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An Integrated Vendor-Buyer Inventory Model with Defective Items and Trade Credit under Holding Cost Reduction

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Abstract

This paper develops an integrated vendor-buyer inventory model by a case study in the precision machine industry from Taiwan. The vendor offers the buyer a permissible delay period, and the buyer receives an arriving lot containing some defective items. In addition, it is assumed that holding cost is a function of capital expenditure. The dispensable assumption that the screening time is less than the permissible delay period is relaxed. The objective is to minimize the joint total expected cost per unit time and to study the effects of trade credit and an added capital expenditure on inventory decisions. A solution procedure is established to find the optimal solutions for the supply chain. Further, we use a couple of numerical examples to illustrate the proposed model and conclude the paper with suggestions for possible future researches.

Keywords: Supply chain coordination, holding cost reduction, defective items, delay in payment, precision machine industry.

1. Introduction

In the last thirty decades, the models for inventory replenishment policies involving defective items have received the attention of several researchers. In practice, many arriving order lots contain defective items as a result of the weak process control, deficient planned maintenance, inadequate work instructions and/or damage in transit. Porteus [21] and Rosenblatt and Lee [22] were among the first researchers integrating the effect of imperfect items into a modified economic production quantity (EPQ) model. Salameh and Jaber [25] developed an extended economic order quantity (EOQ) model by assuming each lot received or produced contains a random fraction of imperfect quality items. Maddah and Jaber [17] used the renewal reward theorem for Salameh and Jaber's model and obtained simple expressions for the expected profit per unit time and the optimal order quantity. Wahab and Jaber [31] presented the optimal lot sizes for an item with imperfect quality based on Salameh and Jaber [25]. Besides, some similar problems related to quality and lot size have been discussed by several authors such as Papachristos and Konstantaras [20], Wee et al. [30], Eroglu and Ozdemir [6], Lin [15], Roy et al. [23], Yassine et al. [32], Hsu and Hsu [8], Khan et al. [13].

The models mentioned above tackled defective items assumed that the buyer must pay for the items at the time of purchase. In fact, it is a common strategy that the supplier permits the retailer a delay of a fixed time period to settle the total amount owed to him/her. Chung and Huang [5] incorporated the concept of inspection of imperfect items with trade credit. Ouyang and Chang [18] studied an EPQ model with imperfect quality and complete backlogging when the supplier offers a permissible delay in payments. Jaggi and Goel [9] developed an inventory model for imperfect quality items under permissible delay in payments with allowable shortages. They relaxed the dispensable assumptions that the screening time is less than the permissible delay period and interest earned per unit is less than interest charged per unit in the work of Chung and Huang [5].

Recently, some researchers dealing with defective items and the trade credit problems have recognized the fact that coordination between both vendors and buyers is better in order to gain competitive advantages through cost reduction. Based on Salameh and Jaber [25], Chen and Kang [4] considered trade credit and imperfect quality in an integrated vendor-buyer supply chain model. Su [27] presented an integrated inventory system with defective items and allowable shortage under trade credit. Lin et al. [16] proposed an integrated supplier-retailer inventory model in which both the supplier and the retailer adopt trade credit policies, and the retailer receives some defective items. Su [27] and Lin et al. [16] treated the percentage of defective items in each deliver as a real number between 0 and 1, and they did not consider the relationship between the screen time and the permissible delay period. Supply chain is major concern in a wide variety of applications on coordinating a multiple-level suppliers and buyiers such as Jaber and Osman [10], Jaber and Goyal [11], Rezaei and Davoodi [24], Jaber and Goyal [12], Khan and Jaber [14].

The classical inventory models assume that holding cost is fixed and not subject to improve. Some practitioners and researchers have questioned its practical applications. In generalization of EOQ models, holding cost is treated as a function of time or the amount of on-hand stock by researchers like Fujiwara and Perera [7], Teng and Yang [29], Alfares [1], Pando et al. [19], Shah et al. [26], and so on. However, some components of holding cost can be reduced. This is particularly true in the storage of deteriorating and perishable items such as food products. For example, the cost of obsolescence and spoilage can be reduced through capital expenditure on acquiring better preserving facility such as refrigeration, freezer or temperature controlling equipment, and drying or vacuum technology. In addition, the cost of handling inventory can be reduced through automation. The cost of capital is often an equivalent cost per period, or a leasing fee per period. For agreement with the practical inventory situation, Billington [3] developed an EOQ model that the retailer is allowed to invest an annual capital cost, instead of a cost that is independent of the period length, to reduce the per-unit holding cost.

The practicality of the proposed model is demonstrated through a case study. Thus, the main contributions of our paper can be summarized as follows:

(1) This paper develops a practical vendor-buyer inventory model with imperfect quality and trade credit within precision machine industry.

(2) The practicality of the proposed model is demonstrated through the real case: the vendor (the manufacturer) of milling cutters in Taiwan; the buyer of Computer Numerical Control (CNC) milling machine in Germany. In addition, policy of trade credit negotiation between the vender and the buyer has emerged as a critical procedure to increase vendor's profit.

(3) The paper concludes with implications for theory, research, and practice.

This paper was motivated by a case study for the buyer of CNC machine processing. The vendor and the buyer are in different countries (Taiwan-Germany) where the trade credit negotiations involve buying and selling. An integrated inventory model is developed involving imperfect items, and holding cost reduction. It is assumed that holding cost is a function of capital expenditure. In addition, the relationship between the screening time and the permissible delay period is considered. The objective is to minimize the joint total expected cost per unit time. A solution procedure is established to determine the lot size per shipment, capital expenditure and the number of shipments from vendor to buyer. Finally, numerical examples are presented to illustrate the proposed model, and concluding remarks are provided.

2. Notation and Assumptions

In this paper, the mathematical model is developed on the basis of the following notation and assumptions.

2.1. Notation

D Demand rate on the buyer (for non-defective items).

P production rate of vendor, $P > D$.

A buyer's ordering cost per order.

S_v setup cost of vender per production run.

F transportation cost per delivery.

c_s the buyer's unit screening cost.

x the buyer's screen rate in units per unit time.

c the unit procurement cost charged by the vendor to the buyer.

s unit retail price of items of good quality charged by the buyer to the customers, $s > c$.

v unit selling price of imperfect quality items, $v < c$.

h_v the vender's holding cost per item per unit time.

T the buyer's replenishment cycle length.

t the required time for screening the defective items, equal to Q/x.

Y random variable representing the percentage of defective items in Q.

$f(y)$ probability density function of Y.

M the buyer's trade credit period offered by the vender per order.

I_{Be} the buyer's interest earned per dollar per unit time.

I_{Bk} the buyer's capital opportunity cost per dollar per unit time.

I_{Vp} the vendor's capital opportunity cost per dollar per unit time.

$EUTCB(Q, B)$ the buyer's total expected cost per unit time.

$EUTCV(n, Q)$ the vendor's total expected cost per unit time.

$JETCU(n, K, Q)$ the joint total expected cost per unit time.

Decision variables:

n Number of shipments from the vendor to the buyer per production run, a positive integer.

K The capital expenditure per unit time for reducing the buyer's holding cost (a decision variable).

Q Size of shipments from the vendor to the buyer in a production batch (a decision variable).

2.2. Assumptions

In addition, the following assumptions are used throughout this paper:

(1) The inventory system consists of a single vendor and a single buyer for one type of item. A long-tern alliance relationship is established between then. Therefore, the related costs are transparent for mnimizing total cost or achieving fair benefit distribution. Some industries satisfy this assumption in the real-world such as electronic and high-precision industries.

(2) Shortages are not allowed.

(3) Defective items are sold as a single batch at a discounted price by the end of the 100% screening process. All the items are so-called "sub-items", such as industrial items, electronic components, consumer electronics and so on.

(4) Both screening as well as demand proceeds simultaneously, but the screening rate is assumed to be greater than demand rate. It is necessary to ensure the assumption (2) is hold.

