International Journal of
Information
and
Management
Sciences

International Journal of Information and Management Sciences
31 (2020), 1-14. DOI:10.6186/IJIMS.202003_31(1).0001

Optimal Control of Postponed Demands in a Continuous Review Inventory System with Two Types of Customers

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Abstract

In this article, we consider a continuous review (s, S) inventory system with two types of customers and exponential lead time. Both types of customers arrive according to independent Poisson processes. When the inventory level is greater than s, both type of customers demands are satisfied. When the inventory level lies between 1 and s, only type-2 customers are satisfied and the type-1 customers are sent to a pool of finite capacity. If the replenished stock is above s, pooled customers are selected one by one and inter-selection time follows exponential distribution. Type-2 customers who arrive during stock out periods leave the system. The problem is to find the optimal selection rate of pool customers at each instant of time so that the long-run total expected cost rate is minimized. The problem is modelled as a semi-Markov decision problem. The stationary optimal policy is computed using the linear programming algorithm.

Keywords: Two types of customers, Postponed demands, Control of selection rate, Semi Markov decision process.

1. Introduction

In most of the literature on inventory models, the authors give equal important to all the demands that arrive to the system. However, in practice, these demands can be treated to have different priorities. Veinott-Jr [20] introduced the concept of multiple demand classes in periodic review inventory system with zero lead time. Kleijn and Dekker [8] have given a summary about the papers which dealt with multiple demand classes in periodic review inventory systems.

In the case of continuous review models, the first paper was by Nahmias and Demmy [14] in which they analyzed an (s, Q) inventory system with two demand classes, stock rationing, Poisson demand, backlogging and derived approximate expressions for costs and service levels. Moon and Kang [13] generalised this model by assuming that demands have a compound Poisson distribution. Arsalan et al. [1] generalised this model by considering more than two demand classes. Also, the model of Nahmias and Demmy [14] was extended by Sivakumar and Arivarignan [17] to a work in which the demands are assumed to follow Markovian arrival process and items in the stock have perishable nature. Ha [5] derived the optimality of critical level policies for a continuous-review model that dealt with Poisson demand processes, a single exponential server for replenishments, and lost sales. Dekker et al. [4] derived exact and heuristic procedures for the generation of an optimal critical level policy for a continuous review model with multiple customer classes, Poisson demands, ample supply, and lost sales. This work was extended by Melchiors et al. [11] to the case of fixed order quantity in which they assumed two demand classes. Sapna-Isotupa [15] considered a lost sales (s, Q) inventory system with two types of customers say ordinary and priority customers whose arrivals are according to independent Poisson arrival processes and exponential lead time. Karthick et al. [7] considered an continuous review (s, S) inventory system with two types of customers. When the inventory level drops to s, the type-1 customers sent to an orbit. The inter retrial times were assumed to have exponential distribution. They derived various performance measures of the system in the steady state and total expected cost rate.

In the literature, many inventory models considered that the demands that arrive when the stock is empty are lost or backlogged. In the later case, the demands that are backlogged are satisfied immediately after the stock is replenished. But in some real life situations, the backlogged demands may have to wait even after the replenishment. This type of inventory problem is called inventory with postponed demands. Berman et al. [2] first introduced the concept of postponed demands in inventory models. Krishnamoorthy and Islam [9] considered an inventory system in which they assumed that the interval time between two successive selections of the customers whose demands are postponed is exponential. Sivakumar and Arivarignan [18] considered a perishable inventory model with Markovian arrival process and Phase-type lead time. They assumed exponential distribution for the time between two selections of the pooled customers. Manuel et al. [10] considered an inventory system with independent Markovian arrival processes for positive and negative customers and exponential distribution whose parameter depends on the number of customers in the pool for the time between the selections of pooled customers. Sivakumar and Arivarignan [19] considered an inventory system with infinite pool size. They assumed that positive and negative demands arrive according to two independent Markovian arrival processes, exponential lead time for the reorders and exponential perishable time for the items in the stock. Jenifer et al. [6] considered the continuous review inventory system with postponed demand consisting of finite waiting hall and a single server. In their paper, under a specified cost structure, the optimal service rate that minimizes the long-run total cost rate had been derived. chitra et al. [3] considered inventory system with postponed demand. they find optimal selection rate that minimizes the total expected cost rate.

