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An Adjusted Robust Optimization Method to an Integrated Production-Distribution Planning Problem in Closed-Loop Supply Chains under Uncertainty

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Abstract

In the last decade, planning of closed-loop supply chains in different strategic, tactical, and operational levels has attracted many interests due to economic reasons, environmental challenges, and government legislations. This paper presents a novel linear programming model for the integrated production and distribution planning in closed-loop supply chains under uncertainty. The proposed model involves multi-product and multi-period which considers multiple transportation modes, direct or indirect shipments, advertising costs, and several customer zones for different types of products and also attempts to integrate production and distribution plans in the forward and reverse sides of the closed-loop supply chain, simultaneously. To deal with uncertain input data, a robust optimization counterpart based on polyhedral uncertainty set is developed to obtain optimal solutions immunizing the problem for any realization of uncertain parameters in the given polyhedral uncertainty set. Computation results for a numerical example under different scenarios are discussed to give insights about the features of the proposed robust optimization model in handling the uncertainty of parameters. Finally, some sensitivity analyses are performed to show the behaviour of the robust and deterministic models respect to changes of uncertainty levels of parameters as well as the amounts of important parameters such as demands and returns.

Keywords: Integrated production and distribution planning, closed-loop supply chain, robust optimization, polyhedral uncertainty set.

1. Introduction

Closed-loop supply chain (CLSC) planning for the long-term and mid-term planning horizon is one of the prominent problems which has attracted many interests by the academic researchers and practitioners. CLSC is the network of organizations, people, activities, information and resources involved in providing new goods from suppliers to the customers and collecting of used products from final customers and remanufacturing, recovering or disposal them in a suitable way [4, 5, 25]. Many companies such as Dell, HP, Kodak, Canon, and Xerox have achieved many economic advantages through collecting and recovering the used products.

The increasing interest in evaluating the performance of supply chain networks over the past years indicates the need for the development of complex optimization models able to answer unsolved questions in the context of production and distribution planning [9, 11, 29]. On the other hand, recovering and redistribution of used products has attracted many attentions by the researchers and practitioners in the last decade due to environmental challenges, governmental limitations and economic reasons [3, 19]. However, many researchers [3, 25] have stated that planning for the forward and reverse supply chain, separately, leads to sub-optimality in the planning of supply chain. Consequently, the main aim of an integrated production and distribution planning model in a closed loop supply chain would be determining the amount of products produced in the plants, the amount of products recovered in recovery centres, the amount of flows between different entities existing in the different echelons of the supply chain, the amount of inventories to be stored in distribution centres, the amount of collected products, and the amount of recoverable and non-recoverable products. Notably, other forms of planning such as master production schedule, capacity requirements planning and material requirements planning follow the outcomes of aggregate production and distribution planning and are determined in a hierarchical way [10, 22] according to supply chain planning matrix [27].

Another important issue which should be addressed in a mid-term planning horizon is handling the uncertainty of parameters emerged from turbulent and competitive environments. Typically, there are three approaches to deal with the uncertainty of such problems namely stochastic programming, fuzzy/ possibilistic programming, and robust optimization methods [25]. Stochastic programming approach is applied when the probability distributions of uncertain parameters are known via sufficient and reliable historical data [1]. Possibilistic programming can be used when the uncertain parameters are expressed based upon insufficient available historical data and subjective opinions of decision makers (DMs). The robust optimization method is applied when historical data about the uncertain parameters and also the infeasibility of the problem cannot be tolerated [2]. In this method, uncertainty of parameters is assumed to be varied within the given set and the robust counterpart optimization model seeks for the solutions immunizing the problem for any potential realizations of uncertain parameters.

In this paper, we present a novel model for multiple products, multiple periods in closed-loop supply chains (IPDPCLSC) consisting of multiple production sites and transportation modes which integrates production and distribution plans in the forward and reverse sides of the closed-loop supply chain, simultaneously. The developed model accounts for the uncertainty of parameters where there is no sufficient historical data. The paper has two major applied and theoretical contributions that differentiate it from those existing in the literature. First, it presents an optimization model for the integrated production and distribution planning in closed-loop supply chains that takes many real-world assumptions into account such as direct or indirect shipments, several customer zones for new, recovered, and non-recovered products, service level of customers, and multiple transportation modes. Second, it introduces utilization of the robust optimization method based on polyhedral uncertainty set to deal with the deep uncertainty of parameters where we just know how they change within a given polyhedral set. Although, some researchers have already applied the robust optimization approach for the worst-case conditions using box uncertainty set in closed-loop supply chain network design problem [8, 25, 32], nevertheless, to the best of our knowledge, there is no research paper applying the robust optimization approach with polyhedral uncertainty set for integrated production and distribution planning in the context of closed-loop supply chains.

The remainder of this paper is organized as follows. The relevant literature is reported in the next Section. In Section 3, we define our notations, describe our assumptions and develop a new linear programming model for the IPDPCLSC. The concepts of the applied robust optimization method for the polyhedral uncertainty set are discussed in Section 4. Computational results with some conducted sensitivity analyses are reported in Section 5. Section 6 states the conclusion and managerial implications of this paper and open some future research directions.

2. Literature Review

Nam and Logendran [21] provided a comprehensive review on aggregate production planning (APP) to investigate the advantages and deficiencies of the proposed models and opened channels for future researches. Mula [18] presented a general review covering different aspects of production planning such as supply chain structure, decision level, modelling approach, purposes, shared information, limitations, novelty and applications in the context of supply chain management. This review mainly focuses on tactical level of decision making in production and distribution echelons including mathematical programming and centralized planning models rather than strategic or operational level of decision making in this context. They argue that the literature suffers from lack of models considering integration of decisions pertaining to transportation modes and recycling/recovering operations with the production and distribution planning in forward supply chain management. Another comprehensive review in this field that covers the strategic, tactical, and operational levels of decision focusing on strategic level is the work of Souza [26]. Govindan et al. [38] presented a comprehensive review to explore the literature gaps in reverse logistics and closed-loop supply chains and suggested some efficient future research directions.

At the following, we review the most relevant works that cover many different aspects of complex modelling and also different approaches used to handle the uncertainty of the parameters. It should be noted that we do not care about solution methods in this review. Therefore, we focus on modelling and the approaches applied to deal with the uncertain parameters.

Torabi and Hassini [30] considered a multi-objective supply chain master planning in fuzzy environments for integrating different aspects in a multi-product and multi-period supply chain. Also, the proposed model covers quantity and time-dependent discount policy for different product in different periods. As an extension to this work, Torabi and Hassini [31] presented a multi-objective supply chain master planning model to integrate procurement and distribution plans with production planning in a multi-site manufacturing setting with a case study in an automaker. Leung et al. [15] presented a scenario-based robust optimization approach, which is a branch of scenario-based stochastic programming, to face with the uncertainty of multi-site production planning problem. Ghasemy Yaghin et al. [7] presented a new approach by introducing pricing (markdown policy) concepts in modelling of APP problems. Although this approach imposes non-linearity to the proposed model, the model would be so compatible with real life assumptions. In their model, demand of products is a function of advertising cost and price in any period. The model maximizes the total profit of manufacturers in the first objective, total profit of retailers in the second objective and qualitative-oriented aspects for retailing in the third one. Jamalnia and Soukhakian [10] presented a fuzzy multi-objective nonlinear model for the APP with imprecise parameters including both quantitative goals (production costs, carrying and back ordering costs and costs of changes in workforce level) as well as a qualitative goal in terms of customer satisfaction regarding the company's commitments about delivery times and quality of products. They also considered the learning curves effects for workers in production planning. Wang and Liang [33] developed a multi-objective APP model to reduce production costs, carrying and backordering costs, and rate of change in labour levels. They accounted for fuzziness in constraints and used fuzzy goal programming technique to deal with the multiple objectives. They also demonstrated the importance of considering the time value of money in production planning models by conducting a sensitivity analysis on the escalating factor. Liu and Papageorgiou [16] presented a multi-objective model for concurrent production, distribution and capacity planning in global supply chains. The proposed model seeks efficient solutions for satisfying the three objectives including minimizing costs, transportation time, and lost sales as well as optimal capacity expansion planning during the planning horizon. Torabi and Moghaddam [29] developed an integrated multi-site production-distribution model under fuzziness of input data while accounting for possibility of having lateral transshipment of products between manufacturing plants. Fahimnia et al. [6] presented multiple products, multiple periods aggregate production and distribution planning model which considers most of real-life assumptions such as possibility of having multiple routes (i.e., direct and/or indirect shipments), multiple plants, multiple distribution centres, outsourcing and inventory holding. Table 1 shows the details of some relevant works.

