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An Exact Rectangular Two-Segment Layout Algorithm with Optimal Same-Shape Strip Generation

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Abstract

Compared with the general three-stage, the general two-segment, the well-known T-shape and TABU500 algorithms, this paper proposes a novel algorithm to solve large-scale rectangular packing problems efficiently. The algorithm in this paper can generate exactly rectangular optimal same-shape strip two-segment layout. The algorithm not only meets practical guillotine-cutting problems, but also is reasonable in computation time consuming. Firstly, the algorithm uses dynamic programming recursion to generate optimal same-shape strips; secondly, it solves knapsack problems to obtain the optimal same-shape strip two-segment layout. The algorithm is tested on 62 large-scale benchmark problems. Experimental results show that the algorithm is efficient and the solutions of the algorithm are better than conventional algorithms in solving large-scale two-dimensional cutting instances.

Keywords: Packing, cutting, same-shape strip, two-segment layout, dynamic programming.

1. Introduction

Packing problem comes from engineering practice, which is an old and famous problem, and mainly related to operations research, engineering technology, mathematics,computer science and logic. From the computation complexity theory, the packing problem has been proven a typical NP-hard problem [1]−[3], [8, 9], [11, 12], [14]. The two-dimensional layout problem is an important branch of packing problem. Rectangular layout is often involved in the processing industry, which is the first step of the manufacturing process, and has direct relationship with improving utilization of raw materials and reducing product cost.

This paper studies the unconstrained two-dimensional cutting (UTDC) problem: m types of rectangular pieces are cut from stock plate $L \times W$ (length × width), the demand of pieces is unconstrained. The objective is to find a cutting pattern that will maximize the summation of area s of all smaller rectangular pieces. Let the length, width and area of piece type i be l_i, w_i and v_i (the area of rectangular pieces), respectively, $i = 1, \dots, m$.

The objective is to find a layout that will maximize the summation of areas of all smaller rectangular pieces.

Assume that a layout P includes d_i number of type i pieces; V is the layout area. The mathematical model for the UTDC problem is:

$$V = \boxed{\left\{ \max \left(\sum_{i=1}^{m} v_i d_i \right); \ P \text{ is a feasible pattern; } d_i \text{ non-negative integers, } i=1,2,\ldots,m \right\}}$$

Although exact algorithm exists for UTDC problem, the computation time cannot be tolerated [3, 4]. Therefore, researchers usually resolve the problem by two ways: one is specific layout, such as Hifi [10] proposed a general three-stage and two-stage layout, Fayard [6] the general two segment layout, Cui [5] the famous T-shape layout; the other method is heuristic algorithm for generating layout, and the results of such algorithms are close to the general layout of exact algorithms, such as Alvarez [1] proposed a TABU heuristic algorithm (TABU500).

This paper proposes the application of the same-shape strip two segment (SS-2SEGMENT) layout in solving the UTDGC problem. The contents are arranged as follows. Section 2 describes SS-2SEGMENT layouts. Section 3 presents the algorithm. Section 4 summarizes the computational results. Section 5 gives the conclusions.

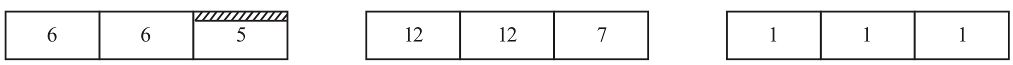
2. Fundamental concept of the SS-2SEGMENT layout

2.1. Piece

The piece direction is in the piece length direction. Assuming a given piece $l \times w$ the piece is in the rectangular sheet $L \times W$ (length L, width W). There are two arrangements: one is horizontal row, which the piece direction has the same direction of panel length (X direction); the other is vertical row, which the piece direction has the same direction of panel width (Y direction). Because each cut is always X or Y direction in the cutting process, the piece direction can only be X or Y direction.

2.2. Strip in layout

Figure 1 shows three type of strips for generating layout, and the number indicates the type of pieces. Where, the first is the general strip consisting of different width piece (Figure 1a), the second is the uniform strip having the same width piece (Figure 1b), and the third is the same-shape strip made of the same size piece in the same direction (Figure 1c).