In many Business sector, the customer pays for the actual profit for the business. Customer uses the inventory and services and judges the quality of those inventories and services. To manage customers, the manager should follow some approaches like division of customers into two classes, generally called ordinary and priority. The priority customers are promote more sales and profit as compared to ordinary customers, as these types of customers demand are completely satisfied. So the inventory level is below some prefixed level, only priority customers demand are satisfied and ordinary customers are sent to the pool, because all customer has to be considered valuable and profitable. These pool customers are selected one by one after some random time, but long time they are not wait in the pool. Hence they want to find the optimal selection rate of pool customer, so that they reduce the pool customer waiting time and also minimize the total expected cost rate.

In this article, we extend Sapna-Isotupa [15] model by assuming that the ordinary customer demands are postponed when the inventory level reaches the prefixed level. We select the customers from the pool one by one with the exponential inter selection time. We focus our study on a system where speeding up or slowing down the selection rate is possible. This problem is modelled as a semi Markov decision problem and the optimal solution is obtained using linear programming method.

The rest of the paper is organized as follows. In section 2, we formulate the model. In section 3, we present the steady state analysis of the problem and calculate the total expected cost rate. In section 4, we derive the linear programming formulation of the problem. Numerical illustration of the results, which provide insights of the behaviour of the system, are provided in the final section.

2. Model Description and Analysis

We consider a continuous review inventory system with two types of customers say type-1 and type-2 customers arriving according to two independent Poisson processes with rates λ_1 and λ_2 respectively. The ordering policy is (s, S) policy, which operates as follows: whenever the inventory level drops to the prefixed level s, an order of $Q(= S - s)$ units is placed, which arrives after an exponential amount of time with parameter $\beta(> 0)$. Demands of both types of customers are satisfied, whenever the inventory level exceeds the prefixed level s, otherwise only type-2 customers demands are satisfied if items are available and demands of type-1 customers are postponed until the ordered items are received. The postponed customers are retained in a pool, which has finite capacity N. After the replenishment and as long as the inventory level is greater than the prefixed level s, the pooled customers are selected according to exponentially distributed time lag whose parameter is chosen from a given set of positive values $\{\mu_1, \mu_2, \ldots, \mu_K\}$. The demands of type-2 customers arriving during the stock out periods are assumed to be lost.

Let $L(t)$ and $X(t)$ denote, respectively the on-hand inventory level and the number of customers in the pool at time t. From the assumption made on the input and output process it may be verified that the stochastic process $\mathcal{X} = \{(L(t), X(t)), t \geq 0\}$ is a Markov process with state space Ω, where

$$\Omega = \{(i, j), 0 \leq i \leq S, 0 \leq j \leq N\}$$

Whenever $L(t) \leq s$ or $X(t) = 0$, we do not select the customer from the pool, for ease of notation in the sequel, we denote a null action by μ_0 during the inventory level is less than or equal to s or the pool customer level is zero. Based on the choice of actions, the state space Ω is partitioned as follows

$$\Omega_1 = \{(i,j); 0 \leq i \leq s, 0 \leq j \leq N\} \cup \{(i,0); s+1 \leq i \leq S\}$$
$$\Omega_2 = \{(i,j); s+1 \leq i \leq S, 1 \leq j \leq N\}$$

Let $\mathscr{A}_n (n = 1,2)$ represent the set of all possible actions of the system when it belongs to the set $\Omega_n (n = 1,2)$. Then, we have

$$\mathscr{A}_1 = \{\mu_0\}, \tag{2.1}$$
$$\mathscr{A}_2 = \{\mu_k, \text{ for some integer k between 1 and K.}\} \tag{2.2}$$
$$\text{and } \mathscr{A} = \mathscr{A}_1 \cup \mathscr{A}_2 \tag{2.3}$$

Let $\mathscr{F}$ be a set of functions f from the state space Ω to the action space $\mathscr{A}$ defined by $f : \Omega \to \mathscr{A}$

$$f(i,j) = \begin{cases} \mu_0, & \text{if } (i,j) \in \Omega_1, \\ \mu_k, \text{ for some integer k between 1 and K.,} & \text{if } (i,j) \in \Omega_2. \end{cases}$$

The function f specifies a policy in terms of the actions taken on the states. For each policy f the infinitesimal generator matrix is defined by

$$Q^f = (q^f(i,j),(i',j'))_{(i,j),(i',j') \in \Omega}$$

Let E_i^j represent the set $\{i, i+1, \ldots, j\}$. In order to write down the rate matrix, we induce an ordering in the state space Ω as follows:

$$(\mathbf{0}, \mathbf{1}, \ldots, \mathbf{S})$$

where $\mathbf{i} = ((i,0),(i,1),\ldots,(i,N))$, $0 \leq i \leq S$. The infinitesimal generator Q^f can be expressed in block-partitioned form.

