat{\sigma}^*) - (1-\rho)\underline{x}F(\hat{\sigma}^*)]}\Big),$$

when $p_O^* \in (0, p_O^*)$, p_O^* increases with the reference effect coefficient η. Otherwise, p_O^* decreases with η. The effects of λ and ρ on the optimal price p_O^* have similar properties. This indicates that the retailer will have a price standard when formulating the price strategy. If the established price strategy does not exceed this price standard, the price increases with the increase of η. If the established price strategy exceeds this price standard, the price decreases as the reference effect coefficient η increases. (ii) and (iii) have similar properties.

The following proposition characterizes the impact of the strength of reference point effects (η), loss aversion factor (λ) and optimism factor (ρ) on the optimal order quantity Q_O^*.

Proposition 6.

(i) *If*
$$\lambda \le \Big\{\frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\overline{F}(Q_O^*)}{\rho[p_O^* - (1-\alpha)m - s]} - 1\Big\}\frac{1}{F(\hat{\sigma}^*)} + 1,$$
then the optimal order quantity Q_O^* increases with η; furthermore, if
$$\lambda > \Big\{\frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\overline{F}(Q_O^*)}{\rho[p_O^* - (1-\alpha)m - s]} - 1\Big\}\frac{1}{F(\hat{\sigma}^*)} + 1,$$
then Q_O^* decreases with η, where $\hat{\sigma}^* = \hat{\sigma}(p_O^*, Q_O^*) = \dfrac{r_O(p_O^*, Q_O^*) + (c-s)Q_O^*}{\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)}$.

(ii) *The optimal order quantity Q_O^* decreases with λ.*

(iii) *The optimal order quantity Q_O^* decreases with ρ.*

Under the omni-channel strategy, Proposition 6 indicates that the optimal structural properties of ordering strategy when pricing decision is endogenous are similar to that when pricing decision is exogenous.

The following conclusion is the direct result of Proposition 6.

Corollary 1.

(i) *If*

$$\lambda \leq \Big\{\frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\overline{F}(Q_O^*)}{\rho[p_O^* - (1-\alpha)m - s]} - 1\Big\}\frac{1}{F(\hat{\sigma}^*)} + 1,$$

then $Q_O^* > Q_O^{\text{no-rp}^*}$*; furthermore, if*

$$\lambda > \Big\{\frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\overline{F}(Q_O^*)}{\rho[p_O^* - (1-\alpha)m - s]} - 1\Big\}\frac{1}{F(\hat{\sigma}^*)} + 1,$$

then $Q_O^* < Q_O^{\text{no-rp}^*}$, *where* $\hat{\sigma}^* = \hat{\sigma}(p_O^*, Q_O^*) = \dfrac{r_O(p_O^*, Q_O^*) + (c-s)Q_O^*}{\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)}$.

(ii) *The optimal order quantity* Q_O^* *satisfies* $Q_O^* < Q_O^{\text{lo-neu}^*}$.

Corollary 1 (i) shows that, when the retailer is highly loss averse, the order quantity considering the reference point effects is less than that without considering the reference point effects. The opposite is true when loss aversion is low. Moreover, Corollary 1 (ii) indicates that the order quantity under loss averse can be less than the results without loss aversion (loss neutral retailer).

Proposition 7.

(i) *If*

$$\lambda \leq \Big\{\frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]Q_O^*\overline{F}(Q_O^*) + \hat{\Omega}}{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\hat{\sigma}^*} - 1\Big\}\frac{1}{F(\hat{\sigma}^*)} + 1,$$

then the expected utility $E[\widetilde{U}_r(X, p_O^*, Q_O^*)]$ *under the optimal price* p_O^* *and order quantity* Q_O^* *increases with* η*; further, if*

$$\lambda > \Big\{\frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]Q_O^*\overline{F}(Q_O^*) + \hat{\Omega}}{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\hat{\sigma}^*} - 1\Big\}\frac{1}{F(\hat{\sigma}^*)} + 1,$$

then $E[\widetilde{U}_r(X, p_O^*, Q_O^*)]$ *decreases with* η, *where* $\hat{\Omega} = [\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)][(\lambda - 1)H(\hat{\sigma}^*) + H(\hat{Q}_O^*)]$ *and* $\hat{\sigma}^* = \hat{\sigma}(p_O^*, Q_O^*) = \dfrac{r_O(p_O^*, Q_O^*) + (c-s)Q_O^*}{\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)}$.