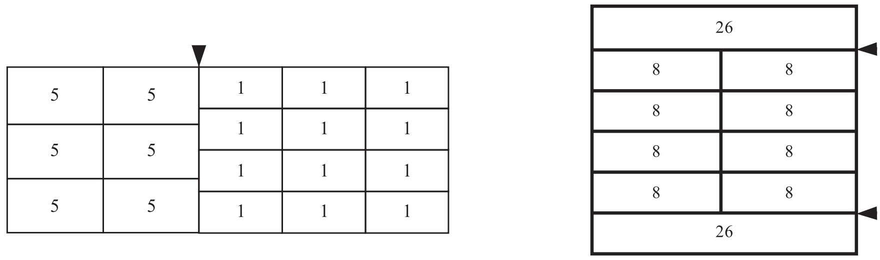


(a) The general strip (b) The uniform strip (c) The same-shape strip

Figure 1: Several kinds of strips.

2.3. Section and segment in layout

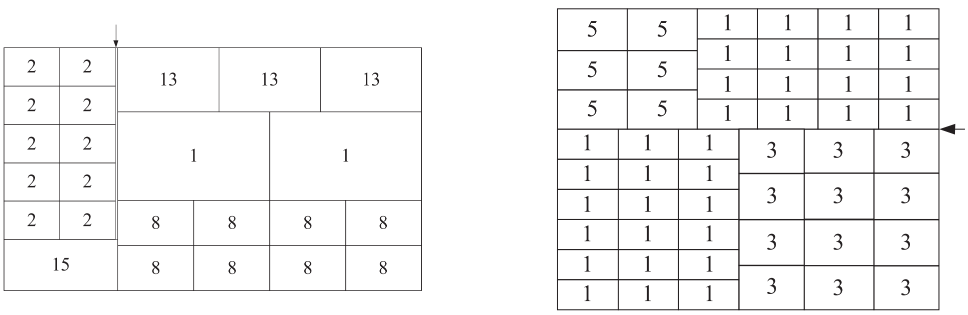
The section consists of several same-shape strips. The X section comprises a series of same-shape strips that are arranged horizontally from left to right, and the Y section includes a series of same-shape strips that are arranged vertically from top to bottom, each section contains same-shape strips of the same length and direction. Figure 2 shows the section of the rectangular pieces, where, an arrow shows the boundary of the same-shape strip. Figure 2a is X section consisting of two same-shape strip; Figure 2b is Y section including three same-shape strips.



(a) The X section (b) The Y section

Figure 2: The sections of rectangular pieces.

The segment is of several sections. The X segment includes a series of Y sections that are arranged horizontally from left to right, and the Y segment comprises a series of X sections that are arranged vertically from top to bottom. Figure 3 shows segment, where, an arrow shows the boundary of the sections. Figure 3a is X segment consisting of two Y section, where, the left Y section consists of two same-shape strips (piece 2 and 15), and the right Y section includes three same-shape strips (piece 13, 1 and 8); Figure 3b is Y segment consisting of two X section, where, the upper X section consists of two same-shape strips (piece 5 and 1), and the bottom X section also includes two same-shape strips (piece 1 and 3).

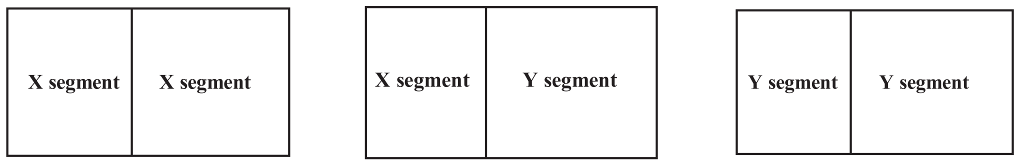


(a) The X segment (b) The Y segment

Figure 3: The segments of rectangular pieces.

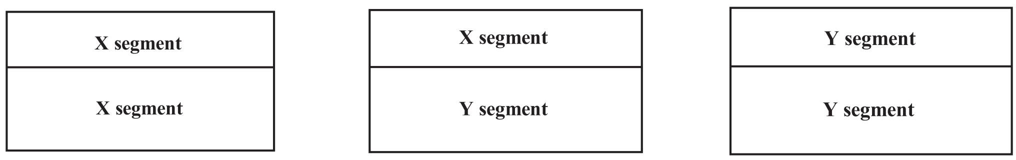
2.4. The SS-2SEGMENT layout

The SS-2SEGMENT layout is of two segments. Figure 4 indicates the types of SS-2SEGMENT layout. On the one hand, when division cut is vertical, the layout is termed as SSX-2SEGMENT; on the other hand, when division cut is horizontal, the layout is referred as SSY-2SEGMENT. In the SSX-2SEGMENT layout, if both the two segments are X segment, the layout is SSX-2SEGMENT-XX (Figure 4a1); if one segment is X segment and the other is Y segment, the layout is SSX-2SEGMENT-XY (Figure 4a2); if both the two segments are Y segment, the layout is SSX-2SEGMENT-YY (Figure 4a3). The naming approach of SSY-2SEGMENT is the same as SSX-2SEGMENT.