(5) To avoid shortages within the screening period t, the on-hand non-defective inventory is larger or equal to the demand, i.e., $(1 - Y)x \geq D$. Therefore, Y is restricted to $Y \leq 1 - D/x$.

(6) The inventory carrying cost rate (excluding interest charges), $i(K)$, is a continuous function of the buyer's capital expenditure, K, where $\lim_{K\to 0^+} i(K) = i_L$, $\lim_{K\to\infty^-} i(K) = i_U$ and $i_L < i_U$.

(7) The vendor offers the buyer a certain credit period M. During the credit period, the buyer sells the items and uses the sales revenue to earn interest at a rate of I_{Be}. It usually occurs, however, these items contract with technical transfer or exclusive license, and so-called "reward". At the end of the permissible delay period, the buyer pays the purchasing cost to the vendor and incurs a capital opportunity cost at a rate of I_{Bk} for the items in stock.

(8) Since the vendor offers retailer a trade credit strategy, the vendor cannot receive the payment immediately after delivery of the items and therefore has to incur an opportunity cost at a rate of I_{Vp}. Trade credit occurs naturally between living company which the supplier may receive the payment with monthly balance or quarterly balance.

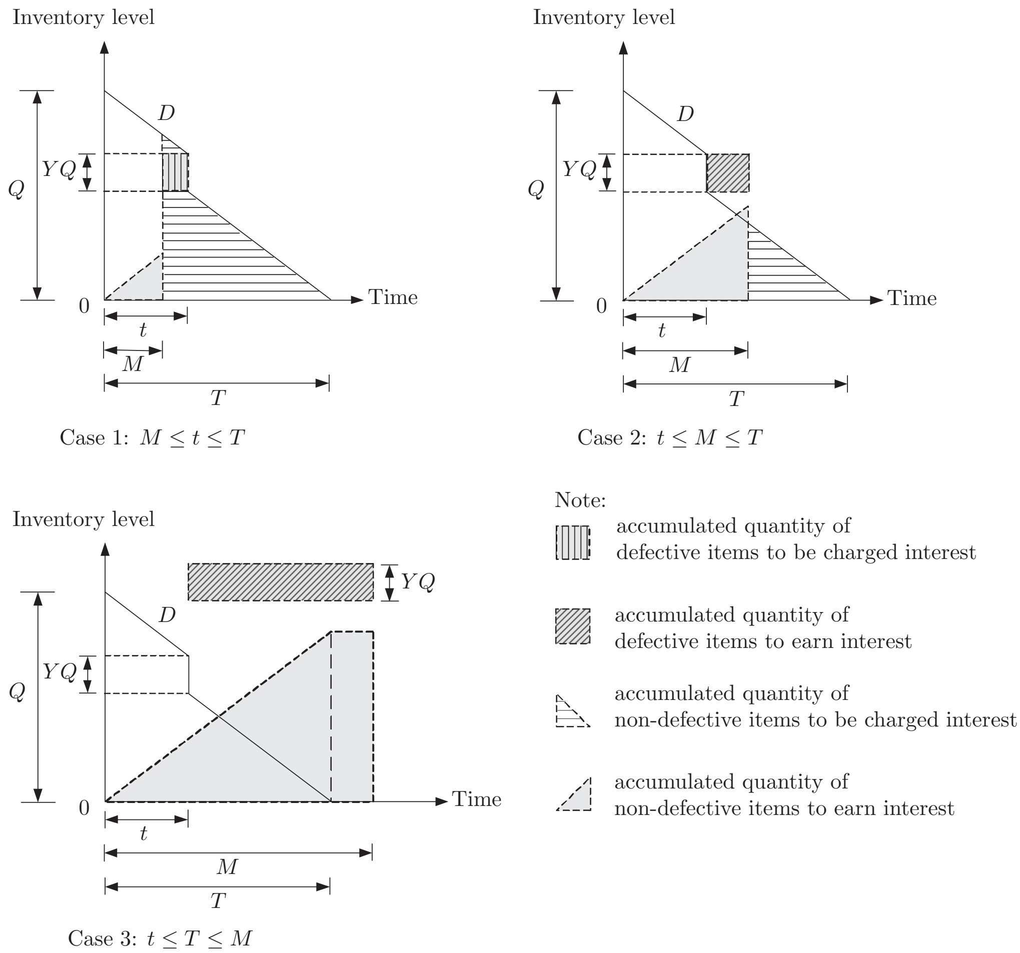

Figure 1: Inventory profile for a buyer's cycle.

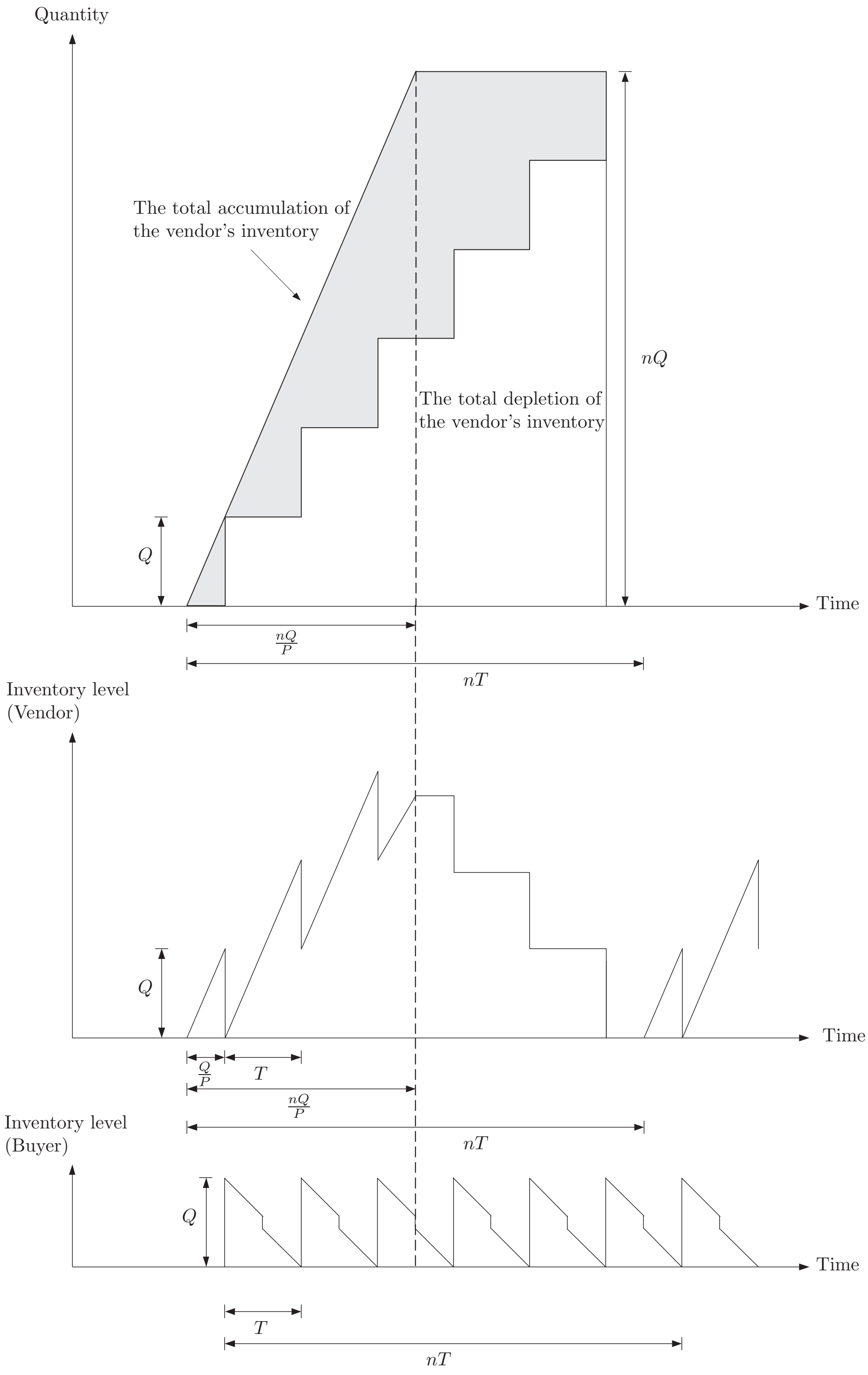

Figure 2: Time-weighted inventory against time for $n = 6$ for vendor and buyer.