(ii) *The expected utility* $E[\widetilde{U}_r(X, p_O^*, Q_O^*)]$ *under the optimal price* p_O^* *and order quantity* Q_O^* *decreases with* λ.

(iii) *The expected utility* $E[\widetilde{U}_r(X, p_O^*, Q_O^*)]$ *under the optimal price* p_O^* *and order quantity* Q_O^* *decreases with* ρ.

From Proposition 7, we find that the structural properties of the total expected utility when pricing decision is endogenous is similar to that when pricing decision is exogenous.

For the case when the demand is uniform, we can determine the optimal price p_O^* and optimal order quantity Q_O^* in Corollary 2.

Corollary 2. *If the demand follows the uniform distribution on* $[\underline{x}, \overline{x}]$, *then the online retailer's optimal price* p_O^* *and order quantity* Q_O^* *are given by*

$$L_2(p_O^*) = \frac{2\eta\rho Q_O^*[\overline{x} - \lambda\underline{x} + (\lambda-1)\hat{\sigma}^*]}{(1+\eta)Q_O^*(2\overline{x} - Q_O^*) - (1+\lambda\eta)\underline{x}^2 + (\lambda-1)\eta\hat{\sigma}^2 - 2\eta(1-\rho)k\underline{x}[\overline{x} - \lambda\underline{x} + (\lambda-1)\hat{\sigma}^*]}$$

and

$$\begin{aligned} Q_O^* = & \frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\overline{x}\{(1+\eta)[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)] - \eta\rho[p_O^* - (1-\alpha)m - s] - (c-s)\}}{(\lambda-1)\eta\rho^2[p_O^* - (1-\alpha)m - s]^2 + (1+\eta)[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]^2} \\ & + \frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\overline{x}\{\eta\rho[p_O^* - (1-\alpha)m - s][\rho(\lambda-1)+1] + (c-s)\}}{(\lambda-1)\eta\rho^2[p_O^* - (1-\alpha)m - s]^2 + (1+\eta)[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]^2} \end{aligned}$$

4. Performance Analysis

In Sections 3, we established the price and inventory decisions of the online retailer with reference point effects under the omni-channel strategy. In this section, we compare the total expected utility of our model with that of Zhang et al. [41] under the omni-channel strategy. For simplicity of notation, we call their model ZH model. It is worth noting that the ZH model doesn't consider the reference point effects (i.e., $\eta = 0$). The corresponding utility comparison is as follows.

Proposition 8. *If*

$$\lambda \leq \left\{\frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]Q_O^*\overline{F}(Q_O^*) + \hat{\Omega}}{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\hat{\sigma}^*} - 1\right\}\frac{1}{F(\hat{\sigma}^*)} + 1,$$

then the total expected utility $E[\widetilde{U}_r(X, p_O^*, Q_O^*)]$ *under the optimal price* p_O^* *and order quantity* Q_O^* *satisfies* $E[\widetilde{U}_r(X, p_O^*, Q_O^*)] > E[\pi_O(X, p_O^*, Q_O^*)]$. *If*

$$\lambda > \left\{\frac{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]Q_O^*\overline{F}(Q_O^*) + \hat{\Omega}}{[\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)]\hat{\sigma}^*} - 1\right\}\frac{1}{F(\hat{\sigma}^*)} + 1,$$

then $E[\widetilde{U}_r(X, p_O^*, Q_O^*)] < E[\pi_O(X, p_O^*, Q_O^*)]$ *holds for the optimal price* p_O^* *and order quantity* Q_O^*, *where* $\hat{\Omega} = [\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)][(\lambda-1)H(\hat{\sigma}^*) + H(Q_O^*)]$ *and* $\hat{\sigma}^* = \hat{\sigma}(p_O^*, Q_O^*)$ $= \dfrac{r_O(Q_O^*) + (c-s)Q_O^*}{\lambda_1 A(p_O^*) + \lambda_2 B(p_O^*)}$.