(1) SSX-2SEGMENT-XX (2) SSX-2SEGMENT-XY (3)SSX-2SEGMENT-YY

(a) SSX-2SEGMENT layout



(1) SSY-2SEGMENT-XX (2) SSY-2SEGMENT-XY (3) SSY-2SEGMENT-YY

(b) SSY-2SEGMENT layout

Figure 4: The types of SS-2SEGMENT layout.

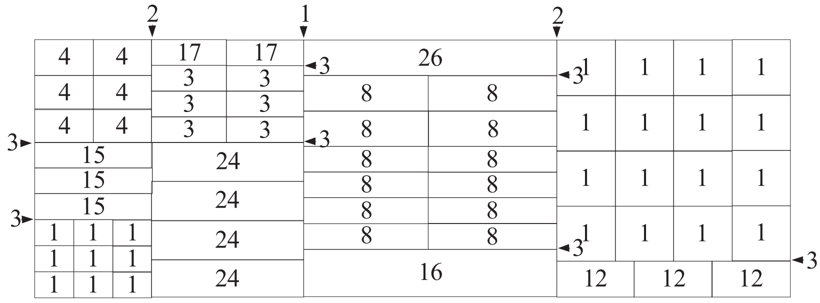
Figure 5 illustrates SSX-2SEGMENT layout and its cutting processes, the area 1 represents the borders of the segments, '2 denotes the borders of the section and 3 is the borders of the same-shape strip. In the SSX-2SEGMENT-XX (Figure 5a), the left X segment comprises two Y section, where, the first Y section includes three same-shape strips (piece 4, 15 and 1), and the second Y section also consists of three same-shape strips (piece 17, 3 and 24); the right X segment comprises two Y section, where, the first Y section includes three same-shape strips (piece 26, 8 and 16), the second Y section only consists of two same-shape strips (piece 1 and 12).

3. The Generating Algorithm for Optimal SS-2SEGMENT Layouts

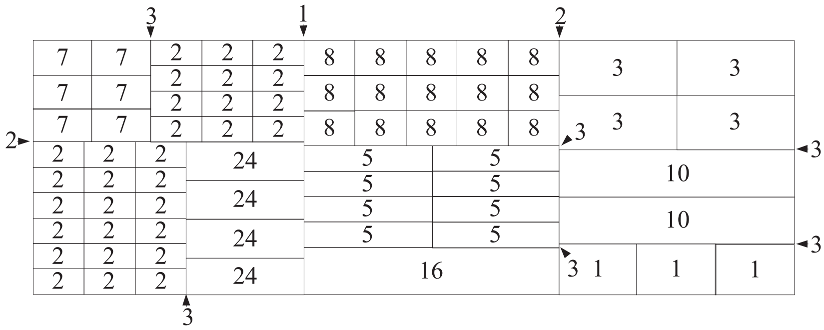
It is assumed that both the plate and pieces have integral sizes, and the pieces direction is fixed. The proposed approach for the SS-2SEGMENT layouts consists of four steps.

Step 1. Determining the normal sets.

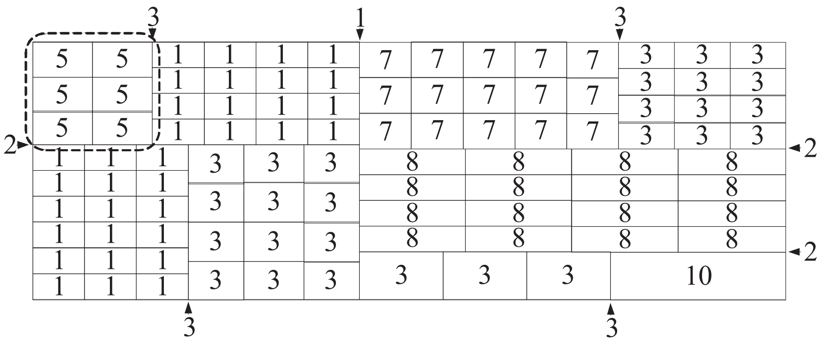
Step 2. Using dynamic programming recursion to determine the optimal same-shape strips.