3. Mathematical Formulation

In this section, we establish an integrated vendor-buyer inventory model with defective goods and trade credit. Given the assumptions mentioned above, Figures 1 and 2 depict the behavior of inventory levels for both the buyer and the vendor. We first derive the buyer's and the vendor's total expected cost per unit time, and then formulated a mathematical programming model for the supply chain system considered. The objective is to determine the optimal shipment size, the optimal number of shipments and the optimal capital expenditure that minimizes the joint total expected cost per unit time for the integrated supply chain.

3.1. Buyer's total expected cost per unit time

The buyer incurs an order quantity $(1-Y)Q$, so the cycle length of the buyer is given by $T = (1-Y)Q/D$ and the expected cycle time of the buyer is $E(T) = (1-E(Y))Q/D$. The inventory profile for a buyer's cycle can be schematized as follows.

The buyer's total expected cost per unit time consists of ordering cost, transportation cost, capital expenditure, screening cost, holding cost, interest earned, and opportunity cost. These components are calculated as follows:

(1) The expected ordering cost per unit time for each order of quantity Q is $A/E(T)$.

(2) The expected transportation cost per unit time for each order of quantity Q is $F/E(T)$.

(3) The capital expenditure is a cost per period. Hence, the total expected capital expenditure per unit time is $E(KT)/E(T) = K$.

(4) The vendor delivers batches of size Q to the buyer, and the buyer's unit screening cost is c_s. Therefore, the total expected screening cost per unit time is $c_sQ/E(T)$.

(5) The cumulated number of non-defective items is $(1-Y)QT/2$. The duration of the screening period is Q/x. The cumulated number of defective items is YQ^2/x. The holding cost per item per unit time is $ci(K)$. Therefore, the total expected holding cost per unit time is $ci(K)E[Q(1-Y)T/2 + YQ^2/x]/E(T)$.

(6) Expected interest earned and opportunity cost for perfect items and defective items are considered in the following three cases based on T, M and t, respectively.

Considering the relationship between M, t, and T, there are three possible cases as: (i) $M \le t \le T$, (ii) $t \le M \le T$ and (iii) $t \le T \le M$. These cases are depicted in Figure 1.

Case 1: $M \le t \le T$

Since the buyer does not pay the vendor until the end of the credit period, the buyer can use the sales revenue during the interval $[0, M]$ at a rate of I_{Be}. The buyer's interest earned for non-defective items per unit time is

$$\frac{1}{E(T)}\frac{sI_{Be}DM^2}{2} = \frac{sI_{Be}D^2M^2}{2Q(1-E(Y))}.$$

The buyer pays to the vendor at the end of the credit period, M, which is before the inventory is depleted completely. Hence, the buyer still has some stock on hand during the time interval $[M, T]$ and has to endure a capital opportunity cost at a rate of I_{Bk}. The capital opportunity cost of non-defective items per cycle is $cI_{Bk}[(1-Y)Q - DM](T-M)/2$. The capital opportunity cost of defective items per cycle is $vI_{Bk}YQ(t-M)$. Therefore, the buy's total expected capital opportunity cost per unit time is

$$\begin{aligned}&\frac{1}{E(T)}\frac{cI_{Bk}}{2}E\{[(1-Y)Q-DM](T-M)\}+\frac{1}{E(T)}vI_{Bk}E(Y)Q(t-M)\\&=\frac{cI_{Bk}E[(1-Y)Q-DM]^2}{2Q(1-E(Y))}+vI_{Bk}D\Big(\frac{Q}{x}-M\Big)\frac{E(Y)}{1-E(Y)}.\end{aligned}$$

Case 2: $t \le M \le T$

In this case, the buyer's interest earned for defective items per cycle is $vI_{Be}YQ(M-Q/x)$. The buyer's interest earned for non-defective items per cycle is $sI_{Be}DM^2/2$. Therefore, the buyer's total expected interest earned per unit time is

$$\begin{aligned}&\frac{1}{E(T)}vI_{Be}E(Y)Q(M-Q/x)+\frac{1}{E(T)}\frac{sI_{Be}DM^2}{2}\\&=vI_{Be}D\Big(M-\frac{Q}{x}\Big)\frac{E(Y)}{1-E(Y)}+\frac{sI_{Be}D^2M^2}{2Q(1-E(Y))}.\end{aligned}$$

The expected capital opportunity cost of non-defective items per unit time is

$$\frac{1}{E(T)}\frac{cI_{Bk}E[(1-Y)Q-DM]^2}{2D}=\frac{cI_{Bk}E[(1-Y)Q-DM]^2}{2Q(1-E(Y))}.$$

Case 3: $t \le T \le M$

In this case, the buyer's interest earned for defective items per cycle is $vI_{Be}YQ(M-t)$. The buyer's interest earned for non-defective items per cycle is $sI_{Be}[DT^2/2+DT(M-T)]$. Therefore, the total expected interest earned per unit time is

$$\begin{aligned}&\frac{1}{E(T)}vI_{Be}E(Y)Q(M-t)+\frac{1}{E(T)}sI_{Be}E\Big[\frac{DT^2}{2}+DT(M-T)\Big]\\&=vI_{Be}D\Big(M-\frac{Q}{x}\Big)\frac{E(Y)}{1-E(Y)}+sI_{Be}\Big[DM-\frac{QE(1-Y)^2}{2(1-E(Y))}\Big].\end{aligned}$$

Since the cycle length T depends on the percentage rate of defective items, T is a random variable. Therefore, in deriving the optimal lot size the expected cycle time of the buyer should be considered as $E(T) = (1-E(Y))Q/D$. Considering the relationship between M, Q/x, and $E(T)$, we have the three cases (i) $Q \ge Mx$, (ii) $MD/(1-E(Y)) \le Q \le Mx$ and (iii) $0 < Q \le MD/(1-E(Y))$.

For notational convenience, let

$$E_1 = 1 - E(Y),$$
$$E_2 = E(1-Y)^2,$$
$$\varphi_1(K) = i(K)c\Big[\frac{E_2}{E_1} + \frac{2D(1-E_1)}{xE_1}\Big] + cI_{Bk}\frac{E_2}{E_1} + 2vI_{Bk}\frac{D(1-E_1)}{xE_1},$$
$$\varphi_2(K) = i(K)c\Big[\frac{E_2}{E_1} + \frac{2D(1-E_1)}{xE_1}\Big] + cI_{Bk}\frac{E_2}{E_1} + 2vI_{Be}\frac{D(1-E_1)}{xE_1},$$

and

$$\varphi_3(K) = i(K)c\Big[\frac{E_2}{E_1} + \frac{2D(1-E_1)}{xE_1}\Big] + sI_{Be}\frac{E_2}{E_1} + 2vI_{Be}\frac{D(1-E_1)}{xE_1}.$$