$$[Q^f]_{ii'} = \begin{cases} B_1 \ i' = i & i = 0 \\ B_2 \ i' = i & 1 \leq i \leq s \\ B_3 \ i' = i & s+1 \leq i \leq S \\ C_1 \ i' = i-1 & 1 \leq i \leq s \\ C_2 \ i' = i-1 & s+1 \leq i \leq S \\ D \ \ i' = i+Q & 0 \leq i \leq s \\ \mathbf{0} \ \ \text{otherwise} \end{cases}$$

The sub matrices are given below

$$[B_1]_{jj'} = \begin{cases} -(\lambda_1 + \beta) & j' = j & 0 \leq j \leq N-1 \\ -\beta & j' = j & j = N \\ \lambda_1 & j' = j+1 & 0 \leq j \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

$$[B_2]_{jj'} = \begin{cases} -(\lambda_1+\lambda_2+\beta) & j'=j \quad 0 \le j \le N-1 \\ -(\lambda_2+\beta) & j'=j \quad j=N \\ \lambda_1 & j'=j+1 \; 0 \le j \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

$$[B_3]_{jj'} = \begin{cases} -(\lambda_1+\lambda_2) & j'=j \quad j=0 \\ -(\lambda_1+\lambda_2+\mu^f) & j'=j \quad 1 \le j \le N \\ 0 & \text{otherwise} \end{cases}$$

$$[C_1]_{jj'} = \begin{cases} \lambda_2 & j'=j \quad 0 \le j \le N \\ 0 & \text{otherwise} \end{cases}$$

$$[C_2]_{jj'} = \begin{cases} (\lambda_1+\lambda_2) & j'=j \quad 0 \le j \le N \\ \mu^f & j'=j-1 \; 1 \le j \le N \\ 0 & \text{otherwise} \end{cases}$$

$$[D]_{jj'} = \begin{cases} \beta & j'=j \quad 0 \le j \le N \\ 0 & \text{otherwise} \end{cases}$$

It may be noted that B_1, B_2, B_3, C_1, C_2 and D are square matrices of order $N+1$.

2.1. Steady state analysis

Let $\mathcal{X}^f = \{(L^f(t), X^f(t)), t \ge 0\}$ denote the Markov process $\{(L(t), X(t)), t \ge 0\}$ when policy f is adopted. A policy f is said to be stationary policy, if it is independent of the history of previous states, decisions taken and transition times. Furthermore, a process is called completely ergodic if the Markov process under consideration is irreducible for every stationary policy. For every stationary policy f, (L^f, X^f) is denoted by $(\bar{L}^f, \bar{X}^f)$. From our assumptions, it can be seen that for every stationary policy f, $(\bar{L}^f, \bar{X}^f)$ is completely ergodic. Since the action space is also finite, a stationary optimal policy exist Mine and Osaki [12]. Hence we consider the class $\mathscr{F}$ of all stationary policy.

For any fixed $f \in \mathscr{F}$ and $(i,j), (r,s) \in \Omega$, define

$$P^f_{(i,j)}(r,s,t) = Pr[\bar{L}^f(t) = r, \bar{X}^f(t) = s \mid \bar{L}^f(0) = i, \bar{X}^f(0) = j]$$

Then $P^f_{(i,j)}(r,s,t)$ satisfies the Kolmogorov forward differential equations, as each policy f results in an irreducible Markov chain. We also note that the state space and the action set are finite. Hence the limit

$$\pi^f(r,s) = \lim_{t \to \infty} P^f_{(i,j)}(r,s,t)$$

exists and is independent of the initial conditions. Hence the stationary vector satisfies the balance equation

$$\sum_{(i,j)\in\Omega} \pi^f(i,j) Q^f(\ (i,j),(r,s)\ ) = 0, \forall (r,s) \in \Omega \tag{2.4}$$

and normalization condition

$$\sum_{(i,j)\in\Omega} \pi^f(i,j) = 1 \tag{2.5}$$

We define

$$\begin{aligned} \pi^f &= (\pi^f(0), \pi^f(1), \dots, \pi^f(S)), \\ \pi^f(i) &= (\pi^f(i,0), \pi^f(i,1), \dots, \pi^f(i,N)), \, i \in E_0^S \end{aligned}$$