According to Proposition 8, we can conclude that when the retailer is highly loss averse, the optimal total expected utility considering the reference point effects is less than that without considering the reference point effects (i.e., the models proposed by Zhang et al. [41]). The opposite is true when loss aversion is low.

5. Conclusions

Our research complements the existing research stream in coordinating pricing and inventory replenishment decisions under omni-channel retail environmental by incorporating the retailer's behavior (i.e., retailer's reference point effects). Specifically, this paper considers the online retailer's omni-channel retail operations under reference point effects in which consumers can cancel their order before payment and return the product after payment if the product does not meet their expectation. The online retailer's optimal pricing and inventory decisions are derived under the omni-channel strategy by maximizing the total expected utility. In addition, the impact of the reference point effects on optimal price and order quantity are studied. Our main results are as follows.

First, our analysis reveals a threshold strategy on the retailer's optimal pricing and inventory decisions as well as the optimal total expected utility while considering the impact of reference point effects. Moreover, with the increase of the retailer's loss aversion or the optimism level, the order quantity and overall expected utility decrease, while the optimal price presents a threshold type.

Second, we investigate how key parameters affect the optimal total expected utility. We find that when the retailer is highly loss averse, the optimal total expected utility considering the reference point effects is less than that without considering the reference point effects (i.e., the models proposed by Zhang et al. [41]). The opposite is true when loss aversion is low.

Though this paper has identified the effects of reference point on the coordination of pricing and ordering decisions for omni-channel retailer, there are still some shortcomings that can be investigated in the future. First, this paper analyzes the pricing and ordering decisions of a single omni-channel retailer under reference point effects, and unaware of the influence of reference point effects on suppliers. An interesting future research topic is to examine the pricing and inventory decisions for suppliers when considering the reference point effects of the suppliers, and to design an appropriate coordination mechanism so that a win-win outcome for both parties can be obtained. Second, in our study, the consumer's valuation of the product is assumed to be a random variable. In view of the difficulty in obtaining the information on product valuation by consumers, demand learning can be incorporated into formulating pricing and inventory strategy in the presence of the reference point effects.

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Appendix

Proof of Proposition 1. The concavity can be shown by proving the non-positivity of the second-order condition. After taking the first order derivative of (3.4) w.r.t. Q, we get

$$\frac{\partial E[\widetilde{U}_r[X, p_O^e, Q]}{\partial Q}$$

$$
\begin{aligned}
&=(1+\eta)[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q)-(c-s)-\lambda\eta[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]F(\sigma)\frac{\partial\sigma}{\partial Q}\\
&\quad-\eta[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)][F(Q)-F(\sigma)]\frac{\partial\sigma}{\partial Q}-\eta[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q)\frac{\partial\sigma}{\partial Q}\\
&=(1+\eta)[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q)-(c-s)-\eta[(\lambda_1-1)F(\sigma)+1][\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\frac{\partial\sigma}{\partial Q}\\
&=(1+\eta)[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q)-(c-s)-\eta\rho[p_O^*-(1-\alpha)m-s][(\lambda-1)F(\sigma)+1]
\end{aligned}
$$

and the second derivative w.r.t. Q is as follows

$$
\begin{aligned}
&\frac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial Q^2}\\
&\qquad=-(1+\eta)[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]f(Q)-\eta\rho(\lambda-1)[p_O^e-(1-\alpha)m-s]f(\sigma)\frac{\partial\sigma}{\partial Q},
\end{aligned}
$$

Consequently, we can conclude that if $\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]/\partial Q^2<0$ if

$$
\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)>\frac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{1+\eta},
$$

which implies the concavity of the expected utility function $E[\widetilde{U}_r(X,p_O^e,Q)]$ in Q.