Summarizing the above cases, the buyer's total expected cost per unit time is as follows:

$$EUTCB(Q,K) = \begin{cases} EUTC_1^b(Q,K), \text{ if } Q \geq Mx, \\ EUTC_2^b(Q,K), \text{ if } MD/E_1 \leq Q \leq Mx, \\ EUTC_3^b(Q,K), \text{ if } 0 < Q \leq MD/E_1, \end{cases}$$

where

$$\begin{aligned} EUTC_1^b(Q,K) = {} & \frac{D}{QE_1}\Big(A + F + cI_{Bk}\frac{DM^2}{2} - sI_{Be}\frac{DM^2}{2}\Big) + \frac{Q}{2}\varphi_1(K) \\ & + \frac{c_sD}{E_1} - cI_{Bk}DM - vI_{Bk}DM\frac{1-E_1}{E_1} + K, \end{aligned} \tag{3.1}$$

$$\begin{aligned} EUTC_3^b(Q,K) = {} & \frac{D}{QE_1}\Big(A + F + cI_{Bk}\frac{DM^2}{2} - sI_{Be}\frac{DM^2}{2}\Big) + \frac{Q}{2}\varphi_2(K) \\ & + \frac{c_sD}{E_1} - cI_{Bk}DM - vI_{Bk}DM\frac{1-E_1}{E_1} + K, \end{aligned} \tag{3.2}$$

and

$$\begin{aligned} UTC_3^b(Q,K) = {} & \frac{D}{QE_1}(A+F) + \frac{Q}{2}\varphi_3(K) \\ & + \frac{c_sD}{E_1} - vI_{Be}DM\frac{1-E_1}{E_1} - sI_{Bk}DM + K. \end{aligned} \tag{3.3}$$

3.2. Vendor's total expected cost per unit time

The vendor's total expected cost per unit time consists of setup cost, holding cost and opportunity cost (vendor cannot receive the payment immediately after delivery of the items). Each element can be calculated as follows:

(1) Setup cost
The vendor incurs a batch setup cost S_v, and each production cycle length is nT. Therefore, the vendor's expected setup cost per unit time is $S_v/(nE(T)) = S_vD/(nQE_1)$.

(2) Holding cost
The accumulation of the vendor's inventory during the production run shown by the shaded area in Figure 2 is determined as follows:

$$\begin{aligned}&\Big[nQ\Big(\frac{Q}{P}+(n-1)T\Big)-\frac{nQ}{2}\frac{nQ}{P}\Big]-[1+2+\cdots+(n-1)]QT\\&=\Big[nQ\Big(\frac{Q}{P}+(n-1)\frac{(1-Y)Q}{D}\Big)-\frac{n^2Q^2}{2P}\Big]-\frac{n(n-1)Q}{2}\frac{(1-Y)Q}{D}\\&=\frac{nQ^2}{2D}\Big[(n-1)\Big(1-Y-\frac{D}{P}\Big)+\frac{D}{P}\Big].\end{aligned}$$

Hence, the vendor's expected holding cost per unit time is

$$\frac{h_v}{nE(T)}\frac{nQ^2}{2D}\Big[(n-1)\Big(1-E(Y)-\frac{D}{P}\Big)+\frac{D}{P}\Big]=h_v\frac{Q}{2}\Big[(n-1)\Big(1-\frac{D}{PE_1}\Big)+\frac{D}{PE_1}\Big].$$

(3) Opportunity cost
Due to the vendor offers credit period M to the buyer, the vendor will not receive the payment until M. Hence, with a finance rate, I_{Vp}, the expected opportunity cost per unit time for the vendor is

$$\frac{E(cI_{Vp}QM)}{E(T)}=\frac{cI_{Vp}DM}{E_1}.$$

Aforementioned, for fixed payment date M, the total expected cost per unit time for the vendor can be expressed as

$$EUTCV(n,Q)=\frac{S_vD}{nE_1Q}+h_v\frac{Q}{2}\Big[(n-1)\Big(1-\frac{D}{PE_1}\Big)+\frac{D}{PE_1}\Big]+\frac{cI_{Vp}DM}{E_1}. \tag{3.4}$$

3.3. Joint total expected cost per unit time

The buyer and the vendor can jointly determine the best policy for both parties once they have built up a long-term strategic partnership. Accordingly, the joint total expected cost per unit time can be obtained as the sum of the buyer's and the vendor's total expected costs per unit time. That is,

$$JETCU(n,Q,K)=\begin{cases}JETCU_1(n,Q,K), & \text{if } Q\geq Mx,\\ JETCU_2(n,Q,K), & \text{if } MD/E_1\leq Q\leq Mx,\\ JETCU_3(n,Q,K), & \text{if } 0<Q\leq MD/E_1,\end{cases}$$

where

$$\begin{aligned}JETCU_1(n,Q,K)&=EUTCV(n,Q)+EUTC_1^b(Q,K)\\&=\frac{D}{QE_1}L+\frac{Q}{2}G_1+\frac{c_sD}{E_1}+DM\Big(\frac{cI_{Vp}}{E_1}-cI_{Bk}-vI_{Bk}\frac{1-E_1}{E_1}\Big)+K, \quad (3.5)\\JETCU_2(n,Q,K)&=EUTCV(n,Q)+EUTC_2^b(Q,K)\end{aligned}$$

$$= \frac{D}{QE_1}L+\frac{Q}{2}G_2+\frac{c_sD}{E_1}+DM\Big(\frac{cI_{Vp}}{E_1}-cI_{Bk}-vI_{Bk}\frac{1-E_1}{E_1}\Big)+K, \quad (3.6)$$

$$\begin{aligned} JETCU_3(n,Q,K) &= EUTCV(n,Q) + EUTC_3^b(Q,K) \\ &= \frac{D}{QE_1}\Big(\frac{S_v}{n}+A+F\Big)+\frac{Q}{2}G_3+\frac{c_sD}{E_1} \\ &\quad +DM\Big(\frac{cI_{Vp}}{E_1}-cI_{Be}-vI_{Be}\frac{1-E_1}{E_1}\Big)+K, \qquad (3.7) \\ L &= \frac{S_v}{n}+A+F-(sI_{Be}-cI_{Bk})\frac{DM^2}{2}, \\ G_1 &= h_v\Big[n\Big(1-\frac{D}{PE_1}\Big)-1+\frac{2D}{PE_1}\Big]+\varphi_1(K)>0, \\ G_2 &= h_v\Big[n\Big(1-\frac{D}{PE_1}\Big)-1+\frac{2D}{PE_1}\Big]+\varphi_2(K)>0, \end{aligned}$$

and

$$G_3 = h_v\Big[n\Big(1-\frac{D}{PE_1}\Big)-1+\frac{2D}{PE_1}\Big]+\varphi_3(K)>0,$$

4. Solution Procedure

The objective is to determine the optimal the number of shipments, lot size per shipment and capital expenditure that minimizes the joint total expected cost per unit time of the integrated supply chain.

First, for given Q and K, to understand the effect of shipment number n on the join total expected cost per unit time, we temporarily relaxes the integer requirement on n, and taking the second partial derivative of $JETCU(n,K,Q)$ with respect to n, it gets

$$\frac{\partial JETCU(n,K,Q)}{\partial n^2}=\frac{\partial^2 JETCU_i(n,K,Q)}{\partial n^2}=\frac{D}{QE_1}\frac{2S_v}{n^3}>0, \quad i=1,2,3.$$

Thus, for fixed Q and K, $JETCU(n,K,Q)$, is a convex function of n. Hence, for given Q and K, the search for the optimal number of shipments, denoted by n^*, is reduced to find a local optimal solution.

Next, for given n and K, the joint total expected cost per unit time is shown to be a convex function of Q and the optimal lot size per shipment is derived first. Then for given n and Q, the joint total expected cost per unit time is shown to be a convex function of K and the optimal capital expenditure is derived.

Based on the derived results, an algorithm is proposed to solve the optimal solutions in the integrated vendor-buyer inventory model.

4.1. Determination of the optimal Q for given n and K

In this section, we first find the optimal lot size per shipment which minimizes $JETCU_i(n,K,Q)$, $i = 1,2,3$, respectively, for a given n and K. Then the optimal number of shipments is derived for a given K.