The balance equation and the normalizing condition can be re-written as

$$\pi^f Q^f = \mathbf{0} \text{ and } \pi^f \mathbf{e} = 1.$$

The first equation of the above yields the following set of equations:

$$\pi^f(0)B_1 + \pi^f(1)C_1 = \mathbf{0} \tag{2.6}$$

$$\pi^f(i)B_2 + \pi^f(i+1)C_1 = \mathbf{0}, i \in E_1^{s-1} \tag{2.7}$$

$$\pi^f(s)B_2 + \pi^f(s+1)C_2 = \mathbf{0} \tag{2.8}$$

$$\pi^f(i)B_3 + \pi^f(i+1)C_2 = \mathbf{0}, i \in E_{s+1}^{Q-1} \tag{2.9}$$

$$\pi^f(i-Q)D + \pi^f(i)B_3 + \pi^f(i+1)C_2 = \mathbf{0}, i \in E_Q^{S-1} \tag{2.10}$$

$$\pi^f(s)D + \pi^f(S)B_3 = \mathbf{0} \tag{2.11}$$

The above set of equations together with the condition $\sum_{i=0}^{S} \pi^f(i)\mathbf{e} = 1$ determine the steady state probability.

2.2. System performance measure

(i) *Expected Inventory Level:* The expected inventory level M_I^f is given by

$$M_I^f = \sum_{i=1}^{S} i \sum_{j=0}^{N} \pi^f(i,j)$$

(ii) *Expected Reorder Rate:* The expected reorder rate M_R^f is given by

$$M_R^f = \sum_{j=0}^{N} (\lambda_1 + \lambda_2 + \mu^f)\pi^f(s+1,j)$$

(iii) *Expected Balking Rate for Type-1 Customer:* The expected balking rate for type-1 customer M_{B1}^f due to the pool is full under the policy f is given

$$M_{B1}^f = \lambda_1 \sum_{i=0}^{s} \pi^f(i,N)$$

(iv) *Expected Balking Rate for Type-2 Customer:* The expected balking rate for type-2 customer M_{B2}^f due to the inventory level is zero under the policy f is given

$$M_{B2}^f = \lambda_2 \sum_{j=0}^{N} \pi^f(0, j)$$

(v) *Expected Number of Customer Waiting in the Pool:* The expected number of customer waiting in the Pool M_W^f is given by

$$M_W^f = \sum_{i=0}^{S} \sum_{j=1}^{N} j\pi^f(i, j)$$

(vi) *Expected Cost for using the Different Selection Rates:* The expected cost due to using the different selection rate M_{SC}^f is given by

$$M_{SC}^f = \sum_{i=s+1}^{S} \sum_{j=1}^{N} \Delta_{(i,j)}^f \pi^f(i, j)$$

where $\Delta_{(i,j)}^f = \tau_k$ if $f(i, j) = \mu_k$ and τ_k : cost associated for choosing selection parameter μ_k.

2.3. Total cost

We construct the total expected cost per unit time based on these system performance measures. Our main objective is to determine the optimal selection rates so that the expected cost rate is minimized. To do this, we define the following cost values:

c_h : inventory carrying cost per unit item.

c_r : setup cost per order.

c_{b1} : balking cost for type - 1 customer per customer.

c_{b2} : balking cost for type - 2 customer per customer.

c_w : waiting time cost of a pool customer per unit time.

Using these cost elements, the expected cost function T^f is given by

$$TC^f = c_h M_I^f + c_r M_R^f + c_{b1} M_{B1}^f + c_{b2} M_{B2}^f + c_w M_W^f + M_{SC}^f$$

3. Linear Programming Formulation

To determine the optimal policy of selecting the rates of selection of pool customer so as to minimize the total expected cost rate, we express the problem in terms of states and policy. Assume that an optimal policy k is implemented that selects the rate μ_k at the state (i, j). let $\phi(i, j, k)$ be the long run probability of the joint event that the system $\mathcal{X}$ is in state (i, j) and the selected rate of selection of pool customer is μ_k. Also let the conditional probability, $D(i,j,k)$, that the decision taken is μ_k *given* that the system is in state (i,j). That is,

$$D(i,j,k) = \Pr[\text{ Decision is } \mu_k | \text{ State is } (i,j)], \qquad (i,j) \in \Omega.$$

where $0 \leq D(i,j,k) \leq 1$ and $\sum_k D(i,j,k) = 1$.