(ii) If $\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)<0$, we have $\dfrac{\partial E[\widetilde{U}_r[X,p_O^*,Q]}{\partial Q}<0$, then the optimal order quantity Q_O^* is zero. If

$$
0\le\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)\le\frac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{1+\eta},
$$

we have $\dfrac{\partial E[\widetilde{U}_r[X,p_O^*,Q]}{\partial Q}<0$, then the optimal order quantity Q_O^* is zero. If

$$
\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)>\frac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{1+\eta},
$$

the second order derivative w.r.t. Q is negative, and the optimal order quantity Q_O^* is characterized by the first order condition

$$
\overline{F}(Q_O^*)=\frac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{(1+\eta)[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]}.
$$

The proof is complete. □

Proof of Proposition 2. When

$$
\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)\le\frac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{1+\eta},
$$

the optimal order quantity $Q_O^* = 0$, then $\dfrac{\partial Q_O^*}{\partial \eta} = 0$, $\dfrac{\partial Q_O^*}{\partial \lambda} = 0$ and $\dfrac{\partial Q_O^*}{\partial \rho} = 0$. Therefore, we only need to prove the case when

$$\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e) > \frac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{1+\eta}.$$

(i) From the implicit function theorem, we have

$$\begin{aligned}\frac{\partial Q_O^*}{\partial \eta} &= -\frac{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial Q\partial\eta}}{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial Q^2}} \\ &= \frac{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q_O^*)-\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]}{-\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial Q^2}}\end{aligned}$$

Therefore, when

$$\lambda \le \left\{\frac{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q_O^*)}{\rho[p_O^e-(1-\alpha)m-s]}-1\right\}\frac{1}{F(\hat{\sigma}^*)}+1, \quad \frac{\partial Q_O^*}{\partial \eta} \ge 0. \text{ Otherwise, } \frac{\partial Q_O^*}{\partial \eta} < 0.$$

(ii) From the implicit function theorem, we get

$$\frac{\partial Q_O^*}{\partial \lambda} = -\frac{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial Q\partial\lambda}}{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial Q^2}} = \frac{-\eta\rho[p_O^e-(1-\alpha)m-s]F(Q_O^*)}{-\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial Q^2}} < 0.$$

Thus, Q_O^* decreases with λ.

(iii) From the implicit function theorem, it follows

$$\begin{aligned}\frac{\partial Q_O^*}{\partial \rho} &= -\frac{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial Q\partial\rho}}{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial Q^2}} \\ &= \frac{\eta[p_O^e-(1-\alpha)m-s]\{[(\lambda-1)F(\sigma^*)+1]+(\lambda-1)\rho f(\sigma^*)\frac{\partial\sigma^*}{\partial\rho}\}}{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial Q^2}}\end{aligned}$$

where

$$\frac{\partial\sigma^*}{\partial\rho} = \frac{[(1-\alpha)(p_O^*-c-m)+\alpha(p_O^e-c)]Q_O^*-\{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\underline{x}-(c-s)Q_O^*\}}{\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)} \ge \underline{x},$$

which follows from $\sigma = \dfrac{r_O(p,Q)+(c-s)Q}{\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)} \geq \underline{x}$. Then we have $\dfrac{\partial Q_O^*}{\partial \rho} < 0$, which implies that Q_O^* is decreasing in ρ. This completes the proof. □

Proof of Proposition 3. Similar to Proposition 2, we only prove the case when

$$\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e) > \frac{\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]+(c-s)}{1+\eta}.$$

(i) Substituting Q_O^* into $E[\widetilde{U}_r(X,p_O^e,Q)]$ in (3.4) and differentiating it w.r.t. η, we have