Taking the first and second order partial derivative of $JETCU_i(n,K,Q)$, $i=1,2,3$, with respect to Q respectively, we have

$$\frac{\partial JETCU_1(n,K,Q)}{\partial Q}=\frac{1}{2Q^2E_1}(-2DL+Q^2E_1G_1),$$
$$\frac{\partial JETCU_2(n,K,Q)}{\partial Q}=\frac{1}{2Q^2E_1}(-2DL+Q^2E_1G_2),$$
$$\frac{\partial JETCU_3(n,K,Q)}{\partial Q}=\frac{1}{2Q^2E_1}\Big[-2D\Big(\frac{S_v}{n}+A+F\Big)+Q^2E_1G_3\Big],$$
$$\frac{\partial^2 JETCU_1(n,K,Q)}{\partial Q^2}=\frac{\partial^2 JETCU_2(n,K,Q)}{\partial Q^2}=\frac{2D}{Q^3E_1}L,$$

and

$$\frac{\partial^2 JETCU_3(n,K,Q)}{\partial Q^2}=\frac{2D}{Q^3E_1}\Big(\frac{S_v}{n}+A+F\Big)>0.$$

By fixing n and K, we have the following results.

Proposition 1. *For given n and K,*

(a) *if $-2DL+M^2x^2E_1G_1\geq 0$, then $JETCU_1(n,K,Q)$ has the minimum value at the lower boundary $Q=Mx$.*

(b) *if $-2DL+M^2x^2E_1G_1<0$, then the optimal value of Q, denoted by $Q_1^{(n)}$, minimizing $JETCU_1(n,K,Q)$ is $Q_1^{(n)}=\sqrt{\frac{2DL}{E_1G_1}}$.*

Proof. See the Appendix A. □

Proposition 2. *For given n and K,*

(a) *if $-2DL+M^2x^2E_1G_2\leq 0$, then $JETCU_2(n,K,Q)$ has the minimum value at the lower boundary $Q=Mx$.*

(b) *if $-2DL+\frac{M^2D^2}{E_1}G_2\geq 0$, then $JETCU_2(n,K,Q)$ has the minimum value at the lower boundary $Q=\frac{MD}{E_1}$.*

(c) *if $\frac{M^2D^2}{E_1}G_2<2DL<M^2x^2E_1G_2$, then the optimal value of Q, denoted by $Q_2^{(n)}$, minimizing $JETCU_2(n,K,Q)$ is $Q_2^{(n)}=\sqrt{\frac{2DL}{E_1G_2}}$.*

Proof. The proof is similar that of Proposition 1, we omit it here. □

Proposition 3. *For given n and K,*

(a) *if $-2D\Big(\frac{S_v}{n}+A+F\Big)+\frac{M^2D^2}{E_1}G_3\leq 0$, then $JETCU_3(n,K,Q)$ has the minimum value at the lower boundary $Q\frac{MD}{E_1}$.*

(b) *if* $-2D\Big(\frac{S_v}{n}+A+F\Big)+\frac{M^2D^2}{E_1}G_3>0$, *then the optimal value of* Q, *denoted by* $Q_3^{(n)}$, *minimizing* $JETCU_3(n,K,Q)$ *is* $Q_3^{(n)}=\sqrt{\frac{2D}{E_1G_3}\Big(\frac{S_v}{n}+A+F\Big)}$.

Proof. The proof is similar that of Proposition 1, we omit it here. □

4.2. Determination of the optimal K for any given n and Q

For given n and Q, the first-order necessary condition of $JETCU(n,K,Q)$ with respect to K. This gives

$$\frac{\partial JETCU_i(n,K,Q)}{\partial K}=\frac{cQ}{2}\Big[\frac{E_2}{E_1}+\frac{2D(1-E_1)}{xE_1}\Big]\frac{\partial i(K)}{\partial K}+1=0,\quad i=1,2,3. \tag{4.1}$$

By fixing n and Q, we have the following result.

Proposition 4. *For given feasible* n *and* Q, *if* $i(K)$ *is a strictly decreasing and convex function of* K, *then there exists a unique* K^* *minimizing* $JETCU_i(n,K,Q), i=1,2,3$.

Proof. See the Appendix B. □

Let n^* be the optimal number of shipments. To avoid using a brute force enumeration for finding n^*, we further simplify the search process by providing an intuitively good starting value for n^*. For simplicity, we may assume the initial capital expenditure, say K_{init}, satisfies $i(K)=(i_U+i_L)/2$. By using the similar method to obtain an estimate of the optimal number of shipments n^* as Equation (18) in Teng et al. [28], we propose an estimate of the number of shipments, say n_{init}, as

$$n_{init}=\begin{cases}1, & \text{if } \varphi_i(K_{init})-h_v\Big(1-\frac{2D}{PE_1}\Big)\le 0,\\[2ex] n_1=\sqrt{\dfrac{S_v\Big[\varphi_1(K)-h_v\Big(1-\frac{2D}{PE_1}\Big)\Big]}{\Big(A+F+cI_{Bk}\frac{DM^2}{2}-sI_{Be}\frac{DM^2}{2}\Big)h_v\Big(1-\frac{D}{PE_1}\Big)}}, & \text{if } \varphi_1(K_{init})-h_v\Big(1-\frac{2D}{PE_1}\Big)>0,\\[2ex] n_2=\sqrt{\dfrac{S_v\Big[\varphi_2(K)-h_v\Big(1-\frac{2D}{PE_1}\Big)\Big]}{\Big(A+F+cI_{Bk}\frac{DM^2}{2}-sI_{Be}\frac{DM^2}{2}\Big)h_v\Big(1-\frac{D}{PE_1}\Big)}}, & \text{if } \varphi_2(K_{init})-h_v\Big(1-\frac{2D}{PE_1}\Big)>0,\\[2ex] n_3=\sqrt{\dfrac{S_v\Big[\varphi_3(K)-h_v\Big(1-\frac{2D}{PE_1}\Big)\Big]}{\Big(A+F\Big)h_v\Big(1-\frac{D}{PE_1}\Big)}}, & \text{if } \varphi_3(K_{init})-h_v\Big(1-\frac{2D}{PE_1}\Big)>0.\end{cases} \tag{4.2}$$

Combining the above arguments, we propose the following algorithm to solve the inventory problem.

Algorithm

Step 1. Choose an initial trial value of n, where $n = \lceil \min\{n_1, n_2, n_3\} \rceil$ is obtained by Equation (4.2).

Step 1.1. Start with $j = 0$ and let K_{init} be the initial trial value of .

Step 1.2. Use Propositions 1-3 to determine

$$\min\{JETCU_1(n, K_j, Q), JETCU_2(n, K_j, Q), JETCU_3(n, K_j, Q)\}$$

and the corresponding value of Q_J.

Step 1.3. Use the result in Step 1.2 to determine the value of K_{j+1} by Equation (4.1).

Step 1.4. If the difference between K_j and K_{j+1} is sufficiently small (for example, set $|K_{j+1} - K_j| < 10^{-4}$), set $(K_n^*, Q_n^*) = (K_{j+1}, Q_{j+1})$ is the optimal solution with the given n.

Step 2. Set $n = n - 1$.

Step 2.1. Use Steps 1.1-1.4 to obtain (K_{n-1}^*, Q_{n-1}^*), and compute the corresponding $JETCU(n-1, K_{n-1}^*, Q_{n-1}^*)$.

Step 2.2. If $JETCU(n-1, K_{n-1}^*, Q_{n-1}^*) > JETCU(n, K_n^*, Q_n^*)$, go to Step 3. Otherwise compute $JETCU(n-2, K_{n-2}^*, Q_{n-2}^*) > JETCU(n-3, K_{n-3}^*, Q_{n-3}^*)$, $\ldots$, until we find $JETCU(m-1, K_{m-1}^*, Q_{m-1}^*) > JETCU(m, K_m^*, Q_m^*)$. Set $(n^*, K^*, Q^*) = (m, K_m^*, Q_m^*)$ and stop.

Step 3. Set $n = n + 1$.