The review of literature shows there are limited research works for tactical (mid-term) planning in reverse and closed-loop supply chains. Kim et al. [12] proposed a MILP model, under deterministic conditions, for the reverse logistics network planning that trades-off between two alternatives including acquiring items from suppliers or supplying items from those provided from disassembling of used products for producing new products. Meanwhile, they avoid integrating decisions related to forward and reverse logistics, simultaneously. Shi et al. [28] studied the production planning optimality for a closed-loop supply chain under uncertainty of demand and returns, which is price-sensitive. They assumed that the uncertain parameters have stochastic behaviour based on normal distribution with known mean and standard deviation and then proposed a solution method based on Lagrangian relaxation method. Kenne et al. [13] developed a production planning model for the forward-reverse logistics under machine failures that uses dynamic stochastic programming method to solve the proposed continuous model.

Pereira Ramos et al. [35] developed a multi-objective model with aim of optimizing economic, environmental and social objectives in a reverse logistics systems. Their proposed model supports tactical and operational level decision making in a reverse logistics system. Niknejad and Petrovic [36] proposed a MILP model to optimize inventory control and production planning decisions in an integrated forward-reverse supply chain network under demand and returns uncertainty. They used fuzzy mathematical programming method to deal with the uncertainty of the problem. Chuang et al. [37] studied closed-loop supply chain models for a high-tech product. They investigated three alternatives for collecting the used product from customers including: (1) collecting the used products by the manufacturer; (2) collecting the used products by the retailer for the manufacturer; and (3) collecting the used products by the third-party firm for the manufacturer.

Niknamfar et al. [39] proposed a stochastic robust optimization method to an aggregate production-distribution planning in a three echelon supply chain. They considered regular time, overtime, outsourcing, hiring, firing, inventory holding, backordering, and machine capacity in modelling the problem. Cheng et al. [40] presented an improved ant colony optimization method to optimize scheduling activities in a production-distribution planning problem with the aim of minimizing total costs. They considered third-party logistic (3PL) provider for distribution of products in supply chain. The proposed heuristic algorithm could solve the proposed large size problem in reasonable time. Ma et al. [41] used a bi-level programming approach to optimize integrated production-distribution planning in a supply chain. Their model considers conflict and coordination in supply chain management. They also proposed a two stage genetic algorithm with a fuzzy logic controller algorithm to solve the problem for real sizes. Camacho-Vallejo et al. [42] extended a heuristic algorithm based on scatter search that considers the Stackelberg's equilibrium to solve a production-distribution planning problem with the aim of optimizing operation and transportation costs.
Notably, some models have tried to optimize strategic decisions of the CLSC, such as determining the number and location of facilities (e.g., [5, 23]). Although in such studies, the optimal decisions about the amount of aggregated amounts of production, inventory and distribution are taken, the main aim is determination of strategic decisions such as facility location, technology selection and capacity sizing. Indeed, the output of these models about the amount of production, inventory and distribution could not be implemented for various periods with their specific features in a mid-term planning horizon.

3. Mathematical Formulation

The following main assumptions are made in developing the proposed IPDPCLSC model:

Table 1: Review of some existing models.

Reference	structure	period	Marketing aspects	Backorder	Transportation mode	Delivery time	Approach
[31]	Forward	Multiple	-	-	-	Considered	Fuzzy
[15]	Forward	Multiple	-	Considered	-	-	SRo.
[7]	Forward	Multiple	Considered	-	-	negligible	Fuzzy
[10]	Forward	Multiple	-	Considered	-	Negligible	Deter.
[33]	Forward	Multiple	-	Considered	-	-	Fuzzy
[16]	Forward	Multiple	-	Considered	-	Considered	Deter.
[29]	Forward	Multiple	-	Considered	-	Considered	fuzzy
[12]	Reverse	Multiple	-	-	-	-	Deter.
[28]	Closed-loop	Single	-	Considered	-	-	Sto.
Our work	Closed-loop	Multiple	Considered	Considered	Considered	Considered	Ro.

Deter. (Deterministic), Sto. (Stochastic), Ro. (Robust), SRo (Robust stochastic).

- Customers are divided in to three groups: new products' customers, recovered products' customers, and materials' customers (customers of non-recoverable products).
- The main parameters directly affected by customer's behaviors including demands of new and recovered products, amount of returns, quality of returned products, selling price of different products, purchasing costs, shortage costs are tainted with uncertainty.
- Due to lack of sufficient historical data, there is no known probability distribution to show the behavior of uncertain parameters in the future. Therefore, they are handled via uncertainty sets in the form of a given interval for each parameter.
- Different transportation modes are utilized to deliver products to customer groups; however, collecting of used products is performed through only one transportation mode.
- Advertising costs are spent in both customer zones and the maximum budget for such costs is limited.
- All demands of new products must be fulfilled. However, shortage is permitted for recovered products but the back-orders of recovered products should be fulfilled in the next period.
- All used products are collected and purchased with reasonable price.
- Sub-contracting is not allowed.
- All facilities in different echelons have limited capacities.
- Manufacturing new products and recovering used products are carried out at the hybrid manufacturing/recovery (HMR) centres.

- Distribution of new products and redistribution of recovered products are performed at the hybrid distribution/redistribution (HDR) centers.
- Besides traditional shipment of both new and recovered products, direct shipment of new products from HMR centers to customer zones is permitted.
- Safety stock is hold only for new products at HDR centers.

3.1. Problem description and notations

The concerned aggregate production-distribution planning model for closed-loop supply chains (IPDPCLSC) is of multi-site, multi-echelon, multi-period, and multi-product network type. Figure 1 illustrates the structure of the problem. The dashed lines show the flow of used and recovered products, while the continuous lines show the flow of new products between facilities. Other notations noted on the Figure 1 have been defined in notations. A practical situation of such problem can be found in several industries such as printers and copiers production, namely Xerox and Kodak companies, or digital cameras production, namely Canon Company. As depicted in Figure 1, new products are produced and also recovered products are recovered in HMR centres and shipped to HDR sites. As it was mentioned by Pishvaee et al. [24], hybrid processing facilities offer potential cost savings compared with separate distribution or collection centres in a closed-loop supply chain network.

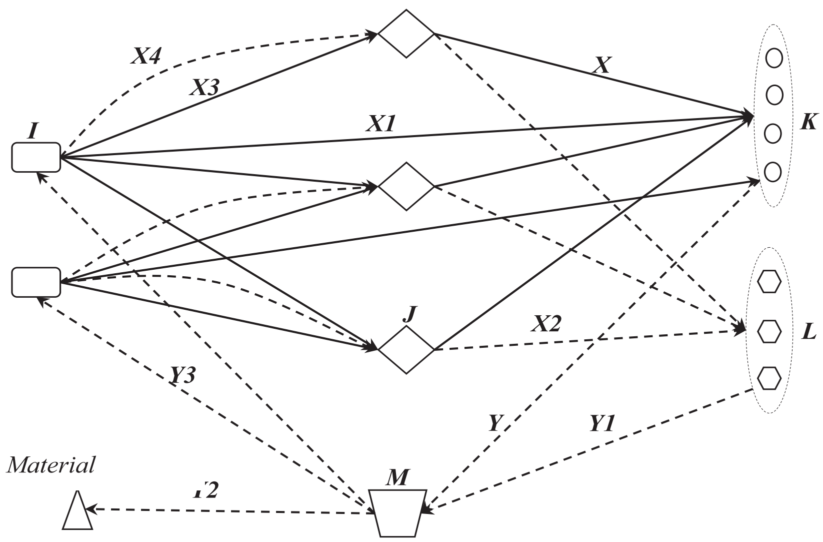

Figure 1: The concerned closed-loop supply chain structure.

In addition, new products could be directly shipped from HMR centres to customer zones to fulfil new products' demands before reaching due dates. At HDR centres, some products are stored and the rest is shipped to customer zones through different transportation modes. Also, some new products are hold as safety stock to face with unscheduled changes in customer needs.

It is worthy to note that new products' customers have higher priority respect to recovered products' customers when assigning resources to customers. Therefore, direct shipment of products and holding safety stock options are presented only for new products. New products are delivered to customers in a pull manner, while demands of recovered products are fulfilled in a push way where their raw material are provided from collected used products and thus are limited. Therefore, some demands of recovered products may not be fulfilled. Used products are purchased from customer zones and shipped to collection centres. After testing and evaluating the quality of used products in collection centres, they are classified into recoverable and non-recoverable products. The recoverable products are shipped to HMR centres and the non-recoverable products are sold to material customers.

In the proposed IPDPCLSC model, delivery time of products to customers might be violated which may lead to deviations from predetermined customer service levels based on senior management's preferences. For example, customers with targeted service level 100

The following notations are used to formulate the problem mathematically.

Indices

- i Index of hybrid manufacturing/recovery centres ($i = 1, \dots, I$).
- j Index of hybrid distribution/redistribution centres ($j = 1, \dots, J$).
- k Index of first markets' customer zones ($newproducts$) ($k = 1, \dots, K$).
- l Index of second markets' customer zones ($recoveredproducts$) ($l = 1, \dots, L$).
- m Index of collection centres ($m = 1, \dots, M$).
- n Index of transportation modes ($n = 1, \dots, N$).
- p Index of products ($p = 1, \dots, P$).
- t Index of time periods ($t = 1, \dots, T$).