$$\begin{aligned}
&\frac{\partial E[\widetilde{U}_r(X,p_O^*,Q_O^*)]}{\partial \eta}\\
=&\{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q_O^*)-(c-s)\}\frac{\partial Q_O^*}{\partial \eta}\\
&-\lambda[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]\int_{\underline{x}}^{\sigma^*} f(x)dx-\lambda\eta\frac{\partial[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]}{\partial\eta}\int_{\underline{x}}^{\sigma^*} f(x)dx\\
&+\lambda[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\int_{\underline{x}}^{\sigma^*} xf(x)dx+[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\int_{\sigma^*}^{Q_O^*} xf(x)dx\\
&-[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]\int_{\sigma^*}^{Q_O^*} f(x)dx-\eta\frac{\partial[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]}{\partial\eta}\int_{\sigma^*}^{Q_O^*} f(x)dx\\
&+\{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]Q_O^*-[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]\}\overline{F}(Q_O^*)\\
&+\eta\Big\{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\frac{\partial Q_O^*}{\partial\eta}-\frac{\partial[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]}{\partial\eta}\Big\}\overline{F}(Q_O^*)\\
=&[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]Q_O^*\overline{F}(Q_O^*)-[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\sigma^*[(\lambda-1)F(\sigma^*)-1]+\Omega,
\end{aligned}$$

where

$$\begin{aligned}
\Omega &= [\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\Big(\lambda\int_{\underline{x}}^{\sigma^*} xf(x)dx+\int_{\sigma^*}^{Q} xf(x)dx\Big)\\
&= [\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)][(\lambda-1)H(\sigma^*)+H(Q_O^*)],
\end{aligned}$$

Thus, when

$$\lambda \leq \Big\{\frac{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]Q_O^*\overline{F}(Q_O^*)+\Omega}{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\sigma^*}-1\Big\}\frac{1}{F(\sigma^*)}+1,\ \ \frac{\partial E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial\eta} \geq 0.$$

Otherwise, $\dfrac{\partial E[\widetilde{U}_r(X,p_O^e,Q_O^*)]}{\partial\eta} < 0$.

(ii) Substituting Q_O^* into $E[\widetilde{U}_r(X,p_O^e,Q)]$ in (3.4) and differentiating it w.r.t. λ, we get

$$\begin{aligned}
&\frac{\partial E[\widetilde{U}_r(X,p_O^*,Q_O^*)]}{\partial \lambda}\\
=&\{[\lambda_1 A(p_O^e)+\lambda_2 B(p_O^e)]\overline{F}(Q_O^*)-(c-s)\}\frac{\partial Q_O^*}{\partial \lambda}
\end{aligned}$$

$$
\begin{aligned}
&-\eta[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]\int_{\underline{x}}^{\sigma^*}f(x)dx-\lambda\eta\frac{\partial[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]}{\partial\lambda}\int_{\underline{x}}^{\sigma^*}f(x)dx\\
&+\eta[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\int_{\underline{x}}^{\sigma^*}xf(x)dx-\eta\frac{\partial[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]}{\partial\lambda}\int_{\sigma^*}^{Q_O^*}f(x)dx\\
&+\eta\Big\{[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\frac{\partial Q_O^*}{\partial\lambda}-\frac{\partial[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]}{\partial\lambda}\Big\}\overline{F}(Q_O^*)\\
=&\eta[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\int_{\underline{x}}^{\sigma^*}(x-\sigma^*)f(x)dx\le 0.
\end{aligned}
$$

Hence, Q_O^* decreases with λ.

(iii) Substituting Q_O^* into $E[\widetilde{U}_r(X,p_O^e,Q)]$ in (3.4) and differentiating it w.r.t. ρ, we have

$$
\begin{aligned}
&\frac{\partial E[\widetilde{U}_r(X,p_O^*,Q_O^*)]}{\partial\rho}\\
=&\{[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\overline{F}(Q_O^*)-(c-s)\}\frac{\partial Q_O^*}{\partial\rho}\\
&+\eta\Big\{[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\frac{\partial Q_O^*}{\partial\rho}-\frac{\partial[r_O(p_O^e,Q_O^*)+(c-s)Q_O^*]}{\partial\rho}\Big\}\overline{F}(Q_O^*)\\
=&\{(1+\eta)[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\overline{F}(Q_O^*)-(c-s)\}\frac{\partial Q_O^*}{\partial\rho}-\eta[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\overline{F}(Q_O^*)\frac{\partial\sigma^*}{\partial\rho}\\
=&\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]\frac{\partial Q_O^*}{\partial\rho}-\eta[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\overline{F}(Q_O^*)\frac{\partial\sigma^*}{\partial\rho}.
\end{aligned}
$$