Step 3.1. Use Steps 1.1-1.4 to obtain (K_{n+1}^*, Q_{n+1}^*), and compute the corresponding $JETCU(n+1, K_{n+1}^*, Q_{n+1}^*)$.

Step 3.2. If $JETCU(n+1, K_{n+1}^*, Q_{n+1}^*) > JETCU(n, K_n^*, Q_n^*)$, then compute $JETCU(n+2, K_{n+2}^*, Q_{n+2}^*)$, $JETCU(n+3, K_{n+3}^*, Q_{n+3}^*)$, $\ldots$, until we find $JETCU(m+1, K_{m+1}^*, Q_{m+1}^*) > JETCU(m, K_m^*, Q_m^*)$. Set $(n^*, k^*, Q^*) = (m, K_m^*, Q_m^*)$ and stop.

5. Computational Results

5.1. Numerical example

The analysis of data is carries out by means of extensive interviews with high-level managers in the manufacturing section. The milling cutters was restricted to supplier of manufactured components.

Example 1. In order to illustrate the above solution procedure, we consider an inventory system with the relevant data:

$A = \$360/\text{cycle}$	$F = \$50/\text{shipment}$	$S_v = \$500/\text{cycle}$	$D = 8000\text{units/year}$
$P = 16000\text{units/year}$	$x = 20000$ units/year	$c = \$80/\text{unit}$	$s = \$180/\text{unit}$
$v = \$20/\text{unit}$	$c_s = \$0.5/\text{unit}$	$h_v = \$2/\text{unit/year}$	$M = 0.03\text{year}$
$I_{Bk} = \$0.15/\$/\text{year}$	$I_{Be} = \$0.10/\$/$ year	$I_{Vp} = \$0.02/\$/$ year	

The percentage defective random variable, Y, can take any value in the range $[\alpha, \beta]$ with $\alpha = 0$, and $\beta = 0.04$. It is assumed that Y is uniformly distributed with the following probability density function:

$$f(y) = \begin{cases} 25, & 0 \le y \le 0.04, \\ 0, & \text{otherwise.} \end{cases}$$

In addition, the inventory carrying cost rate is considered as $i(K) = i_L + (i_U - i_L)e^{-aK}$, where $\alpha > 0$. Therefore, we have $i'(K) = -a(i_U - i_L)e^{-aK} < 0$ and $i''(K) = a^2(i_U - i_L)e^{-aK} > 0$. We set $i_L = \$0.05/\$/\text{year}$, $i_U = \$0.2/\$/\text{year}$ and $\alpha = 0.004$.

Example 2. In this example, the same data in Example 1 are used except putting $M = 0.1/\text{year}$ and $D = 3000$ unit/year.

Table 1 shows the optimal solutions of integrated models for Examples 1 and 2. The solution procedures are implemented using Mathematica Version 11 on a personal computer with Intel Core i7 processor under Microsoft Windows 7 Pro. In order to verify the performance of the algorithm in our problem, Examples 1 and 2 were repeated 100 times of the algorithm. In addition, the optimal solution for the special case without holding cost reduction (i.e., $K = 0$), denoted by $JETCU_0(n_0, Q_0, B_0)$ is also illustrated. From Table 1, with an added capital expenditure, the optimal joint total expected cost per unit time decreases, the optimal number of shipments decreases but the optimal size of shipments increases. The value of $\Delta JETCU$ also reveals that the inventory model with an added capital expenditure is better in terms of cost minimization.

On the other hand, we consider another scenario that the buyer and vendor determine to minimize their own cost separately. We first determine the optimal solutions of Q and K which minimize the total expected cost per unit time for the buyer. By using buyer's optimal ordering quantity, we then determine the optimal solution of n which minimizes the total expected cost per unit time for the vendor. These results are shown in Table 2 (we omitted the proofs because they can be obtained by analogous arguments as Propositions 1-4 and Equation (4.2)).

5.2. Sensitivity analysis

In this subsection, we study the sensitivity of the optimal solution to change in the values of the different parameters associated with the model. For this purpose, sensitivity analyses are performed by varying the coefficients of Example 1, where five have lower parameter values (from -10% to -50%) than the default, and five have higher parameter

Table 1: Computation results of integrated models for Examples 1 and 2.

	with capital expenditure		without capital expenditure	
Example 1	$JETCU^*$	15358.98	$JETCU_0^*$	17683.71
	n^*	5	n_0^*	6
	Q^*	608.19	Q_0^*	478.37
	K^*	669.31	–	–
	$E(T^*)$	0.074	$E(T_0^*)$	0.0586
	Policy	Case 1	Policy	Case 2
	$EUTCB^*$	12123.07	$EUTCB_0^*$	14454.23
	$EUTCV^*$	3235.92	$EUTCV_0^*$	3229.48
	$\Delta JETCU$	-13.15%		
	ACT	5.85695	ACT	0.242397
Example 2	$JETCU^*$	6745.73	$JETCU_0^*$	7823.94
	n^*	4	n_0^*	5
	Q^*	348.86	Q_0^*	278.40
	K^*	527.78	–	–
	$E(T^*)$	0.114	$E(T_0^*)$	0.091
	Policy	Case 2	Policy	Case 3
	C.T.	9.68468	C.T.	0.672351
	$EUTCV^*$	4245.98	$EUTCB_0^*$	5280.76
	$EUTCV^*$	2499.75	$EUTCV_0^*$	2543.18
	$JETCU$	-13.78%		
	ACT	9.68468	ACT	0.672351

Note : ACT: average CPU time (seconds), $\Delta JETCU = 100\% \times (JETCU^* - JETCU_0^*)/JETCU_0^*$.

Table 2: Computation results of independent models for Examples 1 and 2.

	Buyer		Vendor	
Example 1	Policy	Case 1	number of shipments	5
	size of shipments	617.96		
	the capital expenditure	673.29		
	total expected cost per unit time	12121.80	total expected cost per unit time	3238.82
Example 2	Policy	Case 2	number of shipments	4
	size of shipments	337.50		
	the capital expenditure	519.50		
	total expected cost per unit time	4242.93	total expected cost per unit time	2506.94

values (from 10% to 50%). The computed results are shown in Tables 3 and 4. The results obtained for illustrative examples provide certain insights about the problem studies. Some of them are as follows.

(1) From Table 3, as the demand rate D increases, the buyer wants to sell more so that he/she increases the size of shipments, which leads to more inventory on hand. Then the buyer would like to spend more money on reducing holding cost, i.e., a larger value for K^*.

(2) From Table 3, as the ordering cost A increases, the buyer wants to order more so that he/she decreases the number of shipments. Then the buyer would like to spend more money on reducing holding cost, i.e., a larger value for K^*.

(3) From Table 3, as the capital opportunity cost per dollar I_{Bk} increases, the buyer wants to decrease the size of shipments to avoid too much capital opportunity cost, which leads to less inventory on hand. Then the buyer would like to spend less money on reducing holding cost, i.e., a lower value for K^*.

(4) From Table 3, as the interest earned per dollar I_{Be} increases, the buyer wants to decrease the size of shipments so that he/she takes the benefits of the permissible delay more frequently, which leads to lower joint total expected cost per unit time. Then less inventory on hand results in less money on reducing holding cost, i.e., a lower value for K^*.

(5) From Table 3, as β increases, which implies the mean of defective percentage increases, the buyer wants to sell more so that he/she increases the size of shipments, which leads to more inventory on hand. Then the buyer would like to spend more money on reducing holding cost, i.e., a larger value for K^*.

(6) From Table 4, due to the longer credit period, the buyer earns more interest while the opportunity cost for the vendor increases. Then the vendor's cost increases, but the buyer's cost decreases more, which leads to decrease in the cost of entire supply chain. In addition, when $sI_{Be} > cI_{Bk}$, the size of shipments decreases with increase in permissible delay period. It means that the buyer wants to order less quantity so that he/she takes the benefits of the permissible delay more frequently. Then less inventory on hand results in less money on reducing holding cost, i.e., a lower value for K^*. When $sI_{Be} \leq cI_{Bk}$, the size of shipments increases with increase in permissible delay period which implies that the buyer should procure more quantity to avoid higher interest charges after the credit period. Then more inventory on hand results in more money on reducing holding cost, i.e., a larger value for K^*.