Parameters

- $D1_{kpt}$ Demand of customer k for new product p in period t.
- $D2_{lpt}$ Demand of customer l for recovered product p in period t.
- $Re1_{kpt}$ Amount of returns of product p from customer k in period t.
- $Re2_{lpt}$ Amount of returns of product p from customer l in period t.
- $\beta1_p$ Recoverable percentage of product p collected from the first markets' customers in period t.
- $\beta2_p$ Recoverable percentage of product p collected from the second markets' customers in period t.

$SS1_{jpt}$ Safety stock level of new product p at distribution/redistribution centre j in period t.

Pr_{jknpt} Unit Selling price of new product p shipped from distribution/redistribution centre j to customer k by transportation mode n in period t.

$Pr1_{iknpt}$ Unit Selling price of new product p shipped from manufacturing/recovery centre i to customer k by transportation mode n in period t.

$Pr2_{jlnpt}$ Unit Selling price of recovered product p shipped from distribution/redistribution centre j to customer l by transportation mode n in period t.

$Pr3_{pt}$ Unit Selling price of non-recoverable product p sold to material customers in period t.

$Pur1_{kpt}$ Unit Purchasing cost of used product p from customer k in period t.

$Pur2_{lpt}$ Unit Purchasing cost of used product p from customer l in period t.

Pc_{ipt} Manufacturing cost of new product p at manufacturing/recovery centre i in period t.

Rc_{ipt} Unit Recovery cost of used product p at manufacturing/recovery centre i in period t.

Hpc_{mpt} Unit Processing and quality test costs of used product p at collection centre m in period t.

$Hic1_{jpt}$ Unit holding cost of new product p at distribution/redistribution centre j in period t.

$Hic2_{jpt}$ Unit holding cost of recovered product p at distribution/redistribution centre j in period t.

sc_{lpt} Unit shortage cost of recovered product p for customer l in period t.

$Ha1_{kpt}$ Unit advertising cost of product p at customer zone k in period t.

$Ha2_{lpt}$ Unit advertising cost of product p at customer zone l in period t.

r Interest rate.

BC_t Maximum budget assigned for advertising and marketing activities in period t.

Tc_{ijnpt} Unit transportation cost of new product p shipped from manufacturing/recovery centre i to distribution/redistribution centre j by transportation mode n in period t.

$Tc1_{iknpt}$ Unit transportation cost of new product p shipped from manufacturing/recovery centre i to customer k by transportation mode n in period t.

$Tc2_{jknpt}$ Unit transportation cost of new product p shipped from distribution/redistribution centre j to customer k by transportation mode n in period t.

$Tc3_{jlnpt}$ Unit transportation cost of recovered product p shipped from distribution/redistribution centre j to customer l by transportation mode n in period t.

$Tc4_{kmpt}$ Unit transportation cost of used product p shipped from customer k to collection centre m in period t.

$Tc5_{lmpt}$ Unit transportation cost of used product p shipped from customer l to collection centre m in period t.

$Tc6_{mipt}$ Unit transportation cost of used product p shipped from collection centre m to manufacturing/recovery centre i in period t.

Td_{ikn} Delivery time from manufacturing/recovery centre i to customer k by transportation mode n.

Te_{kp} Expected delivery time of customer k for new product p in any period.

$Td1_{jkn}$ Delivery time from distribution/redistribution centre j to customer k by transportation mode n (in days).

$Td2_{jln}$ Delivery time from distribution/redistribution centre j to customer l by transportation mode n (in days).

$Te1_{lp}$ Expected delivery time of customer l for recovered product p in any period (in days).

$Sl1_k$ Average predetermined service level for customer k (the percentage of on-time deliveries).

$Sl2_l$ Average predetermined service level for customer l (the percentage of on-time deliveries).

b_p Required storage capacity per unit of product p (volume).

$b1_p$ Required production capacity per unit of product p (machine-hour/unit).

$b2_p$ Required recovery capacity per unit of product p (machine-hour/unit).

$b3_p$ Required handling capacity per unit of product p at collection centres (machine-hour/unit).

$Ca1_i$ Maximum capacity of hybrid manufacturing/recovery centre i.

$Ca2_j$ Maximum capacity of hybrid distribution/redistribution centre j.

$Ca3_m$ Maximum capacity of collection centre m.

Variables

x_{jknpt} Quantity of new product p shipped from distribution/redistribution j to customer k by transportation mode n in period t.

$x1_{iknpt}$ Quantity of new product p shipped from manufacturing/recovery centre i to customer k by transportation mode n in period t.

$x2_{jlnpt}$ Quantity of recovered product p shipped from distribution/redistribution centre j to customer l by transportation mode n in period t.

$x3_{ijnpt}$ Quantity of new product p shipped from manufacturing/recovery centre i to distribution/redistribution centre j by transportation mode n in period t.

$x4_{ijnpt}$ Quantity of recovered product p shipped from manufacturing/recovery centre i to distribution/redistribution centre j by transportation mode n in period t.

xe_{ipt} Quantity of new product p manufactured at manufacturing/recovery centre i in period t.

y_{kmpt} Quantity of returned product p shipped from customer k to collection centre m in period t.

$y1_{lmpt}$ Quantity of returned product p shipped from customer l to collection centre m in period t.

$y2_{pt}$ Quantity of scraped product p sold to material customers in period t.

$y3_{mipt}$ Quantity of recoverable product p shipped from collection centre m to manufacturing/recovery centre i in period t.

$IC1_{jpt}$ Inventory level of product p at distribution/redistribution centre j in period t.

$IC2_{jpt}$ Inventory level of recovered product p at distribution/redistribution centre j in period t.

λ_{lpt} Backorder quantity of recovered product p for customer l in period t.

Rp_{pt} Quantity of recoverable product p in period t.

3.2. Problem formulation

Objective function The proposed IPDPCLSC aims to maximize the net present value of total profit (that is, total profit = total revenues − total costs). The total revenues are resulted from products sold in different customer zones including customers of new products, customers of recovered products, and customers of non-recoverable products and thus can be formulated as follows:

$$\sum_{j}\sum_{k}\sum_{n}\sum_{p}\sum_{t}\overline{Pr}_{jknpt}x_{jknpt}+\sum_{i}\sum_{k}\sum_{n}\sum_{p}\sum_{t}\overline{Pr1}_{iknpt}x2_{iknpt}$$
$$+\sum_{j}\sum_{l}\sum_{p}\sum_{n}\sum_{t}\overline{Pr2}_{jlnpt}x2_{jlnpt}+\sum_{p}\sum_{t}\overline{Pr3}_{pt}x_{pt}$$

Hereafter, a bar sign is used to show each uncertain parameter (for example, selling prices of products are uncertain parameters in the total revenue function). Note that new products could be directly shipped from the HMR sites to customers or shipped through HDR centres. Although the costs incurred by direct shipments are higher than traditional indirect shipments, a particular case of interest is that the selling price in both types of shipments to be equal. In fact, direct shipment strategy is used to fulfil customer expectations within their maximal allowable times and thus customers should not charge more costs due to direct shipments utilized by the companies in supply chain. On the other hand, utilizing direct shipment strategy via different transportation modes boosts customer's beliefs about the delivery times obligated by the supply chain members.

The total costs which is a common efficiency criterion to optimize decisions made to use different resources in supply chain planning models efficiently [38] include transportation costs, production and recovering costs, quality testing costs of collected products, inventory holding costs, purchasing the used products, advertisement costs, and shortage costs.