Furthermore, since

$$
\frac{\partial\sigma^*}{\partial\rho}=\frac{[p_O^e-(1-\alpha)m-s]-[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\underline{x}+\rho[p_O^e-(1-\alpha)m-s]\frac{\partial Q_O^*}{\partial\rho}}{\lambda_1A(p_O^e)+\lambda_2B(p_O^e)}
$$

then

$$
\begin{aligned}
&\frac{\partial E[\widetilde{U}_r(X,p_O^*,Q_O^*)]}{\partial\rho}\\
=&\eta\rho[p_O^e-(1-\alpha)m-s][(\lambda-1)F(\sigma^*)+1]\frac{\partial Q_O^*}{\partial\rho}-\eta[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\overline{F}(Q_O^*)\frac{\partial\sigma^*}{\partial\rho}\\
=&\eta\rho(\lambda-1)[p_O^e-(1-\alpha)m-s]F(\sigma^*)\frac{\partial Q_O^*}{\partial\rho}\\
&-\eta\overline{F}(Q_O^*)\{[p_O^e-(1-\alpha)m-s]Q_O^*-[\lambda_1A(p_O^e)+\lambda_2B(p_O^e)]\underline{x}\}\\
&+\eta\rho[p_O^e-(1-\alpha)m-s][1-\overline{F}(Q_O^*)]\frac{\partial Q_O^*}{\partial\rho}
\end{aligned}
$$

It follows from $\sigma\ge\underline{x}$ and Proposition 2(iii) that $\frac{\partial E[\widetilde{U}_r(X,p_O^*,Q_O^*)]}{\partial\rho}<0$, which proves that $E[\widetilde{U}_r(X,Q_O^*)]$ decreases with ρ. This completes the proof. □

Proof of Proposition 4. After taking the first order derivative of (3.4) w.r.t. p, we have

$$\begin{aligned}
&\frac{\partial E[\widetilde{U}_r(X,p,Q)]}{\partial p}\\
=&L_2(p)[Q\overline{F}(Q)+H(Q)]-\lambda\eta[\rho Q+(1-\rho)L_2(p)\underline{x}]F(\hat{\sigma})\\
&+\lambda\eta L_2(p)\int_{\underline{x}}^{\hat{\sigma}}xf(x)dx+\eta L_2(p)\int_{\hat{\sigma}}^{Q}xf(x)dx-\eta[\rho Q+(1-\rho)L_2(p)\underline{x}][F(Q)-F(\hat{\sigma})]\\
&+\eta L_2(p)Q\overline{F}(Q)-\eta[\rho Q+(1-\rho)L_2(p)\underline{x}]\overline{F}(Q)\\
=&L_2(p)\{(1+\eta)[Q\overline{F}(Q)+H(Q)]+(\lambda-1)\eta H(\hat{\sigma})\}-\eta[\rho Q+(1-\rho)L_2(p)\underline{x}][(\lambda-1)F(\hat{\sigma})+1],
\end{aligned} \tag{A.1}$$

It follows from (A.1) and the first order condition that the optimal price p_O^* solves (3.6). Moreover, we can see that $L_2(p_O^*)>0$.

For any p satisfying Equation (3.4), differentiating (A.1) twice w.r.t. p, we have

$$\begin{aligned}
&\frac{\partial^2 E[\widetilde{U}_r(X,p,Q)]}{\partial p^2}\\
=&\{(1+\eta)[Q\overline{F}(Q)+H(Q)]+(\lambda-1)\eta H(\hat{\sigma})\}\frac{dL_2(p)}{dp}\\
&-\eta(1-\rho)\underline{x}[(\lambda-1)F(\hat{\sigma})+1]\frac{dL_2(p)}{dp}+\eta(\lambda-1)f(\hat{\sigma})\{[\hat{\sigma}-(1-\rho)\underline{x}]L_2(p)-\rho Q\}\frac{\partial\hat{\sigma}}{\partial p}\\
=&\{(1+\eta)[E(X\wedge Q)+(\lambda-1)\eta H(\hat{\sigma})-\eta(1-\rho)\underline{x}[(\lambda-1)F(\hat{\sigma})+1]\}\frac{dL_2(p)}{dp}\\
&+\eta(\lambda-1)f(\hat{\sigma})\{[\hat{\sigma}-(1-\rho)\underline{x}]L_2(p)-\rho Q]\frac{\partial\hat{\sigma}}{\partial p},
\end{aligned}$$