(a) The SSX-2SEGMENT-XX layout



(b) The SSX-2SEGMENT-XY layout



(c) The SSX-2SEGMENT-YY layout

Figure 5: The SSX-2SEGMENT layout.

Step 3. Step3. Solving the knapsack problem of same-shape strips to obtain the optimal sections and segments.

Step 4. Solving the knapsack problem of sections to obtain the optimal SSX-2SEGMENT layout and the SSY-2SEGMENT layout.

Step 5. Determining the optimal SS-2SEGMENT layout.

3.1. Notations and functions

Some notations and functions are listed in Table 1. Most of them will be re-

Table 1: Notations and functions.

L, W	Length and width of the stock sheet
l_i, w_i, v_i	Length, width and area of the ith rectangular piece, $i = 1, 2, \ldots, m$
$P^1_{(i)}, P^2_{(i)}$	The set of normal same-shape strip length and width
$Q^1_{(i)}, Q^2_{(i)}$	The set of normal section length and width
P_3	The set of normal segments
$n^{(i)}(x, y)$	The maximum number of the ith pieces in same-shape strip $x \times y$
$u(x, y)$	The maximum area of same-shape strip $x \times y$
$f_1(x, y)$	The optimal area of the X section $x \times y$
$f_2(x, y)$	The optimal area of the Y section $x \times y$
$g_1(x, y)$	The area of X segment $x \times W$
$g_2(x, y)$	The area of Y segment $L \times y$
$v_{\text{SSX}-\text{2SEGMENT}-\text{XX}}$	The area of the optimal SSX-2SEGMENT-XX pattern
$v_{\text{SSX}-\text{2SEGMENT}-\text{XY}}$	The area of the optimal SSX-2SEGMENT-XY pattern
$v_{\text{SSX}-\text{2SEGMENT}-\text{YY}}$	The area of the optimal SSX-2SEGMENT-YY pattern
$v_{\text{SSY}-\text{2SEGMENT}-\text{XX}}$	The area of the optimal SSY-2SEGMENT-XX pattern
$v_{\text{SSY}-\text{2SEGMENT}-\text{XY}}$	The area of the optimal SSY-2SEGMENT-XY pattern
$v_{\text{SSY}-\text{2SEGMENT}-\text{YY}}$	The area of the optimal SSY-2SEGMENT-YY pattern
$v_{\text{SSX}-\text{2SEGMENT}}$	The area of the optimal SSX-2SEGMENT pattern
$v_{\text{SSY}-\text{2SEGMENT}}$	The area of the optimal SSY-2SEGMENT pattern
$v_{\text{SS}-\text{2SEGMENT}}$	The area of the optimal SS-2SEGMENT pattern

introduced where they are used for the first time. The readers can find it is more convenient to look for the notations definitions in the table than in the text.

3.2. Three main normal sets

Many authors have used normal sets to develop algorithms for layouts [2]-[4],[9], [11, 12]. This paper uses the normal sets as follows.

Definition 1. Normal sets of same-shape strips

According to 2.2 descriptions, the normal sets of same-shape strips is defined from each piece. For piece type i, $i = 1, \ldots, m$, suppose $P^1_{(i)}$ is the set of the same-shape strip normal length, and $P^2_{(i)}$ the set of the same-shape strip width

$$P^1_{(i)} = \{x = z_i l_i; z_i \in N; 0 \le x \le L\};$$
$$P^2_{(i)} = \{y = z_i w_i; z_i \in N; 0 \le y \le L\}.$$

The $P^1_{(i)} = p^1_1, p^1_2, \ldots, p^1_M$ represents the same-shape strip length normal size of piece, and M is the number of normal size; and the $P^2_{(i)} = p^2_1, p^2_2, \ldots, p^2_M$ represents ith the same-shape strip width normal size of ith piece, and N is the number of normal size.

