

## Application of a Mathematical Programming Model to Solve the Confidence Interval of Process Capability Index $S_{pk}$

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### Abstract

This study developed a mathematical programming model to determine confidence intervals of  $S_{pk}$  by converting index  $S_{pk}$  into a function of  $\mu_y = (\mu - T)$  and  $\sigma_y = \sigma/d$ , constructing the feasible region of joint confidence interval with  $\mu_y$  and  $\sigma_y$ , and then regarding  $S_{pk}(\mu_y, \sigma_y)$  as an objective function, to overcome the shortage of point-estimate and interval-estimate calculations of the past process capability index. Then, Monte Carlo simulation was used to analyze the coverage rate in order to validate the accuracy of the proposed method. Our results demonstrate the efficacy of the proposed evaluation model using quartz crystal oscillators, a passive component commonly used in communication devices. The proposed method eliminates the complex complexity of statistical methods, and the results are optimal values largely robust to errors. The proposed model can also be applied to other complex process evaluation indices, thereby presenting manufacturers with an efficient and convenient method for the assessment of process capability.

*Keywords:* Mathematical programming, process capability index, Monte Carlo simulation, coverage rate.

### 1. Introduction

Process yield and expected process loss are two basic tools used to evaluate the quality and performance of processing. The former is traditionally employed to assess the quality of products or processes and the latter proposed by Taguchi [20] is used to define the quality of products or processes based on the concept of loss. Each of these tools has its specific benefits and shortcomings. Kane [14] combined the advantages of the two tools within the process capability index (PCI), resulting in a convenient tool for the evaluation of processing quality. A PCI enables design engineers and onsite manufacturing personnel to quickly identify processing issues and reach a consensus, thereby forming an efficient quality improvement system.

The concept of PCIs involves the presentation of process capability using simple quantification methods that manufacturers can use to create products to fulfill the upper