Consider

$$\begin{aligned} \phi(i,j,k) &= \Pr[\text{ System is at } (i,j) \cap \text{Decision is } \mu_k] \\ &= \Pr[\text{Decision is } \mu_k | \text{ System is at } (i,j)] \times \Pr[\text{ System is at } (i,j)] \\ &= D(i,j,k)\pi^f(i,j) \end{aligned} \tag{3.1}$$

On summing both sides on k, we get

$$\pi^f(i,j) = \sum_{k=1}^{K} \phi(i,j,k) \tag{3.2}$$

as $\sum_k D(i,j,k) = 1$. we have

$$\pi^f(i,j) = \begin{cases} \phi(i,j,0), & (i,j) \in \Omega_1 \\ \sum\limits_{k=1}^{K} \phi(i,j,k), & (i,j) \in \Omega_2 \end{cases}$$
$$\Theta = (\mu_1, \mu_2, \ldots, \mu_K)^T.$$

By using the above in the total expected cost rate, we can express the TC in terms of ϕ's.

Minimize

$$\begin{aligned} TC = c_h &\left(\sum_{i=1}^{S} i\phi(i,0,0) + \sum_{i=1}^{s}\sum_{j=1}^{N} i\phi(i,j,0) + \sum_{i=s+1}^{S}\sum_{j=1}^{N}\sum_{k=1}^{K} i\phi(i,j,k) \right) \\ &+ c_w \left(\sum_{i=0}^{s}\sum_{j=1}^{N} j\phi(i,j,0) + \sum_{i=s+1}^{S}\sum_{j=1}^{N}\sum_{k=1}^{K} j\phi(i,j,k) \right) \\ &+ c_r \left(\sum_{j=1}^{N}\sum_{k=1}^{K} (\lambda_1 + \lambda_2 + \mu_k)\phi(s+1,j,k) + (\lambda_1+\lambda_2)\phi(s+1,0,0) \right) \\ &+ c_{b1}\lambda_1 \sum_{i=0}^{s} \phi(i,N,0) + c_{b2}\lambda_2 \sum_{j=0}^{N} \phi(0,j,0) + \sum_{i=s+1}^{S}\sum_{j=1}^{N}\sum_{k=1}^{K} \tau_k \phi(i,j,k) \end{aligned}$$

The $\phi's$ must also satisfy constraints and these are obtained from the balance equations and the normalizing condition. Thus we get the constraints as

$$\phi^f(0)B_1 + \phi^f(1)C_1 = \mathbf{0} \tag{3.3}$$
$$\phi^f(i)B_2 + \phi^f(i+1)C_1 = \mathbf{0}, i \in E_1^{s-1} \tag{3.4}$$
$$\phi^f(s)B_2 + \phi^f(s+1)\tilde{C}_2 = \mathbf{0} \tag{3.5}$$

$$\phi^f(i)\tilde{B}_3 + \phi^f(i+1)\tilde{C}_2 = \mathbf{0}, i \in E_{s+1}^{Q-1} \tag{3.6}$$

$$\phi^f(i-Q)D + \phi^f(i)\tilde{B}_3 + \phi^f(i+1)\tilde{C}_2 = \mathbf{0}, i \in E_{Q}^{S-1} \tag{3.7}$$

$$\phi^f(s)D + \phi^f(S)\tilde{B}_3 = \mathbf{0} \tag{3.8}$$

Where

$$[\tilde{B}_3]_{ij} = \begin{cases} -(\lambda_1+\lambda_2) & j=i \quad\quad i=0 \\ -(\lambda_1+\lambda_2)\mathbf{e}+\Theta & j=i \quad\quad 1\le i\le N \\ 0 & \text{otherwise} \end{cases}$$

$$[\tilde{C}_2]_{ij} = \begin{cases} (\lambda_1+\lambda_2) & j=i \quad\quad i=0 \\ (\lambda_1+\lambda_2)\mathbf{e} & j=i \quad\quad 1\le i\le N \\ \Theta & j=i-1 \quad 1\le i\le N \\ 0 & \text{otherwise} \end{cases}$$

It may be noted that the matrices $\tilde{B}_3$ and $\tilde{C}_2$ are of order $(NK+1)\times(N+1)$.

The normalizing constraint becomes

$$\sum_{(i,j)\in\Omega_1} \phi(i,j,0) + \sum_k \sum_{(i,j)\in\Omega_2} \phi(i,j,k) = 1$$

and finally we have

$$\phi(i,j,k) \ge 0 \text{ for } (i,j)\in\Omega_n,\ k\in\mathscr{A}_n,\ n=1,2$$

For the completely ergodic process the rank of the coefficient matrix associated with the constraints is $M-1$, where M is the total number of constraints. As one of the constraints is redundant, we omit one constraint. The remaining constraints are the constraints of the linear programming model.