Definition 2. Normal sets of sections

According to 2.3 descriptions, the normal section length is the set of each piece length, and the normal section width is the set of each piece width. Suppose $Q^1_{(i)}$ is the set of the section normal length and $Q^2_{(i)}$ the set of the section normal width

$$Q^1_{(i)} = \Big\{x = \sum_{i=1}^{m} z_i l_i; z_i \in N; i = 1, \ldots, m; 0 \le x \le L\Big\};$$
$$Q^2_{(i)} = \Big\{y = \sum_{i=1}^{m} z_i w_i; z_i \in N; i = 1, \ldots, m; 0 \le y \le W\Big\}.$$

The $q^1_1, q^1_2, \ldots, q^1_M$ represents the section length normal size, and M is the number of normal size; the $q^2_1, q^2_2, \ldots, q^2_M$ represents the section width normal size, and N is the number of normal size.

Definition 3. Normal sets of segments

According to 2.3 description, the segment consists of same direction sections, therefore, the normal segment length is the set of section normal length. Suppose P_3 is the set of segment normal length.

$$P_3 = Q^1_{(i)} = \Big\{x = \sum_{i=1}^{m} z_i l_i; z_i \in N; i = 1, \ldots, m; 0 \le x \le L\Big\}.$$

Application of the above normal sets can be greatly reduced the amount of same-shape strip, section and segment, which can greatly improve the efficiency of the algorithm.

3.3. The generating algorithm for optimal same-shape strip

(1) Solving the maximum amount of piece in the same-shape strip $x \times y$

Suppose that $n^{(i)}(x, y)$ is the maximum number of piece in the same-shape strip $x \times y$:

$$n^{(i)}(x, y) = \{\mathrm{int}\lfloor x/l_i \rfloor \times \mathrm{int}\lfloor y/w_i \rfloor; x \in P^1_{(i)}; y \in P^2_{(i)}\}.$$

Above formula means that two paths may lead to the layout on $x \times y$:

(i) As shown in Figure 6a, lay an X direction same-shape strip along the upper side of rectangle $x \times (y - l_i)$. The strip is of length $x \times y$ and includes $\mathrm{int}\lfloor x/w_i \rfloor$ pieces. The new rectangle $x \times y$ d contains $n^{(i)}(x, y - l_i) + \mathrm{int}\lfloor x/w_i \rfloor$ pieces.

(ii) As shown in Figure 6b, lay an Y direction same-shape strip along the upper side of rectangle $(x - l_i) \times y$. The strip is of length $x \times y$ and includes $\mathrm{int}\lfloor y/w_i \rfloor$ pieces. The new rectangle $x \times y$ d contains $n^{(i)}(x - l_i, y) + \mathrm{int}\lfloor y/w_i \rfloor$ pieces.

(2) Solving optimal the same-shape strip $x \times y$

Suppose that $u(x, y)$ is piece area in the same-shape strip $x \times y$:

$$u(x, y) = \{\max[n^{(i)}(x, y) \times v_i]; x \in P^1_{(i)}; y \in P^2_{(i)}\}.$$

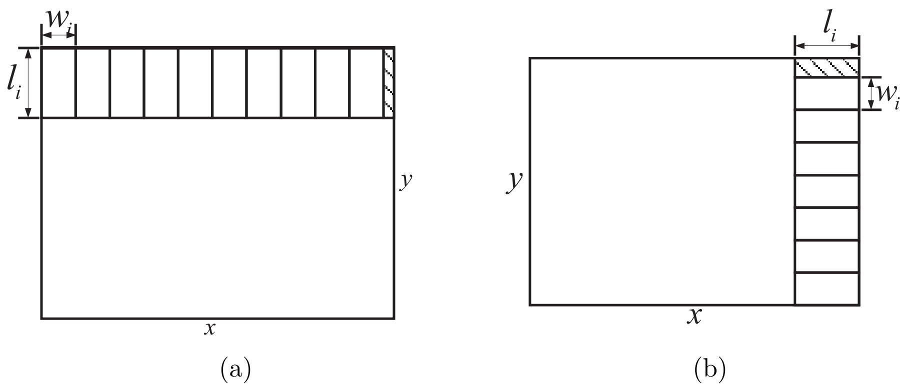


Figure 6: Two paths lead to rectangle $x \times y$. (a) From $x \times (y - l_i)$ and (b) From $(x - l_i) \times y$.