(7) From Table 4, as the unit procurement cost c increases, the size of shipments decreases while the number of shipments increases. From economic point of view, if the vendor provides a higher unit procurement cost, the buyer will order lower quantity in order to avoid too much capital opportunity cost, which leads to less inventory on hand. In order to reduce the per-unit holding cost, $ci(K)$, the buyer would like to increase cost in capital expenditure with increase in the unit procurement cost.

5.3 Managerial implication

Milling cutter is a kind of cutting tools, and be used in milling machines or machining centres to perform milling operations (and occasionally in other machine tools) in precision machine industry. In fact, milling cutters are widely applied in aero industrial, automobile industrial, medical industrial, mold industrial. For the reason that milling cutters are often expensive component, these parts will be inspected by the end of the 100% screening process.

In order to reflect real integrate supply chain situations, the selected one manufacturer is a popular and small and medium enterprise in Taichung around 20 years of excellent background, own 160 sale centers around the world. In the case of the biggest buyer in Germany (annual purchase volume: 1 million units), an election is made volume of sale pursuant to $ 3 million. The buyer usually requires the vender ISO 9001 certification, for insure the quality is less than 5% of defective items in a single batch. Viewed in this light of ISO 9001, the first objective might be: "to improve on-time delivery from 90% to 95% within the next year" and the second could be : "to reduce field escapes to the customer from 4% to 3% within the next year".

International trade increases global income that results in more international tourist travel and shipment of higher value goods. For this reason, the cost (transportation cost) for shipping milling cutters has never been as low as now. It is necessary that the case-study company allow Germany enterprise trade credit for long-term cooperative relationship in CNC industrial. Moreover, in order to reduce the holding cost, the case-study company will take 100% surface inspection for defective milling cutters before shipments or sell those defective items to other buyers at low price.

A laser gauge makes use of an inspect object like milling cutters, ground parts metal tube in precision machine industry. When the customer's demands are broken down, the reduction of defective items will be the important technology for inspect process.

In order to prevent the barrel roll will shift from in-control state to out-of-control state, Badami et al. [2] describes the development of a portable three-dimensional (3-D) stylus-based surface profiler with scan range of 4.5 mm X 5.5 mm and a maximum vertical of 150μm. Moreover, the specification changed in new product development play a critical role in guiding that vender-buyer relationship. In the process of new product development (short life-cycle), the product launch time is crucially important to meet changing customer desires. As mentioned above, the vender will be consolidated into the fewest number of shipments possible through air transport or ocean carriage. On account of high air transport cost, the buyer will order more and decrease the shipping times.

Table 3: Effects of parameters on optimal solution.

		percentages of parameter change										
		-50%	-40%	-30%	-20%	-10%	0	10%	20%	30%	40%	50%
D	Policy	Case 2	Case 2	Case 2	Case 2	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1
	n^*	4	4	4	4	4	5	5	5	6	6	7
	$JETCU^*$	11047.07	12058.44	12982.76	13837.17	14633.63	15358.98	16032.30	16665.12	17240.04	17769.43	18240.00
	Q^*	442.6	487.2	528.5	567.3	603.8	608.2	641.5	673.8	681.6	712.9	725.5
	K^*	587.82	612.22	632.99	651.10	667.11	669.31	683.05	695.75	699.01	710.65	715.46
	$EUTCB^*$	8596.43	9403.37	10148.95	10845.94	11503.12	12123.07	12715.58	13283.47	13833.53	14357.30	14869.81
	$EUTCV^*$	2450.64	2655.07	2833.81	2991.23	3130.50	3235.92	3316.72	3381.64	3406.51	3412.13	3370.19
	ACT	5.79909	5.85303	5.28878	6.72386	7.85289	5.85695	4.33734	4.97889	3.96678	3.93158	3.83287
A	Policy	Case 2	Case 2	Case 2	Case 2	Case 2	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1
	n^*	6	6	6	5	5	5	4	4	4	4	4
	$JETCU^*$	12600.79	13224.52	13814.08	14353.60	14866.47	15358.98	15833.16	16270.38	16694.12	17105.54	17505.65
	Q^*	457.1	485.2	511.8	560.9	585.1	608.2	661.3	683.0	704.1	724.5	744.4
	K^*	597.93	612.83	626.15	649.07	659.65	669.31	690.24	698.32	705.91	713.07	719.85
	$EUTCB^*$	9368.09	9994.84	10578.57	11121.29	11634.60	12123.07	12589.18	13032.48	13459.21	13871.05	14269.42
	$EUTCV^*$	3232.70	3229.68	3235.51	3232.31	3231.87	3235.92	3243.98	3237.91	3234.91	3234.48	3236.23
	ACT	8.30292	8.26784	5.52644	5.21495	4.95199	5.85695	6.39624	6.8898	6.88315	5.96316	3.0886

I_{Bk}	Policy	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 2	Case 2	Case 2	Case 2	Case 2
	n^*	4	4	4	4	5	5	5	5	5	5	5
	$JETCU^*$	14544.24	14734.34	14911.70	15077.81	15227.67	15358.98	15483.29	15601.30	15713.50	15820.37	15922.34
	Q^*	743.6	717.6	694.6	674.1	623.1	608.2	594.8	582.3	570.8	560.1	550.2
	K^*	719.55	710.68	702.54	695.04	675.36	669.31	663.76	658.45	653.45	648.72	644.25
	$EUTCB^*$	11308.13	11499.98	11675.79	11837.80	11987.06	12123.07	12250.22	12369.64	12482.00	12587.97	12688.15
	$EUTCV^*$	3236.11	3234.37	3235.91	3240.01	3240.61	3235.92	3233.07	3231.66	3231.51	3232.41	3234.19
	ACT	11.018	5.04794	7.8356	8.0385	6.3 44	5.85695	6.018	6.46898	5.2373	5.15447	8.87875
I_{Be}	Policy	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 2	Case 2	Case 2	Case 2
	n^*	5	5	5	5	5	5	5	5	5	5	5
	$JETCU^*$	15786.75	15702.31	15617.32	15531.78	15445.67	15358.98	15271.71	15183.84	15095.37	15006.26	14916.49
	Q^*	682.4	624.4	620.4	616.4	612.3	608.2	604.1	600.0	595.8	591.6	587.3
	K^*	677.49	675.90	674.28	672.65	670.99	669.31	667.61	665.92	664.17	662.38	660.56
	$EUTCB^*$	12544.09	12461.20	12377.67	12293.48	12208.62	12123.07	12036.82	11949.83	11862.13	11773.67	11684.42
	$EUTCV^*$	3242.66	3241.11	3239.65	3238.30	3237.05	3235.92	3234.89	3234.01	3233.24	3232.58	3232.07
	ACT	3.27744	3.47527	3.54759	3.74312	3.87529	5.85695	3.61587	3.40536	3.79428	4.53604	5.18386
β	Policy	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1
	n^*	5	5	5	5	5	5	5	5	5	5	5
	$JETCU^*$	15294.98	15307.64	15320.37	15333.17	15346.04	15358.98	15372.00	15385.09	15398.26	15411.50	15424.82
	Q^*	603.2	604.2	605.2	606.2	607.2	608.2	609.2	610.2	611.2	612.2	613.3
	K^*	667.65	667.98	668.31	668.64	668.97	669.31	669.65	669.99	670.33	670.68	671.03
	$EUTCB^*$	12068.60	12079.35	12090.17	12101.06	12112.03	12123.07	12134.18	12145.37	12156.64	12167.98	12179.39
	$EUTCV^*$	3226.39	3228.30	3230.20	3232.11	3234.01	3235.92	3237.82	3239.72	3241.62	3243.53	3245.43
	ACT	3.06549	3.09467	3.10989	3.65419	4.14094	5.85695	5.88248	5.34019	4.85631	3.84415	3.71566

Note : ACT: average CPU time (seconds).