In this regard, the transportation costs encompass shipping costs between different echelons of the closed-loop supply chain in both forward and reverse sides via various transportation modes. The total transportation costs (which are deterministic) can be formulated as follows:

$$\sum_i\sum_j\sum_n\sum_p\sum_t Tc_{ijnpt}x3_{ijnpt} + \sum_i\sum_k\sum_n\sum_p\sum_t Tc1_{iknpt}x1_{iknpt}$$
$$+\sum_j\sum_k\sum_n\sum_p\sum_t Tc2_{jknpt}x_{jknpt} + \sum_j\sum_l\sum_n\sum_p\sum_t Tc3_{jlnpt}x2_{jlnpt}$$
$$+\sum_k\sum_m\sum_p\sum_t Tc4_{kmpt}y_{kmpt} + \sum_l\sum_m\sum_p\sum_t Tc5_{lmpt}y1_{lmpt}$$
$$+\sum_m\sum_i\sum_p\sum_t Tc6_{mipt}y3_{mipt} + \sum_i\sum_j\sum_n\sum_p\sum_t Tc_{ijnpt}x4_{ijnpt}$$

The other types of costs could be written as follows:
Production and recovering costs in HMR centres:

$$\sum_i\sum_p\sum_t Pc_{ipt}xe_{ipt} + \sum_i\sum_p\sum_t Rc_{ipt}\Big(\sum_m y3_{mipt}\Big).$$

Handling costs in collection centres including the testing and evaluating of used products:

$$\sum_m\sum_p\sum_t Hpc_{mpt}\Big(\sum_k y_{kmpt} + \sum_l yl_{lmpt}\Big).$$

Inventory holding costs in HDR centres:

$$\sum_m\sum_i\sum_p\sum_t Tc6_{mipt}y3_{mipt} + \sum_i\sum_j\sum_n\sum_p\sum_t Tc_{ijnpt}x4_{ijnpt}.$$

Purchasing costs of used products from customers (which are uncertain):

$$\sum_k\sum_p\sum_t \overline{Pur1}_{kpt}\Big(\sum_m y_{kmpt}\Big) + \sum_l\sum_p\sum_t \overline{Pur2}_{lpt}\Big(\sum_m y1_{lmpt}\Big).$$

Advertisement costs in different customer zones for new and recovered products:

$$\sum_k\sum_p\sum_t Ha1_{kpt}\Big(\sum_j\sum_n x_{jknpt}+\sum_i\sum_n x1_{iknpt}\Big)+\sum_l\sum_p\sum_t Ha2_{kpt}\Big(\sum_j\sum_n x2_{jlnpt}\Big).$$

Finally, shortage costs related to recovered products (which are uncertain):

$$\sum_l\sum_p\sum_t \overline{sc}_{lpt}\lambda_{lpt}.$$

It is worthy to note that to calculate the net present values of the total revenues and total costs, the coefficient $\frac{1}{(1+r)^t}$ in which r denotes the interest rate, should be multiplied by the above-mentioned equations. Consequently, we would have the following objective function which maximizes the total profit of the proposed IPDPCLSC model:

$$
\text{Max } Z = \sum_t \frac{1}{(1+r)^t} \left(\begin{array}{l}
\Big[\sum_j \sum_k \sum_n \sum_p \overline{Pr}_{jknpt} x_{jknpt} + \sum_i \sum_k \sum_n \sum_p \overline{Pr1}_{iknpt} x1_{iknpt} \\
+ \sum_j \sum_l \sum_p \sum_n \overline{Pr2}_{jlnpt} x2_{jlnpt} + \sum_p \overline{Pr3}_{pt} y2_{pt} \Big] \\
- \Big[\sum_i \sum_j \sum_n \sum_p Tc_{ijnpt} x3_{ijnpt} + \sum_i \sum_k \sum_n \sum_p Tc1_{iknpt} x1_{iknpt} \\
+ \sum_j \sum_k \sum_n \sum_p Tc2_{jknpt} x_{jknpt} + \sum_j \sum_k \sum_n \sum_p Tc3_{jknpt} x3_{jknpt} \\
+ \sum_k \sum_m \sum_p Tc4_{kmpt} y_{kmpt} + \sum_l \sum_m \sum_p Tc5_{lmpt} y1_{lmpt} \\
+ \sum_m \sum_i \sum_p Tc6_{mipt} y3_{mipt} + \sum_i \sum_j \sum_n \sum_p Tc_{ijnpt} x4_{ijnpt} \\
+ \sum_i \sum_p Pc_{ipt} xe_{ipt} + \sum_i \sum_p Rc_{ipt} \Big(\sum_m y3_{mipt} \Big) \\
+ \sum_m \sum_p Hpc_{mpt} \Big(\sum_k y_{kmpt} + \sum_l y1_{lmpt} \Big) \\
+ \sum_j \sum_p HIc1_{jpt} Ic1_{jpt} + \sum_l \sum_p HIc2_{jpt} Ic2_{jpt} \\
+ \sum_k \sum_p \overline{Pur1}_{kpt} \Big(\sum_m y_{kmpt} \Big) + \sum_l \sum_p \overline{Pur2}_{lpt} \Big(\sum_m y1_{lmpt} \Big) \\
+ \sum_k \sum_p Ha1_{kpt} \Big(\sum_j \sum_n x_{jknpt} + \sum_i \sum_n x1_{iknpt} \Big) \\
+ \sum_l \sum_p Ha2_{lpt} \Big(\sum_j \sum_n x2_{jlnpt} \Big) + \sum_l \sum_p \overline{sc}_{lpt} \lambda_{lpt} \Big]
\end{array} \right) \tag{3.1}
$$

Model constraints

Inventory balance constraints in the forward side: The following constraints express the inventory-related and demand satisfaction constrains in the HMR and HDR centres in the forward side.

$$\sum_j \sum_n x_{jknpt} + \sum_i \sum_k x1_{iknpt} = \overline{D1}_{kpt}, \qquad \forall\ k, p, t \tag{3.2}$$

$$Ic1_{jp,t-1} + \sum_i \sum_n x3_{ijnpt} - Ic1_{jpt} = \sum_k \sum_n x_{jknpt}, \qquad \forall\ j, p, t \tag{3.3}$$

$$Ic1_{jpt} \geq SS1_{jpi}, \qquad \forall\ j, p, t \tag{3.4}$$

$$xe_{ipt} = \sum_j \sum_n x3_{ijnpt} + \sum_k \sum_n x1_{iknpt}, \qquad \forall\ i, p, t \tag{3.5}$$

$$\sum_j \sum_n x2_{jlnpt} + \lambda_{lpt} - \lambda_{ip,t-1} = \overline{D2}_{lpt}, \qquad \forall\ l, p, t \tag{3.6}$$

$$ic2_{jp,t-1} + \sum_i \sum_n x4_{ijnpt} - Ic2_{jpt} = \sum_l \sum_n x2_{jlnpt}, \qquad \forall\ j, p, t \tag{3.7}$$

Constraints (3.2) ensure that all demands of customers for new products are satisfied. Constraints (3.3) and (3.4) are inventory balancing equations and safety stock levels at HDR centres. It is worthy to note that determination of the safety stock levels could be performed via the forward inventory coverage concept [14]. That is, the safety stock levels in current period are calculated according to the demands of customers in the next period as follows: $SS1_{jpt} = \alpha_p D1_{kp,t+1}$, where α_p and $D1_{kp,t+1}$ indicate the forward inventory coverage factor of new product p and the most possible value of demands for new products at the next period, respectively. Obviously, the safety stock levels for the last period T is achieved based on the first period demands for the new product. Constraint (3.5) show the total new products produced at HMR centres in any period. Constraints (3.6) and (3.7) are the demand constraint, being satisfied or being left as back-orders, and inventory balancing equation for the recovered products at HDR centres.

Inventory balance constraints in the reverse side:

$$\sum_m y_{kmpt} = \overline{Re1}_{kpt}, \qquad \forall\, k, p, t \tag{3.8}$$

$$\sum_m y1_{lmpt} = \overline{Re2}_{lpt}, \qquad \forall\, l, p, t \tag{3.9}$$

$$Rp_{pt} = \overline{\beta 1}_p \sum_k \sum_m y_{kmpt} + \overline{\beta 2}_p \sum_l \sum_m y1_{lmpt}, \qquad \forall\, p, t \tag{3.10}$$

$$y2_{pt} = (1 - \overline{\beta 1}_p) \sum_k \sum_m y_{kmpt} + (1 - \overline{\beta 2}_p) \sum_l \sum_m y1_{lmpt}, \qquad \forall\, p, t \tag{3.11}$$

$$Rp_{pt} = \sum_m \sum_i y3_{mipt}, \qquad \forall\, p, t \tag{3.12}$$

$$\sum_j \sum_n x4_{ijnpt} = \sum_m y3_{mipt}, \qquad \forall\, i, p, t \tag{3.13}$$

Constraints (3.8) and (3.9) assure that all of the used products are collected from both customer types. Constraints (3.10) and (3.11) distinguish the collected products into the recoverable and non-recoverable products based on their qualities. Constraint (3.12) represents that all recoverable products shipped from collection centres to the HMR centres are recovered. Constraint (3.13) links the amount of recovered products in the forward side with the recoverable products shipped to HMR sites in the reverse side. Indeed, the balance of recovered products is established at HMR centres.

Delivery time constraints:

$$Td_{ikn} x1_{iknpt} - (1 - sl_k) Te_{kp} x1_{iknpt} \le Te_{kp} x1_{iknpt}, \qquad \forall\, i, k, n, p, t \tag{3.14}$$

$$Td1_{jkn} x1_{jknpt} - (1 - sl_k) Te_{kp} x_{jknpt} \le Te_{kp} x1_{jknpt}, \qquad \forall\, j, k, n, p, t \tag{3.15}$$

$$Td2_{jln} x2_{jlnpt} - (1 - sl1_l) Te1_{lp} x2_{jlnpt} \le Te1_{lp} x2_{jlnpt}, \qquad \forall\, j, l, n, p, t. \tag{3.16}$$

Constraints (3.14) and (3.15) state that the new products are delivered to corresponding customers according to predetermined customer service levels about the new products' deliveries. For example, customers with targeted service level 100% (i.e., $sl_k = 1$) will receive new products in their expectation time. Constraint (3.16) is similar to constraints (3.14) and (3.15) but for the recovered products.