3.4. The optimal layout same-shape strip on section

(1) Solving the maximum area X section $x \times y$

Suppose that $f_1(x, y)$ is the maximum area X section $x \times y$:

$$f_1(x,y) = \Big\{ \max \Big[\sum_{i=1}^{M} k_i u(p_i^2, y) \Big]; \sum_{i=1}^{M} k_i p_i^2 \leq x; k_i \in N, x \in Q^1_{(i)}, y \in Q^2_{(i)} \Big\}. \tag{3.1}$$

(2) Solving the maximum area Y section $x \times y$

Suppose that $f_2(x, y)$ is the maximum area Y section $x \times y$:

$$f_2(x,y) = \Big\{ \max \Big[\sum_{i=1}^{N} k_i u(x, q_i^2) \Big]; \sum_{i=1}^{N} k_i q_i^2 \leq y; k_i \in N, x \in Q^1_{(i)}, y \in Q^2_{(i)} \Big\}. \tag{3.2}$$

Where, the k_i is the number of the same-shape strips.

The solution of above knapsack problem can refer to [13].

3.5. The optimal layout section on segment

(1) Solving the maximum area X segment $x \times y$

Suppose that $g_1(x, y)$ is the maximum area X segment $x \times y$:

$$g_1(x,y) = \Big\{ \max \Big[\sum_{i=1}^{N} k_i f_2(x, q_i^2) \Big]; \sum_{i=1}^{N} k_i q_i^2 \leq y; k_i \in N, x, y \in Q^1_{(i)} \Big\}. \tag{3.3}$$

(2) Solving the maximum area Y segment $x \times y$

Suppose that $g_2(x, y)$ is the maximum area X segment $x \times y$:

$$g_2(x,y) = \Big\{ \max \Big[\sum_{i=1}^{M} k_i f_1(p_i^2, y) \Big]; \sum_{i=1}^{M} k_i p_i^2 \leq x; k_i \in N, x, y \in Q^1_{(i)} \Big\}. \tag{3.4}$$

3.6. The optimal SSX-2SEGMENT layout

(1) Solving the maximum area SSX-2SEGMENT

For SSX-2SEGMENT layout, x is the vertical cut line, $x \in P_3$. There are three layout types, SSX-2SEGMENT-XX, SSX-2SEGMENT-XY and SSX-2SEGMENT-YY. Suppose that $v_{\mathrm{SSX-2SEGMENT-XX}}$, $v_{\mathrm{SSX-2SEGMENT-XY}}$ and $v_{\mathrm{SSX-2SEGMENT-YY}}$ are the maximum area of the above layouts.

$$v_{\mathrm{SSX-2SEGMENT-XX}} = \max[g_1(x, W) + g_1(L - x, W)] \tag{3.5}$$

$$v_{\mathrm{SSX-2SEGMENT-XY}} = \max[g_1(x, W) + g_2(L - x, W)] \tag{3.6}$$

$$v_{\mathrm{SSX-2SEGMENT-YY}} = \max[g_1(x, W) + g_2(L - x, W)] \tag{3.7}$$

Assemble $v_{\mathrm{SSX-2SEGMENT}}$ is the maximum area of the SSX-2SEGMENT layout.

$$v_{\mathrm{SSX-2SEGMENT}} = \max(v_{\mathrm{SSX-2SEGMENT-XX}}, v_{\mathrm{SSX-2SEGMENT-XY}}, v_{\mathrm{SSX-2SEGMENT-YY}}). \tag{3.8}$$

(2) Solving the maximum area SSY-2SEGMENT

For SSY-2SEGMENT layout, y is the horizontal cut line, $y \in P_3$. There are three layout types, SSY-2SEGMENT-XX, SSY-2SEGMENT-XY and SSY-2SEGMENT-YY. Suppose that $v_{\mathrm{SSY-2SEGMENT-XX}}$, $v_{\mathrm{SSY-2SEGMENT-XY}}$ and $v_{\mathrm{SSY-2SEGMENT-YY}}$ are the maximum area of the above layouts.

$$v_{\mathrm{SSY-2SEGMENT-XX}} = \max[g_1(L, y) + g_1(L, W - y)] \tag{3.9}$$

$$v_{\mathrm{SSY-2SEGMENT-XY}} = \max[g_1(L, y) + g_2(L, W - y)] \tag{3.10}$$

$$v_{\mathrm{SSY-2SEGMENT-YY}} = \max[g_2(L, y) + g_2(L, W - y)] \tag{3.11}$$

Assemble $v_{\mathrm{SSY-2SEGMENT}}$ is the maximum area of the SSY-2SEGMENT layout.

$$v_{\mathrm{SSY-2SEGMENT}} = \max(v_{\mathrm{SSY-2SEGMENT-XX}}, v_{\mathrm{SSY-2SEGMENT-XY}}, v_{\mathrm{SSY-2SEGMENT-YY}}). \tag{3.12}$$