Since $dL_2(p)/dp<0$ by Lemma 4 in Zhang et al. [41],

$$\begin{aligned}
&\eta(\lambda-1)f(\hat{\sigma})\{[\hat{\sigma}-(1-\rho)\underline{x}]L_2(p)-\rho Q\}\frac{\partial\hat{\sigma}}{\partial p}\\
&\quad=-\frac{\eta(\lambda-1)f(\hat{\sigma})\rho^2Q^2\{[\lambda_1A(p)+\lambda_2B(p)]-[p-(1-\alpha)m-s]L_2(p)\}^2}{[\lambda_1A(p)+\lambda_2B(p)]^3}<0
\end{aligned}$$

and

$$\begin{aligned}
&(1+\eta)E(X\wedge Q)+(\lambda-1)\eta H(\hat{\sigma})-\eta(1-\rho)\underline{x}[(\lambda-1)F(\hat{\sigma})+1]\\
&\geq(1+\eta)E(X\wedge Q)+(\lambda-1)\eta\underline{x}F(\hat{\sigma})-\eta(1-\rho)\underline{x}[(\lambda-1)F(\hat{\sigma})+1]\\
&\geq(\lambda-1)\eta\underline{x}F(\hat{\sigma})-\eta(1-\rho)(\lambda-1)\underline{x}F(\hat{\sigma})\\
&=(\lambda-1)\eta\rho F(\hat{\sigma})>0
\end{aligned}$$

where the first inequality follows from $H(\hat{\sigma})\int_{\underline{x}}^{\hat{\sigma}}xf(x)dx\geq\underline{x}\int_{\underline{x}}^{\hat{\sigma}}f(x)dx=\underline{x}F(\hat{\sigma})$ and the second inequality follows from the fact that $(1+\eta)E(X\wedge Q)-\eta(1-\rho)\underline{x}>0$. We can thus conclude that $\partial^2 E[\widetilde{U}_r(X,p,Q)]/\partial p^2 < 0$. Hence, $E[\widetilde{U}_r(X,p,Q)]$ is concave for all p satisfying Equation (3.6). This yields the uniqueness of the optimal price p_O^*.

(ii) The proof is similar to that of Proposition 1, which is omitted. □

Proof of Proposition 5. (i) By applying the implicit function theorem, we get

$$\begin{aligned}\frac{\partial p_O^*}{\partial \eta} &= -\frac{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p \partial \eta}}{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}} \\ &= \frac{L_2(p_O^*)\{[Q\overline{F}(Q)+H(Q)]+(\lambda-1)H(\hat{\sigma})\}-[\rho Q+(1-\rho)\underline{x}L_2(p_O^*)][(\lambda-1)F(\hat{\sigma})+1]}{-\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}} \\ &= \frac{L_2(p_O^*)\{E(X\wedge Q)+(\lambda-1)H(\hat{\sigma})\}-[\rho Q+(1-\rho)\underline{x}L_2(p_O^*)][(\lambda-1)F(\hat{\sigma})+1]}{-\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}} \\ &= \frac{\{[E(X\wedge Q)-(1-\rho)\underline{x}]+(\lambda-1)[H(\hat{\sigma})-(1-\rho)\underline{x}F(\hat{\sigma})]\}L_2(p_O^*)-\rho Q[(\lambda-1)F(\hat{\sigma})+1]}{-\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}}\end{aligned}$$

Thus, if $L_2(p_O^*) > \dfrac{\rho Q[(\lambda-1)F(\hat{\sigma})+1]}{E(X\wedge Q)-(1-\rho)\underline{x}+(\lambda-1)[H(\hat{\sigma})-(1-\rho)\underline{x}F(\hat{\sigma})]}$, then $\dfrac{\partial p_O^*}{\partial \eta} > 0$. Otherwise, $\dfrac{\partial p_O^*}{\partial \eta} < 0$.