Lemma 1. *There exists a basic feasible solution to the above linear programming model with the property that for each $(i,j)\in\Omega$, there is only one k such that*

$$\phi(i,j,k) > 0$$
$$\text{and } \phi(i,j,k') = 0 \text{ for } k' \ne k$$

Proof. See Mine and Osaki [12]. □

Corollary 1. *Any basic feasible solution of the linear programming Problem yields a pure stationary strategy.*

Proof. Since from (3.1) and (3.2) we have

$$\phi(i,j,k) = D(i,j,k)\pi^f(i,j) > 0$$

and

$$\pi^f(i,j) = \sum_k \phi(i,j,k).$$

Hence, we get

$$D(i,j,k) = \phi(i,j,k)/\sum_{k'} \phi(i,j,k')$$

Thus $D(i,j,k) = 0$ or 1 from the above lemma. □

4. Numerical Illustration

In this section, we illustrate the method described in the above section through numerical examples. In figure 1, L represents the inventory level and X represents the customer level in the pool and we use $S = 30, s = 10, N = 15, \mu_1 = 3, \mu_2 = 5, \mu_3 = 7, \mu_4 = 9$. We present the optimal policy for a specific value for the parameter and the cost.

If (i, j) lies in a shaded vertical bar corresponding to μ_k then the rate of pool customer selection must be selected as μ_k. As an illustration, consider in figure 1, if the arrival rate $\lambda_1 = 4$, the inventory level is 13 and if the customer level in the pool is between 0 to 3, one has to select at the rate μ_1; if pool customer level is above 3 but less than or equal to 5, the optimal selection rate is μ_2; if the pool customer level is above 5 and less than or equal to 10, the selection rate is μ_3; if the pool customer level lies from 11 to 15, then the selection rate is μ_4.

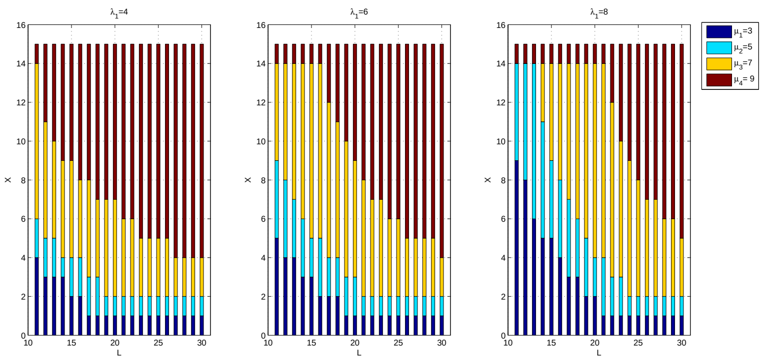
The main objective of these figures (Figure 1 to Figure 8) are to help one to choose the optimal selection rate for a given inventory level and given number of customers in the pool.

The pool customer selection rate depends on both the inventory level and the number of customers in the pool for all the costs and all the system parameters. If maximum number of customers are in the pool, then we must select maximum rate for pool customer selection while the inventory level is near to the reorder level. If more units are available in the stock, one has to increase the selection rate.

When each of type - 2 customer's arrival rate (Figure 2) , pool customer waiting cost (Figure 6) and balking cost of type - 1 customer (Figure 7) increases, then one has to use same or higher selection rate.

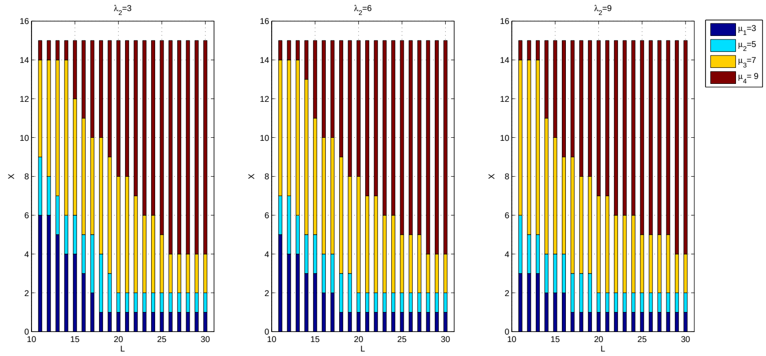
When each of type - 1 customer's arrival rate (Figure 1) and ordering cost (Figure 5) increases, then one has to use same or lower selection rate.

The optimal policy is insensitive to changes in the balking cost for type - 2 customer (Figure 8).