(3) Solving the maximum area SS-2SEGMENT

Supposing $v_{\mathrm{SS-2SEGMENT}}$ is the maximum area of SS-2SEGMENT layout.

$$v_{\mathrm{SS-2SEGMENT}} = \max(v_{\mathrm{SSX-2SEGMENT}}, v_{\mathrm{SSY-2SEGMENT}}). \tag{3.13}$$

4. Computational Results

To the best of our knowledge, the SS-2SEGMENT layout has never been reported. In this section, the algorithm SS-2SEGMENTA is the algorithm for generating optimal SS-2SEGMENT layout, and SS-2SEGMENTA compares with five efficient algorithms through two groups benchmark problems. Suppose $V_{\mathrm{SS-2SEGMENTA}}$, V_{3STAGEA}, $V_{\mathrm{2SEGMENTA}}$, $V_{\mathrm{T-shapeA}}$, V_{TABU500A} and V_{GENERALA} is layout area of the above six algorithms respectively. The problems of the first two groups are available on the internet at [15]. The SS-2SEGMENTA is carried on a computer with Pentium 4 2.8GHz CPU and

Table 2: Layouts and algorithms abbreviation.

Layout Name	Layout abbreviation	Layout algorithms abbreviation
Same-shape strip two segment layout	SS-2SEGMENT	SS-2SEGMENTA
General three stage layout [10]	3STAGE	3STAGEA
General two segment layout [6]	2SEGMENT	2SEGMENTA
General T-shape layout [5]	T-shape	T-shapeA
TABU500layout [1]	TABU500	TABU500A
General layout [4]	GENERAL	GENERALA

512MB main memory. Various layouts and algorithms abbreviations are listed in Table 2.

(1) The first group problems

The first group includes 42 benchmark problems which are given in [10]. We compare SS-2SEGMENT with general three stage layout, general two segment layout, general T-shape layout, general two stage layout, TABU500 layout and general layout. Table 3 and Table 4 are the statistical results, and Table 5 is the calculated results. According to section 1, general layout is the optimal general layout, the marker "▲" denotes that the layout area is equal to that of the optimal general layout in Table 5. We can draw conclusions:

(a) The SS-2SEGMENTA results are equal or very close to those of the GENERALA.

(b) The layout area of SS-2SEGMENT are better than 3STAGE, 2SEGMENT and T-shape.

Therefore, compared to current layouts, the SS-2SEGMENT can improve material usage efficiently.

Table 3: Number of problems of different layouts in optimal results (42 problems).

Layout Type	SS-2SEGMENT	3STAGE	2SEGMENT	T-Shape
Number of optimal solutions (out of 42)	38	32	32	31

Table 4: Number of problems better or equal in different layouts (42 problems).

	3STAGE		2SEGMENT		T-Shape	
	better	equal	better	equal	better	equal
SS-2SEGMENT	8	34	9	33	10	32

Table 5: Computational results of different layout patterns (42 problems).