Capacity constraints:

$$\sum_p b1_p xe_{ipt} + \sum_j \sum_n \sum_p b2_p x4_{ijnpt} \leq ca1_i, \qquad \forall\, i,t \tag{3.17}$$

$$\sum_i \sum_n \sum_p b_p x3_{ijnpe} + \sum_i \sum_n \sum_p b_p x4_{ijnpt} + \sum_p b_p Ic1_{jpt} + \sum_p b_p Ic2_{jpt} \leq ca2_j, \ \forall j,t \tag{3.18}$$

$$\sum_k \sum_p b3_p y_{kmpt} + \sum_l \sum_p b3_p y1_{lmpt} \leq ca3_m, \qquad \forall\, m,t. \tag{3.19}$$

Constraint (3.17) represents the maximum capacity level utilizations in HMR centres for both new and recovered products. Constraints (3.18) and (3.19) are similar to constraint (3.17) for the HDR and collection centres, respectively. Constraint (3.20) demonstrates that the amount of new products directly shipped is restricted. This could be explained due to budget limitations about direct shipment of products.

$$\sum_i \sum_k \sum_n \sum_p \sum_t x1_{iknpt} \leq UB\Big(\sum_k \sum_p \sum_t D1_{kpt}\Big). \tag{3.20}$$

Budget limitation:

$$\sum_i \sum_k \sum_p Ha1_{kpt}\Big(\sum_j \sum_n x_{jknpt} + \sum_i \sum_n x1_{iknpt}\Big) \sum_j \sum_l \sum_n \sum_p Ha2_{lpt} x2_{jlnpt} \leq BC_t, \forall\ t. \tag{3.21}$$

Constraint (3.21) considers the budget limitation for advertisement activities in different customer zones in any period. Finally, constraint (3.22) indicates the non-negativity and type of different decision variables.

$$x_{jknpt}, x1_{iknpt}, x2_{jlnpt}, x3_{ijnpt}, x4_{ijnpt}, \lambda_{lpt}, xe_{ipt}, y_{kmpt}, y1_{lmpt}, y2_{pt}, y3_{mipt}, Ic1_{jpt}, Ic2_{jpt} \geq 0 \qquad \forall\ i,j,k,n,p,l,m,t. \tag{3.22}$$

It is worthy to note that constraints (3.2) and (3.6) could be written in inequality form to reduce computational complexity of the proposed model. Meanwhile, since the robust counterpart of equality form is different from the inequality form we have to write these constraints in equality form. Indeed, as described later (see Section 4) when these constraints are written in inequality form the degree of robustness of the model is increased through paying unnecessary costs.

We have considered demand of new and recovered products, the selling price and also shortage cost as uncertain parameters in the forward side of the proposed IPDPCLSC model. The selling price have uncertain nature, since it is influenced by different factors such as inflation rate, interest rate, fluctuation of raw material costs, production costs, and etc. Also, since shortage cost is associated with backorder and lost sale costs, it is an uncertain parameter. According to Hasani et al. [8] demand of products has high degree of uncertainty which has effects on total performance of a supply chain. The uncertainty of the price of recovered products is affected by the purchasing and collection costs.

In the reverse side, we have considered the amount of returns and recoverable percentage of products as uncertain parameters. According to Pishvaee et al. [24], these parameters are really uncertain parameters with high impact on the reverse planning of a closed-loop supply chain. In fact, the amount of defective returned products is an unknown and uncertain parameter.

4. Robust Counterpart Based on the Polyhedral Uncertainty Set

In this section, we pursuit the concepts of the set-induced robust optimization method for the polyhedral uncertainty set based on the recent advances in the field (see [2, 17, 25]).

In set-induced robust optimization, it is assumed that the uncertain parameters are varied in a given uncertainty set and the model seeks for those solutions that immunize the system for all potential realizations from uncertainty set. Indeed, the best solutions obtained by the robust optimization should be feasible for all realizations of possible values of uncertain parameters in the given uncertainty set. Unlike the box uncertainty set (see Figure 2), which enforces the robust optimization model to find the worst-case feasible solutions, assuming the polyhedral uncertainty set (see Figure 3) for uncertain data leads to realistic feasible solutions with less degree of conservatism immunizing the system for reasonable realizations of uncertain parameters. In fact, assuming the polyhedral uncertainty set for uncertain parameters implies that all uncertain parameters cannot get their worst-case values in the given set, simultaneously. But, the possible values of uncertain parameters are varied within a polyhedral set.

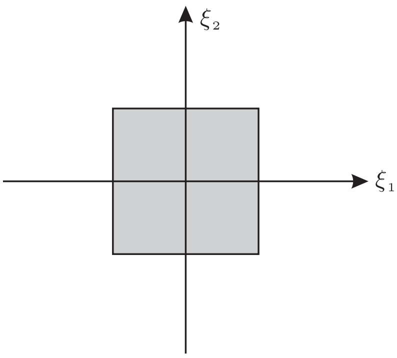

Figure 2. Box uncertainty set.

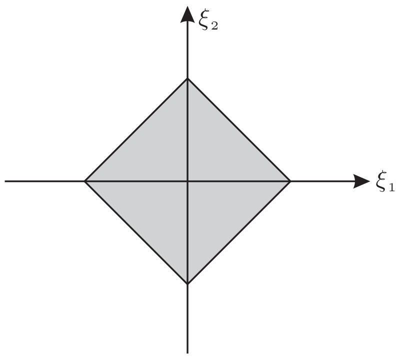

Figure 3: Polyhedral uncertainty.

It should be noted that the formulation of a robust counterpart model depends upon the given uncertainty set assumed for uncertain parameters. Here, the robust counterpart for the original model is formulated by assuming the polyhedral uncertainty set for uncertain parameters.

Consider the following well-known mathematical programming model with uncertain parameters including c_j, a_{ij}, and b_i. Assume that the values of these parameters vary in a bounded polyhedral uncertainty set, say U.

$$\begin{aligned} \text{Max } & \sum_j \bar{c}_j x_j \\ \text{S.t. } & \sum_j \bar{a}_{ij} x_j \le \bar{b}_i, \qquad \forall\ i, \\ & c, a, b \in U_{\text{Polyhedral}} \end{aligned} \tag{4.1}$$

The bar sign is used to show that the corresponding parameters are subject to uncertainty. The parameters $\bar{c}_j$, $\bar{a}_{ij}$, and $\bar{b}_i$ can be written as follows:

$$\bar{c}_j = c_j + \rho_j G^c_j \xi_j, \tag{4.2}$$

$$\bar{a}_{ij} = a_{ij} + \rho_{ij} G^a_{ij} \xi_{ij}, \tag{4.3}$$

$$\bar{b}_i = c_i + \rho_i G^b_i \xi_i \tag{4.4}$$

Where c_j, a_{ij}, and b_i are the nominal values of the corresponding uncertain parameters, ρ, which is a positive number, represents the uncertainty level for the related uncertain parameters, G indicates the uncertainty scale for the related uncertain parameters, and ξ is a random variable. Note that if the variable ξ is bounded, the polyhedral uncertainty set will be bounded. Hereafter, the indices of the above-mentioned parameters are eliminated for simplicity. The uncertainty level expresses the perturbation percentage of uncertain parameters around their nominal values. A particular case of interest is that the uncertainty scale is assumed to be equal to the nominal values [25]. Under the given polyhedral set, finding a robust solution for the problem (4.1) means that all constraints remain feasible for all realizations of uncertain parameters varied within the given polyhedral set and its objective function value is not worse than the objective function values under all realizations. It is noteworthy that for the sake of simplicity and clarity we have applied some little changes on the formulation of robust model presented by Li et al. [17].

4.1. Polyhedral uncertainty set The polyhedral uncertainty set is defined using the 1-norm of the uncertain data vector as follows,

$$U_1 = \{\xi \mid \|\xi\|_1 \le \Gamma\} = \{\xi \mid \sum_{j \in J_i} |\xi_j| \le \Gamma\}. \tag{4.5}$$

Where Γ is an adjustable parameter that controls the size of uncertainty set. J_i represents the index subset including the variable indices that their corresponding coefficients are subject to uncertainty. Indeed, $|J_i|$ indicates the number of variables whose corresponding coefficients are subject to uncertainty in the ith constraint. Without loss of generality, for bounding the polyhedral uncertainty set consider that ξ_j is varied in the range $[-1, 1]$. In this case, representing of uncertain parameters can be easily done by changing the uncertainty level in the given bounded polyhedral set. In order to avoid covering the overall uncertain space the adjustable parameter should be less than or equal to the cardinality of the uncertainty set in any constraint (i.e. $\Gamma \leq |J_i|$).