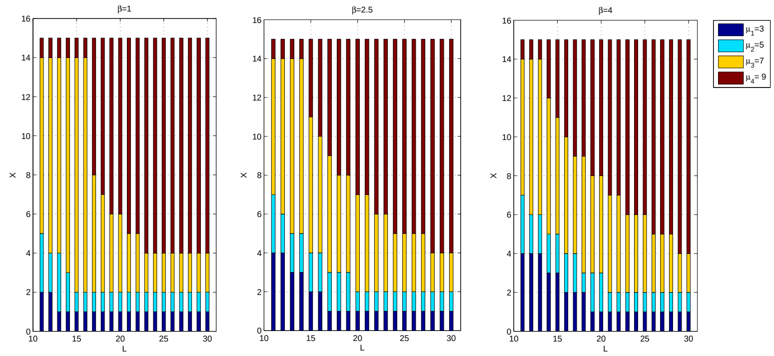
$\lambda_2 = 7, \beta = 3, c_h = 1, c_w = 2, c_{b1} = 2, c_{b2} = 2, c_r = 30$

Figure 1: Influence of type - 1 demand rate on the optimal policy



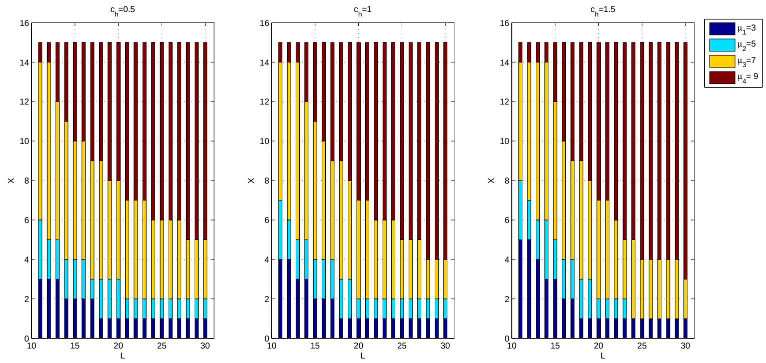
$\lambda_1 = 5, \beta = 3, c_h = 1, c_w = 2, c_{b1} = 2, c_{b2} = 2, c_r = 30$

Figure 2: Influence of type - 2 demand rate on the optimal policy



$\lambda_1 = 5, \lambda_2 = 7, c_h = 1, c_w = 2, c_{b1} = 2, c_{b2} = 2, c_r = 30$

Figure 3: Effect of the lead time rate on the optimal policy

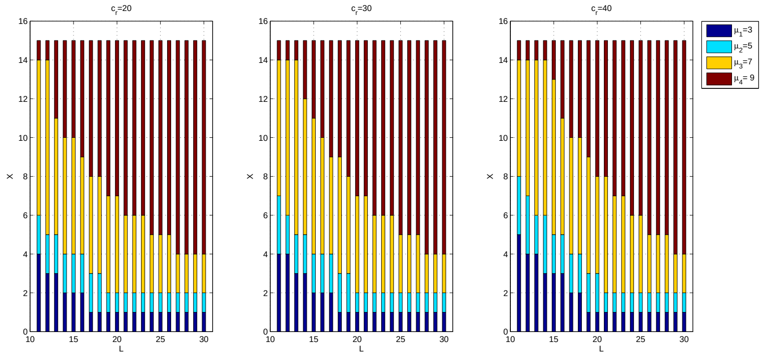


$\lambda_1 = 5, \lambda_2 = 7, \beta = 3, c_w = 2, c_{b1} = 2, c_{b2} = 2, c_r = 30$

Figure 4: Influence of holding cost on the optimal policy

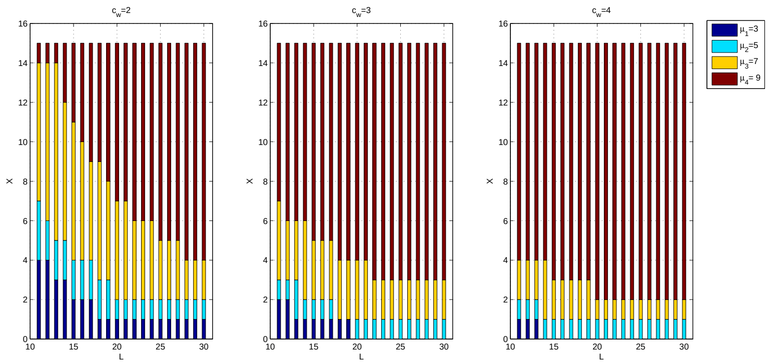
5. Conclusion

In this paper, we consider a continuous review (s, S) inventory system with two types of customers. For the main objective of this work is to find an optimal selection rate of