Table 4: Impact of M on optimal solution.

		percentages of parameter change										
		-50%	-40%	-30%	-20%	-10%	0	10%	20%	30%	40%	50%
$c = 80$	Policy	Case 1	Case 1	Case 1	Case 1	Case 1	Case 1	Case 2	Case 2	Case 2	Case 2	Case 2
$sI_{Be} > cI_{Bk}$	n^*	5	5	5	5	5	5	5	5	5	5	5
	$JETCU^*$	16826.05	16544.36	16256.86	15963.51	15664.24	15358.98	15048.12	14731.17	14407.98	14078.44	13742.42
	Q^*	618.4	616.9	615.1	613.1	610.8	608.2	605.5	602.4	598.9	595.1	591.0
	K^*	673.47	672.87	672.15	671.32	670.38	669.31	668.22	666.90	665.46	663.88	662.16
	$EUTCB^*$	13783.00	13462.62	13136.50	12804.58	12466.80	12123.07	11773.69	11418.29	11056.64	10688.59	10313.99
	$EUTCV^*$	3043.05	3081.74	3120.36	3158.93	3197.44	3235.92	3274.43	3312.88	3351.34	3389.85	3428.43
	ACT	4.22807	3.46658	6.62699	4.69799	6.99301	2.05044	5.97894	7.19567	6.55815	4.79385	6.55015
$c = 120$	Policy	Case 1	Case 1	Case 1	Case 1	Case 2	Case 2	Case 2	Case 2	Case 2	Case 2	Case 2
	n^*	6	6	6	6	6	6	6	6	6	6	6
	$JETCU^*$	18626.07	18251.37	17876.68	17501.98	17127.51	16753.30	16379.10	16004.89	15630.69	15256.49	14882.28
	Q^*	513.4	513.4	513.4	513.4	513.5	513.5	513.5	513.5	513.5	513.5	513.5
	K^*	728.30	728.30	728.30	728.30	728.37	728.37	728.37	728.37	728.37	728.37	728.37
	$EUTCB^*$	15487.92	15054.46	14620.99	14187.52	13754.21	13321.23	12888.25	12455.27	12022.29	11589.31	11156.33
	$EUTCV^*$	3138.14	3196.92	3255.69	3314.47	3373.30	3432.07	3490.85	3549.62	3608.40	3667.17	3725.95
	ACT	5.14855	4.55339	3.81208	4.66818	6.51896	3.87063	4.84446	4.84732	7.9938	5.52778	4.11205
$c = 160$	Policy	Case 1	Case 1	Case 1	Case 2	Case 2	Case 2	Case 2	Case 2	Case 2	Case 2	Case 2
$sI_{Be} < cI_{Bk}$	n^*	6	6	6	6	6	6	6	6	6	6	6
	$JETCU^*$	20075.87	19618.92	19169.50	18727.69	18293.61	17866.86	17447.32	17034.92	16629.55	16231.10	15839.45
	Q^*	459.6	460.7	462.0	463.6	465.3	467.2	469.3	471.6	474.0	476.6	479.5
	K^*	772.56	773.16	773.87	774.73	775.65	776.66	777.78	778.98	780.28	781.67	783.14
	$EUTCB^*$	16843.84	16308.81	15781.34	15261.50	14749.39	14244.59	13746.99	13256.48	12772.95	12296.26	11826.30
	$EUTCV^*$	3232.02	3310.11	3388.16	3466.18	3455.22	3622.27	3700.33	3778.44	3856.60	3934.83	4013.15
	ACT	5.71303	5.03975	7.61818	5.70483	6.92823	4.74418	8.30994	6.10931	6.06042	5.61733	7.25711

Note : ACT: average CPU time (seconds).

6. Concluding Remarks

This paper formulated an integrated vendor-buyer inventory model with imperfect quality, trade credit and controllable holding cost. We will relax the dispensable assumptions that the screening time is less than the permissible delay period ($Q/x < M$) and interest earned per unit is less than interest charged per unit ($I_{Be} \leq I_{Bk}$). We analyze the effect of capital expenditure on inventory strategy. With an added capital expenditure reducing the holding cost, the expected cost of the entire supply chain will decrease. In addition, we find little evidence that the vendor and the buyer will share in profits through trade credit negotiation. The buyer will order more and decrease the number of shipments. A comprehensive sensitivity analysis is also conducted to explore the effects of parameters on the optimal results. When $sI_{Be} > cI_{Bk}$, with increase in permissible delay period, the buyer will decrease capital expenditure and order less to avail the benefit of permissible delay more frequently. When $sI_{Be} \leq cI_{Bk}$, with increase in permissible delay period, the buyer will increase capital expenditure and order more to avoid higher interest charged after the grace period.

There are several ways to extend the proposed model. For example, future research could consider the deterministic demand function to stock-dependent demand patterns. Another extension of this work may be set in the direction of considering stock dependent and stochastic demand with partial-trade credit. Finally, it would be interesting to incorporate the quantity discount, and the learning curve phenomenon into the model.

Appendix A

The proof of Proposition 1 (a).

For a given n and K, if $-2DL + M^2x^2E_1G_1 \geq 0$, we have $\dfrac{\partial JETCU_1(n,K,Q)}{\partial Q} \leq 0$ for $Q \in [Mx, \infty)$, which implies that $JETCU_1(n,K,Q)$ is a strictly increasing function of Q in $Q \in [Mx, \infty)$. Hence, $JETCU_1(n,K,Q)$ has a minimum value at the lower boundary point $Q = Mx \cdot \dfrac{\partial^2 JETCU_1(n,K,Q)}{\partial Q^2} < 0$.

The proof of Proposition 1 (b).

For a given n and K, if $-2DL + M^2x^2E_1G_1 < 0$, implying $L > 0$, we have $\dfrac{\partial^2 JETCU_1(n,K,Q)}{\partial Q^2} > 0$. Therefore, $\dfrac{\partial JETCU_1(n,K,Q)}{\partial Q}$ is strictly increasing on $Q > Mx$. Because $\lim\limits_{Q\to\infty}(-2DL + Q^2E_1G_1) = \infty$ and $-2DL + M^2x^2E_1G_1 < 0$, by the intermediate value Theorem, there exists a unique $Q = Q_1^{(n)} \in (Mx, \infty)$ such that $\dfrac{\partial JETCU_1(n,K,Q)}{\partial Q} = 0$. By solving $\dfrac{\partial JETCU_1(n,K,Q)}{\partial Q} = 0$, we obtain $Q_1^{(n)} = \sqrt{\dfrac{2DL}{E_1G_1}}$. This completes the proof. $\square$

Appendix B

The proof of Proposition 4.

First, from Equation (4.1), because $\partial i(K)/\partial K < 0$, it is clear that Equation (4.1) holds. Next, for given n and Q, taking the second partial derivative of $JETCU(n, K, Q)$ with respect to K yields

$$\frac{\partial^2 JETCU_i(n, K, Q)}{\partial K^2} = \frac{cQ}{2}\Big[\frac{E_2}{E_1} + \frac{2D(1 - E_1)}{xE_1}\Big]\frac{\partial^2 i(K)}{\partial K^2}, \qquad i = 1, 2, 3.$$

Because $i(K)$ is a convex function in K, we obtain $\partial^2 JETCU_i(n, K, Q)/\partial K^2 > 0$. Combining the results above, we know that if $i(K)$ is a strictly decreasing and convex function of K, then there exists a unique K^* minimizing $JETCU_i(n, K, Q)$, $i = 1, 2, 3$. This completes the proof. □

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