4.2. The equivalent linear robust counterpart optimization model

In order to apply the uncertainty on the coefficients of the objective function, it should be considered as a constraint. By replacing the equations (4.2)-(4.4) in the problem (4.1) we have:

$$\begin{aligned} \text{Max } & \sum_j (c_j + \rho_j G_j^c \xi_j) x_j \\ \text{S.t. } & \sum_j (a_{ij} + \rho_{ij} G_{ij}^a \xi_{ij}) x_j \leq b_i + \rho_i G_i^b \xi_i, \qquad \forall\ i. \end{aligned} \tag{4.6}$$

Which can be rewritten as follows:

$$\begin{aligned} \text{Max } & z \\ \text{S.t. } & \sum_j (c_j + \rho_j G_j^c \xi_j) x_j \geq z, \qquad \forall\ i \\ & \sum_j (a_{ij} + \rho_{ij} G_{ij}^a \xi_{ij}) x_j \leq b_i + \rho_i G_i^b \xi_i, \qquad \forall\ i. \end{aligned} \tag{4.7}$$

Or, equivalently,

$$\begin{aligned} \text{Max } & z \\ \text{S.t. } & -\sum_j c_j x_j - \sum_j \rho_j G_j^c \xi_j x_j \leq -z, \\ & \sum_j a_{ij} x_j + \sum_j \rho_{ij} G_{ij}^a \xi_{ij}) x_j - \rho_i G_i^b \xi_i \leq b_i, \qquad \forall\ i. \end{aligned} \tag{4.8}$$

In problem (4.8), the goal is to find solutions immunizing the feasibility of the model for all possible values of ξ in the range [-1, 1] for the given polyhedral set. Therefore, in order to enable problem (4.8) to find such robust solutions, it should be transformed to the following problem, which is the robust counterpart of problem (4.1), for the polyhedral uncertainty set.

$$\begin{aligned} \text{Max } & z \\ \text{S.t. } & -\sum_j c_j x_j - \max_{\xi \in U_{\text{Pal.}}} \Big\{ \sum_j k \rho_j G_j^c \xi_j x_j \Big\} \leq -z, \\ & \sum_j a_{ij} x_j + \max_{\xi \in U_{\text{Pal.}}} \Big\{ \sum_j \rho_{ij} G_{ij}^a \xi_{ij}) x_j - \rho_i G_i^b \xi_i \Big\} \leq b_i, \qquad \forall\ i. \end{aligned} \tag{4.9}$$

The problem (4.9) is computationally intractable due to too many possible values of uncertain parameters within the polyhedral set. Note that we dont know the behaviour of uncertain parameters within the polyhedral set; however, we are aware that the uncertain parameters are varied within the bounded polyhedral uncertainty set. In other words, the probability distribution or possibility distribution of uncertain parameters is not clear in the bounded polyhedral set. Therefore, problem (4.9) is a computationally intractable NP-hard problem. However, it could be transformed to the tractable convex and linear programming model [17].

The equivalent non-linear form of problem (4.9) can be stated as follows:

$$\begin{aligned}
\text{Max } & z \\
\text{S.t. } & z - \sum_j c_j x_j + \Gamma w \le 0 \\
& w \ge \rho_j G_j^c |x_j|, \qquad \forall\, j, \\
& \sum_j a_{ij} x_j + \Gamma_i w_i \le b_i, \qquad \forall\, i, \\
& w_i \ge \rho_{ij} G_{ij}^a |x_j|, \qquad \forall\, j, \\
& w_i \ge \rho_i G_i^b, \qquad \forall\, i.
\end{aligned} \tag{4.10}$$

Note that the adjustable parameter shows the number of uncertain parameters in the corresponding constraint. In addition, w is an auxiliary variable. The absolute form in problem (4.10) imposes non-linearity to the model. Meanwhile, it could be easily transformed to the linear form as follows:

$$\begin{aligned}
\text{Max } & z \\
\text{S.t. } & z - \sum_j c_j x_j + \Gamma w \le 0 \\
& w \ge \rho_j G_j^c y_j, \qquad \forall\, j, \\
& \sum_j a_{ij} x_j + \Gamma_i w_i \le b_i, \qquad \forall\, i, \\
& w_i \ge \rho_{ij} G_{ij}^a y_j, \qquad \forall\, j, \\
& w_i \ge \rho_i G_i^b, \qquad \forall\, i, \\
& -y_i \le x_j \le y_j, \qquad \forall\, j,
\end{aligned} \tag{4.11}$$

It is worthy to note that if the variables (i.e. x_j) are non-negative, the problem (4.10) is solved without absolute forms to find robust solutions.

Another important point which is not stated by Li et al. [17] is developing the equivalent robust counterpart for those constraints that are in equality form. Let's consider the following constraint that its parameters have been tainted with uncertainty:

$$\sum_l \bar{h}_{el} x_l = \bar{k}_e, \qquad \forall\, e. \tag{4.12}$$

The equivalent robust counterpart form for the constraint (4.12) can be stated as follows:

$$
\begin{aligned}
&\sum_l h_{el} x_l + \Gamma_e w_e \geq k_e, \qquad \forall\ e,\\
&\sum_l h_{el} x_l - \Gamma_e w_e \leq k_e, \qquad \forall\ e,\\
&w_e \geq \rho_{el} G^h_{el} |x_j|,\\
&w_e \geq \rho_e G^k_e.
\end{aligned}
\tag{4.13}
$$

According to above descriptions and considering this fact that all variables of the problem are non-negative, the equivalent robust counterpart optimization model for the proposed integrated production-distribution planning model in closed-loop supply chains under uncertainty of some input parameters in the form of polyhedral uncertainty set, can be stated as follows:

$$
\operatorname{Max} z1 \tag{4.14}
$$

$$
\sum_t \frac{1}{(1+r)^t}
\left(
\begin{aligned}
&\Bigg[\sum_j\sum_k\sum_n\sum_p \overline{Pr}_{jknpt} x_{jknpt} - \Gamma^{\mathrm{Pr}} u_0\\
&+\sum_i\sum_k\sum_n\sum_p \overline{Pr1}_{iknpt} x1_{iknpt} - \Gamma^{\mathrm{Pr1}} u_0\\
&+\sum_j\sum_l\sum_p\sum_n \overline{Pr2}_{jlnpt} x2_{jlnpt} - \Gamma^{\mathrm{Pr2}} u_0 + \sum_p \overline{Pr3}_{pt} y2_{pt} - \Gamma^{\mathrm{Pr3}} u_0\Bigg]\\
&\Bigg[\sum_i\sum_j\sum_n\sum_p Tc_{ijnpt} x3_{ijnpt} + \sum_i\sum_k\sum_n\sum_p Tc1_{iknpt} x1_{iknpt}\\
&+\sum_j\sum_k\sum_n\sum_p Tc2_{jknpt} x3_{jknpt} + \sum_j\sum_l\sum_n\sum_p Tc3_{ilnpt} x1_{ilnpt}\\
&+\sum_k\sum_m\sum_p Tc4_{kmpt} y_{kmpt} + \sum_m\sum_i\sum_p Tc6_{mipt} y3_{mipt}\\
&+\sum_i\sum_j\sum_n\sum_p Tc_{ijnpt} x4_{ijnpt} + \sum_i\sum_p Pc_{ipt} xe1_{ipt}\\
&+\sum_i\sum_p Rc_{ipt}\Big(\sum_m y3_{mipt}\Big) + \sum_m\sum_p Hpc_{mpt}\Big(\sum_k y_{kmpt} + \sum_l y1_{lmpt}\Big)\\
&+\sum_j\sum_p HIc1_{jpt} Ic1_{jpt} + \sum_j\sum_p HIc2_{jpt} Ic2_{jpt}\\
&+\sum_k\sum_p \overline{Pur1}_{kpt}\Big(\textstyle\sum_m y_{kmpt}\Big) + \Gamma^{\mathrm{Pur1}} u_0\\
&+\sum_l\sum_p \overline{Pur2}_{lpt}\Big(\textstyle\sum_m y1_{lmpt}\Big) + \Gamma^{\mathrm{Pur2}} u_0\\
&+\sum_k\sum_p Ha1_{kpt}\Big(\sum_j\sum_n x_{jknpt} + \sum_i\sum_n x1_{iknpt}\Big)\\
&+\sum_l\sum_p Ha2_{lpt}\Big(\sum_j\sum_n x2_{jlnpt}\Big) + \sum_l\sum_p \overline{sc}_{lpt}\lambda_{lpt} + \Gamma^{sc} u_0\Bigg]
\end{aligned}
\right) \geq Z1
\tag{4.15}
$$