ID	V_{GENERALA}	$V_{\text{SS}-\text{2SEGMENTA}}$	V_{3STAGEA}	$V_{\text{2SEGMENTA}}$	$V_{\text{T}-\text{ShapeA}}$
H	12,348▲	▲	12,192	12,192	12,132
HZ1	5,226▲	▲	▲	▲	▲
M1	15,024▲	▲	▲	▲	▲
M2	73,176▲	▲	72,564	72,564	72,564
M3	142,817▲	▲	▲	▲	▲
M4	265,768▲	▲	▲	▲	▲
M5	577,882▲	▲	▲	▲	▲
B	8,997,780▲	▲	▲	▲	▲
U1	22,370,130▲	22,363,541	22,351,950	22,351,950	22,351,950
U3	48,142,840▲	48,095,058	48,095,058	48,042,264	48,029,748
UU1	242,919▲	▲	241,260	241,260	241,260
UU2	595,288▲	▲	▲	▲	▲
UU3	1,072,764▲	▲	▲	▲	▲
UU4	1,179,050▲	1,178,295	1,178,295	1,178,295	1,178,295
UU5	1,868,999▲	▲	1,868,985	1,868,985	1,868,985
UU6	2,950,760▲	▲	▲	▲	▲
UU7	2,930,654▲	▲	▲	▲	▲
UU8	3,959,352▲	▲	▲	▲	▲
UU9	6,100,692▲	▲	▲	▲	▲
UU10	11,955,852▲	▲	▲	▲	▲
UU11	13,157,811▲	13,147,305	13,146,050	13,141,175	13,127,726
HZ2	8,226▲	▲	▲	▲	▲
MW1	3,882▲	▲	▲	▲	▲
MW2	24,950▲	▲	▲	▲	▲
MW3	37,068▲	▲	▲	▲	▲
MW4	59,576▲	▲	▲	▲	▲
MW5	189,924▲	▲	▲	▲	▲
BW	2,307,817▲	▲	▲	▲	▲
W1	162,867▲	▲	▲	▲	161,424
W2	35,159▲	▲	34,656	34,656	34,656
W3	234,108▲	▲	▲	▲	▲
UW1	6,036▲	▲	▲	▲	▲
UW2	8,468▲	▲	▲	▲	▲
UW3	6,302▲	▲	6,226	6,226	6,226
UW4	8,326▲	▲	▲	▲	▲
UW5	7,780▲	▲	▲	▲	▲
UW6	6,615▲	▲	▲	▲	▲
UW7	10,464▲	▲	▲	▲	▲
UW8	7,692▲	▲	▲	▲	▲
UW9	7,038▲	▲	▲	▲	▲
UW10	7,507▲	▲	▲	▲	▲
UW11	15,747▲	▲	▲	▲	▲

From Table 4, we can see that:

There are 8 problems that the SS-2SEGMENT layout is better than 3STAGE, 9 problems for and 2SEGMENT and 10 problems for T-shape; the other problems are equal to above tree layouts.

The computation time of SS-2SEGMENTA is 0.399s, which is reasonable in practical application. Because there is no description of the time for above algorithms, so, we do not compare in time.

(2) The second group problems

The second group includes 20 benchmark problems which are given in [1]. We compare SS-2SEGMENT with TABU500 layout and general layout. Table 4 to Table 6 is the statistical results, and Table 4 to Table 7 is the calculatedresults. We can draw conclusions:

Table 6: Number of problems better than different algorithms (20 problems).

	TBAU500A	
	better	equal
S-2SEGMENTA	16	3

Table 7: Computational results of different layout patterns (20 problems).

ID	V_{GENERALA}	$V_{\text{ss}-\text{2SEGMENTA}}$	V_{TABU500A}
APT10	3,589,703	3,589,455	3,585,450
APT11	4,188,915	4,187,668	4,148,798
APT12	5,156,065	5,153,818	5,137,069
APT13	3,498,302	3,495,944	3,483,722
APT14	4,463,550	▲	▲
APT15	6,047,188	6,044,283	5,997,899
APT16	7,566,719	7,560,189	7,513,717
APT17	4,535,302	4,535,262	4,512,417
APT18	5,825,956	5,820,472	5,759,831
APT19	6,826,674	6,825,808	6,763,810
APT20	5,545,818	5,532,197	5,521,885
APT21	3,484,406	▲	▲
APT22	4,145,317	4,140,487	4,116,075
APT23	3,546,535	3,539,116	3,535,623
APT24	3,948,037	3,943,235	3,939,485
APT25	3,507,615	▲	3,500,380
APT26	2,683,689	2,664,507	2,656,729
APT27	2,438,174	▲	2,435,046
APT28	4,065,011	4,055,181	▲
APT29	3,652,858	▲	▲

(a) The SS-2SEGMENTA results are equal or very close to those of the GENERALA.

(b) Among 20 problems, there are 16 problems that the SS-2SEGMENT layout is better than TABU500, 3 problems are equal to TABU500, and 1 is worse than TABU500.

The computation time of SS-2SEGMENTA is 2.031s, which is reasonable in practical application. Because there is no description of the time for TABU500, so, we do not compare in time.

5. Conclusions

Despite exact algorithm for large-scale UTDGC problems exists, but its computational complexity is unknown. According to practical results show that this algorithm in solving large-scale problems cannot be tolerated in calculation time. Usually researchers use specific layouts, which can solve the problem effectively within reasonable time. We propose specific layout SS-2SEGMENT, and we compare the SS-2SEGMENT with the general three-stage, two segment, T-shape and TABU500 layout, and the computation results show that the SS-2SEGMENTA can effectively improve sheet utilization and calculation time is reasonable in practical application.

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