(ii) By applying the implicit function theorem, we get

$$\begin{aligned}\frac{\partial p_O^*}{\partial \lambda} = -\frac{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p \partial \lambda}}{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}} &= \frac{\eta H(\hat{\sigma})L_2(p_O^*)-\eta[\rho Q+(1-\rho)\underline{x}L_2(p_O^*)]F(\hat{\sigma})}{-\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}} \\ &= \frac{\eta[H(\hat{\sigma})-(1-\rho)\underline{x}F(\hat{\sigma})]L_2(p_O^*)-\eta\rho Q F(\hat{\sigma})}{-\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}}\end{aligned}$$

Therefore, if $L_2(p_O^*) > \dfrac{\eta\rho Q F(\hat{\sigma})}{H(\hat{\sigma})-(1-\rho)\underline{x}F(\hat{\sigma})}$, then $\dfrac{\partial p_O^*}{\partial \lambda} > 0$. Otherwise, $\dfrac{\partial p_O^*}{\partial \lambda} < 0$.

(iii) By applying the implicit function theorem, we get

$$\frac{\partial p_O^*}{\partial \rho} = -\frac{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p \partial \rho}}{\dfrac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}}$$

$$= \frac{(\lambda-1)\eta\hat{\sigma}f(\hat{\sigma})L_2(p_O^*)\frac{\partial\hat{\sigma}}{\partial\rho} - \eta[Q - \underline{x}L_2(p_O^*)][(\lambda-1)F(\hat{\sigma})+1]}{-\frac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}}$$
$$+ \frac{-\eta(\lambda-1)f(\hat{\sigma})[\rho Q-(1-\rho)\underline{x}L_2(p_O^*)]\frac{\partial\hat{\sigma}}{\partial\rho}}{-\frac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}}$$
$$= \frac{-\eta Q[(\lambda-1)F(\hat{\sigma})+1] + \eta\underline{x}[(\lambda-1)F(\hat{\sigma})+1]L_2(p_O^*)-\eta\rho(\lambda-1)Qf(\hat{\sigma})\frac{\partial\hat{\sigma}}{\partial\rho}}{-\frac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}}$$
$$+ \frac{\eta(\lambda-1)f(\hat{\sigma})[\hat{\sigma}-(1-\rho)\underline{x}]L_2(p_O^*)\frac{\partial\hat{\sigma}}{\partial\rho}}{-\frac{\partial^2 E[\widetilde{U}_r(X,p_O^*,Q)]}{\partial p^2}}$$

Hence, if $L_2(p_O^*) > \dfrac{\eta Q[(\lambda-1)F(\hat{\sigma})+1]+\eta\rho(\lambda-1)Qf(\hat{\sigma})\frac{\partial\hat{\sigma}}{\partial\rho}}{\eta(\lambda-1)f(\hat{\sigma})[\hat{\sigma}-(1-\rho)\underline{x}]\frac{\partial\hat{\sigma}}{\partial\rho} + \eta\underline{x}[(\lambda-1)F(\hat{\sigma})+1]}$, then $\dfrac{\partial p_O^*}{\partial\rho} > 0$. Otherwise, $\dfrac{\partial p_O^*}{\partial\rho} < 0$. The proof is complete. □

Proof of Proposition 6. By applying the implicit function theorem, the proof is similar to that of Proposition 9, which we omit here. □

Proof of Proposition 7. By applying the implicit function theorem, and note that $\dfrac{\partial E[\widetilde{U}_r(X,p_O^*,Q_O^*)]}{\partial p_O^*} = \dfrac{\partial E[\widetilde{U}_r(X,p_O^*,Q_O^*)]}{\partial Q_O^*} = 0$, the proof is thus similar to that of Proposition 10, which we omit here. □

Proof of Proposition 8. This result is directly follows from Proposition 7. □

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