$$
\rho_{Pr} G^{Pr}_{jknpt} x_{jknpt} \leq u_0, \qquad \forall\ j,k,n,p,t \tag{4.16}
$$

$$\rho_{Pr1}G^{Pr1}_{jknpt}x1_{jknpt} \leq u_0, \qquad \forall\ i,k,n,p,t \tag{4.17}$$

$$\rho_{Pr2}G^{Pr2}_{jlnpt}x2_{jlnpt} \leq u_0, \qquad \forall\ j,l,n,p,t \tag{4.18}$$

$$\rho_{Pr3}G^{Pr3}_{pt}y2_{pt} \leq u_0, \qquad \forall\ p,t \tag{4.19}$$

$$\rho_{Pur1}G^{Pur1}_{kpt}\Big(\sum_m y_{kmpt}\Big) \leq u_0, \qquad \forall\ k,p,t \tag{4.20}$$

$$\rho_{Pur2}G^{Pur2}_{lpt}\Big(\sum_m y1_{lmpt}\Big) \leq u_0, \qquad \forall\ l,p,t \tag{4.21}$$

$$\rho_{sc}G^{sc}_{lpt}\lambda_{lpt} \leq u_0, \qquad \forall\ l,p,t \tag{4.22}$$

$$\sum_j\sum_n x_{jknpt} + \sum_i\sum_k x1_{iknpt} - \Gamma1_{kpt}\rho_{D1}G^{D1}_{kpt} \leq D1_{kpt}, \qquad \forall\ k,p,t \tag{4.23}$$

$$\sum_j\sum_n x_{jknpt} + \sum_i\sum_k x1_{iknpt} - \Gamma1_{kpt}\rho_{D1}G^{D1}_{kpt} \geq D1_{kpt}, \qquad \forall\ k,p,t \tag{4.24}$$

$$\sum_j\sum_n x2_{jlnpt} + \lambda_{lpt} - \lambda_{lp,t-1} - \Gamma2_{lpt}\rho_{D2}G^{D2}_{lpt} \leq D2_{lpt}, \qquad \forall\ l,p,t \tag{4.25}$$

$$\sum_j\sum_n x2_{jlnpt} + \lambda_{lpt} - \lambda_{lp,t-1} - \Gamma2_{lpt}\rho_{D2}G^{D2}_{lpt} \geq D2_{lpt}, \qquad \forall\ l,p,t \tag{4.26}$$

$$\sum_m y_{kmpt} - \Gamma4_{kpt}\rho_{Re1}G^{Re1}_{kpt} \leq Re1_{kpt}, \qquad \forall\ k,p,t \tag{4.27}$$

$$\sum_m y_{kmpt} + \Gamma4_{kpt}\rho_{Re1}G^{Re1}_{kpt} \geq Re1_{kpt}, \qquad \forall\ k,p,t \tag{4.28}$$

$$\sum_m y1_{lmpt} - \Gamma5_{lpt}\rho_{Re2}G^{Re2}_{lpt} \leq Re2_{lpt}, \qquad \forall\ l,p,t \tag{4.29}$$

$$\sum_m y1_{lmpt} + \Gamma5_{lpt}\rho_{Re2}G^{Re2}_{lpt} \geq Re2_{lpt}, \qquad \forall\ l,p,t \tag{4.30}$$

$$Rp_{pt} \geq \overline{\beta1}_p\sum_k\sum_m y_{kmpt} + \overline{\beta2}_p\sum_l\sum_m y1_{lmpt} - \Gamma6_{pt}u1_{pt}, \qquad \forall\ p,t \tag{4.31}$$

$$Rp_{pt} \leq \overline{\beta1}_p\sum_k\sum_m y_{kmpt} + \overline{\beta2}_p\sum_l\sum_m y1_{lmpt} - \Gamma6_{pt}u1_{pt}, \qquad \forall\ p,t \tag{4.32}$$

$$y2_{pt} \geq (1-\overline{\beta1}_p)\sum_k\sum_m y_{kmpt} + (1-\overline{\beta2}_p)\sum_l\sum_m y1_{lmpt} - \Gamma6_{pt}u1_{pt}, \qquad \forall\ p,t \tag{4.33}$$

$$y2_{pt} \leq (1-\overline{\beta1}_p)\sum_k\sum_m y_{kmpt} + (1-\overline{\beta2}_p)\sum_l\sum_m y1_{lmpt} + \Gamma6_{pt}u1_{pt}, \qquad \forall\ p,t \tag{4.34}$$

$$\rho_{\beta1}G^{\beta1}_p\sum_k\sum_m y_{kmpt} \leq u1_{pt}, \qquad \forall\ p,t \tag{4.35}$$

$$\rho_{\beta2}G^{\beta2}_p\sum_l\sum_m y_{lmpt} \leq u1_{pt}, \qquad \forall\ p,t \tag{4.36}$$

Constraints (3.3), (3.4), (3.5), (3.7), and (3.12)-(3.22).

Table 2: Random generation of nominal data.

parameter	value	parameter	value
$D1_{kpt}$	∼Unif (250, 400) units	$Hic1_{jtp}$	∼Unif (30, 40)$
$D2_{ltp}$	∼Unif (130, 200) units	$Hic2_{jtp}$	∼Unif (15, 25)$
$D3_{pt}$	∼Unif (50, 200) units	pc_{ipt}	∼Unif (500, 600)$
$Re1_{kpt}$	∼Unif (60,140) units	Rc_{ipt}	∼Unif (140, 200)$
$Re2_{ltp}$	∼Unif (80,170) units	Hpc_{mpt}	∼Unif (40, 80)$
SS_{jtp}	∼Unif (60,100) units	Tc_{ijnpt}, $Tc2_{jknpt}$,	∼Unif (15, 25)$
$Pr1_{iknpt}$, Pr_{jknpt}	∼Unif (1000, 1300)$	$Tc4_{kmpt}$, $Tc5_{lmpt}$	,∼Unif (10, 20)$
$Pr2_{jlnpt}$	∼Unif (690, 900)$	$Tc3_{jlnpt}$, $Tc6_{mipt}$	∼Unif (10, 20)$
Pr_{pt}	∼Unif (450, 600)$	$Tc1_{iknpt}$	∼Unif (20, 30)$
BC_t	∼Unif (700000, 750000)$	Td_{ikn}	∼Unif (4, 8) days
$Pur1_{kpt}$	∼Unif (200, 290)$	$Td1_{jkn}$	∼Unif (6, 10) days
$Pur2_{ltp}$	∼Unif (130,190)$	$Te1_{lp}$	∼Unif (15, 20) days
$Ha1_{kpt}$	∼Unif (20, 30)$	$Td2_{jln}$	∼Unif (6, 18) days
$Ha2_{ltp}$	∼Unif (15, 20)$	Te_{kp}	∼Unif (6, 10) days
Sc_{ltp}	∼Unif (60,80)$	$\beta1_p$, $\beta2_p$	∼Unif (0.65, 0.85)
sl_k	∼Unif (0.8, 0.9)	$Ca1_i$	∼Unif (10000,15000) units
$sl1_l$	∼Unif (0.7, 0.8)	$Ca2_j$	∼Unif (3500,6000) units
$b_p, b1_p, b2_p$	∼Rand$\{1, 2\}$	$Ca3_m$	∼Unif (4500,6000) units
$\rho_{pr}, \rho_{pr1}, \rho_{pr2}, \rho_{pr3}, \rho_{pur1}, \rho_{pur2}, \rho_{sc}$	∼Unif (0.01, 0.06)	$\rho_{D1}, \rho_{D2}, \rho_{D3}, \rho_{Re1}, \rho_{Re2}, \rho_{\beta1}, \rho_{\beta2}$	∼Unif (0.09, 0.12)

5. Computational Experiments

In this section, a numerical example with reasonable size is presented to investigate the applicability and appropriateness of the proposed robust framework for the IPDPCLSC model. The proposed robust approach was coded in Lingo 11.0 optimization software and solved on a Pentium dual-core 2.60 GHZ computer with 4 GB RAM. It is worthy to note that the conservatism degree of polyhedral robust optimization method depends on the choice of adjustable parameter (i.e., Γ), controlling the size of uncertainty set, and desired uncertainty level (i.e., ρ) for each uncertain parameter. The adjustable parameter controls the uncertainty space covered by the polyhedral uncertainty set, while the uncertainty level controls the range of uncertain parameters changes. As mentioned in the previous section, when the adjustable parameter is set to be equal or larger than the number of uncertain parameters in any constraint (i.e., $\Gamma \geq |J_i|$), the overall uncertainty space is covered by the polyhedral set. In particular case, if $\Gamma = |J_i|$, the intersection between the polyhedral and box uncertainty set is exactly the box [17]. It is worthy to note that the box uncertainty set is assumed for uncertain parameters when the decision maker prefers to achieve over-conservative robust solutions for the worst-case conditions. Meanwhile, the realistic robust solutions can be obtained when $\Gamma < |J_i|$.