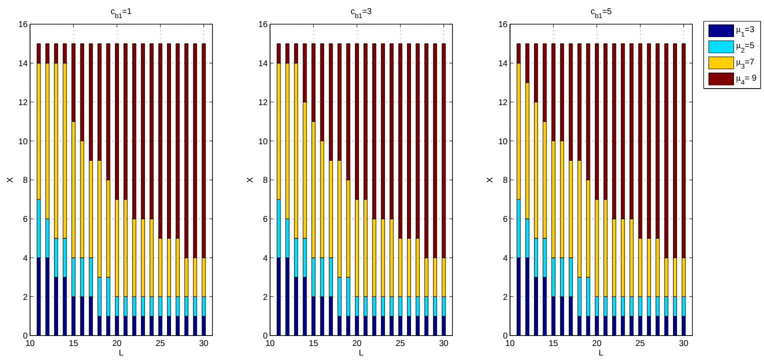
$\lambda_1 = 5, \lambda_2 = 7, \beta = 3, c_h = 1, c_w = 2, c_{b1} = 2, c_{b2} = 2$

Figure 5: Optimal policy for various ordering cost



$\lambda_1 = 5, \lambda_2 = 7, \beta = 3, c_h = 1, c_{b1} = 2, c_{b2} = 2, c_r = 30$

Figure 6: Effect of pool customer waiting time cost on the optimal policy

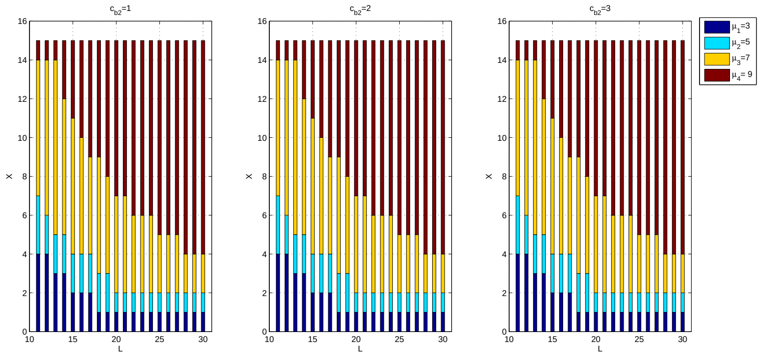


$\lambda_1 = 5, \lambda_2 = 7, \beta = 3, c_h = 1, c_w = 2, c_{b2} = 2, c_r = 30$

Figure 7: Effect of c_{b1} on the optimal policy

pool customers to be adopted at a given inventory level and the number of customer in the pool.

One possible situation for this model is the following: In automobile industries, the



$\lambda_1 = 5, \lambda_2 = 7, \beta = 3, c_h = 1, c_w = 2, c_{b1} = 2, c_r = 30$

Figure 8: Effect of c_{b2} on the optimal policy

orders from the customer who is having annual contracts should be considered on a priority basis than the orders from the non-contract customers. Since all customer has to be considered valuable and profitable. The customers in the pool should be selected one by one after some random time based on the inventory availability and number of customer waiting in the pool.

A similar case of Business sector like internet sales portals such as amazon, flipkart etc. there are two types of customers were found, the one who subscribed for the special membership service and the other one places the order without any subscription. To manage both customers, the delivery system should give first priority to the membership service subscribers who promotes more sales and non-subscriber customers shall sent to the pool and delivery date could be promised based on the inventory level and number of non-subscribers waiting in the pool.

The scope of application of this model is quite wide. The model is analysed using semi Markov decision process and optimal decision rule for the selection of pool customer rate are derived through linear programming formulation. From the numerical illustrations, The optimal policy can be found from the given figures for a prefixed set of parameters and costs. we have observed the pool customer selection rate depends on both inventory level and number of customer in the pool. If number of customer in the pool is maximum, one can choose maximum selection rate while the inventory level is near to the reorder level. If more inventory is available in the system, one has to increase the selection rate.

Acknowledgement

The first author research was supported by DST-INSPIRE Fellowship Scheme No. DST/INSPIRE Fellowship/2011/137. The second author research was supported by DST-INSPIRE Fellowship Scheme No. DST/INSPIRE Fellowship/2012/78.

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(Received April 2018; accepted May 2019)