The size of the considered numerical example for the problem in question is as follows: 4 HMR centres, 8 HDR centres, 3 collection centres, 3 transportation modes, 3 types of products, 3 periods, 20 customer zones for new products, and 15 customer zones for recovered products. The nominal values of the input parameters, which are actually the most likely values, are randomly generated according to Table 2. Table 3 illustrates the cardinality of uncertainty set for the uncertain parameters of the concerned problem. These values are achieved by multiplying the corresponding indices of the uncertain parameters existing in any equation (for example for Pr_{jknpt}, $|J_i| = 8\times20\times3\times3\times3 = 4320$). Since the objective function is considered as a constraint in robust counterpart optimization model (See previous section), all indices of uncertain parameters are multiplied together to achieve the cardinality of uncertainty set. In addition, since the constraints are separately considered for each set of indices, the cardinality of uncertainty set is equal to 1. Indeed, the cardinality of uncertainty set is calculated for any constraint based on the number of uncertain parameters involved in.

Table 3: Cardinality of uncertainty set ($|J_i|$).

Cardinality of uncertainty set	Uncertain parameters of Objective function							Uncertain parameters of Constraints
	Pr_{jknpt}	$Pr1_{iknpt}$	$Pr2_{jlnpt}$	$Pr3_{pt}$	$Pur1_{kpt}$	$Pur2_{lpt}$	Sc_{lpt}	$D1_{kpt}$, $D2_{lpt}$, $Re1_{kpt}$, $Re2_{lpt}$, $\beta1_p$, $\beta2_p$
$\|J_i\|$	4320	2160	3240	9	180	135	135	1

We have specified 11 different scenarios for adjustable uncertainty parameter to evaluate the efficiency and effectiveness of the proposed robust optimization model for the IPDPCLSC under uncertainty. Table 4 shows the defined scenarios. In fact, the cardinalities of uncertainty set for different parameters, specified in Table 3, are divided into 10 equal parts. Obviously, when the cardinalities of uncertainty are set to 0, they describe deterministic condition and when they are set to the number of uncertain parameters in any equation, the worst-case condition for realization of uncertain parameters is envisioned. We mean the scenario 6 as a representative of realistic condition. Clearly, the degree of conservatism of the scenarios is increased shifting from scenario 1 to scenario11. Notably, since the problem is linear, all run times of different problems are less than 2 minutes.

Objective function values and different types of revenues for different specified scenarios have been demonstrated in Table 5. From this Table, the objective function values and revenues, resulted from selling of products being new, recovered and non-recovered, are decreased as long as the degree of conservatism of the scenarios are increased. Indeed, the lost profit is based on the robustness of the scenarios. In addition, reduction gradient of objective function values is larger than revenues. As shown in Table 6, among

Table 4: Specifying different scenarios.

Scenario	Cardinality of uncertainty set for uncertain parameters ($\lvert J_i \rvert$)							
	Pr_{jknpt}	$Pr1_{iknpt}$	$Pr2_{jlnpt}$	$Pr3_{pt}$	$Pur1_{kpt}$	$Pur2_{lpt}$	Sc_{lpt}	$D1_{kpt}$, $D2_{lpt}$, $Re1_{kpt}$, $Re2_{lpt}$, $\beta 1_p$, $\beta 2_p$
1 (Deterministic)	0	0	0	0	0	0	0	0
2	432	216	324	0.9	18	13.5	13.5	0.1
3	864	432	648	1.8	36	27	27	0.2
4	1296	648	972	2.7	54	40.5	40.5	0.3
5	1728	864	1296	3.6	72	54	54	0.4
6 (Realistic)	2160	1080	1620	4.5	90	67.5	67.5	0.5
7	2592	1296	1944	5.4	108	81	81	0.6
8	3024	1512	2268	6.3	126	94.5	94.5	0.7
9	3456	1728	2592	7.2	144	108	108	0.8
10	3888	1944	2916	8.1	162	121.5	121.5	0.9
11 (Worst-case)	4320	2160	3240	9	180	135	135	1

the share of different costs in the objective function, transportation costs, production costs, advertising costs, and shortage costs are increased when the degree of robustness of scenarios are intensified. However, quality testing costs and inventory holding costs for recovered products are decreased and inventory holding costs for new products remain fixed for all specified scenarios. Recovering costs has a reduction in scenario 2 and then is increased. But, purchasing costs has an increase in scenario 2 and then is decreased. Although the more used products are purchased in scenario 2, the recovery costs are less than those in scenario 1. This observation could be explained owing to uncertainty of percentage of recoverable returned products. Consequently, the pure profit is decreased as long as the robustness of the model is increased. Meanwhile, the behaviour of different components in objective function may be different when compared with each other under different scenarios.

Description of abbreviations used in Tables 5 and 6 are as follows: O.F.V. (Objective function value), RV1 (Revenue resulted from selling new products via indirect shipment), RV2 (Revenue resulted from selling new products via direct shipment), RV3 (Revenue resulted from selling recovered products), RV4 (Revenue resulted from selling non-recoverable products), Tr. Costs (Transportation costs), Pr. Costs (Production costs), Re. costs (Recovering costs), NIn costs (Holding costs of new products), RIn. costs (Holding costs of recovered products), Pu. costs (Purchasing costs of used products), Ad. costs (Advertisement costs), Sh. costs (shortage costs).

Increasing costs in Table 6 is due to robustness price paid to deal with the uncertainty. The zero values for inventory holding costs for recovered products illustrate that

Table 5: The objective function values and share of different revenues.

Scenarios	O.F.V.	RV1	RV2	RV3	RV4
Sc. 1	43470110	44629220	19066370	19670200	4624728
Sc. 2	37839840	44913400	17465860	19208410	835533
Sc. 3	36765440	44401250	17655340	19014550	671270
Sc. 4	35776070	44161060	17562520	18745360	663701
Sc. 5	34814890	44096800	17313200	18485200	656158
Sc. 6	33881320	44122980	16987910	18232420	648641
Sc. 7	32975570	44166590	16666560	17991230	641151
Sc. 8	32097640	44220190	16350960	17758210	633687
Sc. 9	31247690	44279410	16046500	17533870	626249
Sc. 10	30425040	44352130	15745550	17319210	618837
Sc. 11	29629330	44441820	15446190	17114520	611452

Table 6: The share of different costs in the objective function.

Scenarios	Tr. Costs	Pr. Costs	Re. Costs	Te. Costs	NIn. costs	RIn. costs	Pu. costs	Ad. costs	Sh. costs
Sc. 1	3137340	27899640	3178786	1562775	188841	77745	6795373	1659455	20453
Sc. 2	3119948	28336400	2884123	1550629	188841	0	6798913	1677863	26641
Sc. 3	3145349	28669510	2918352	1537304	188841	0	6778224	1696496	42898
Sc. 4	3171027	28973150	2952421	1523979	188841	0	6768274	1715129	63758
Sc. 5	3199584	29277630	2984364	1510655	188841	0	6757406	1733761	84224
Sc. 6	3222198	29578990	3020960	1497330	188841	0	6745621	1752394	104297
Sc. 7	3250541	29882960	3055683	1484005	188841	0	6732919	1771027	123977
Sc. 8	3278373	30185960	3089336	1470680	188841	0	6719299	1789659	143264
Sc. 9	3308328	30487750	3120852	1457356	188841	0	6704762	1808292	162158
Sc. 10	3332296	30791400	3157226	1444031	188841	0	6689307	1826925	180659
Sc. 11	3359867	31096380	3191591	1430706	188841	0	6672935	1845558	198766

demands of recovered products are faced with shortage when the uncertainty of scenarios is increased. The amounts of shortage costs confirm this claim. Fixed inventory holding costs for all scenarios imply that the all demands of new products are satisfied by existing capacity of facilities and inventory holding costs are charged because of holding safety stocks.

The amount of new products, recovered products, amount of non-recovered products sold to material customers, and inventory of new and recovered products under deterministic condition (Sc. 1), realistic condition (Sc. 6), and worst-case condition (Sc. 11) have been summarized in Table 7. Since the shortage is not possible for new product demands, the amount of new products in the forward side of the closed-loop supply chain is increased when the scenarios moves toward robust ones. Meanwhile, in view of the fact that some demand of customers for recovered products can be unfulfilled, the amount of recovered products in the reverse side is determined according to a trade-off between shortage costs and revenues in the objective function. The amount of non-recovered products strictly depends on the percentage of recoverable products and related uncertainty. As shown in Table 7, the amount of non-recovered products is strictly decreased by increasing the uncertainty level. The amount of inventories for new product is the same for all scenarios because of holding safety stocks in HDR centres. The impact of uncertainty on the amount products and inventory in the forward and reverse sides can be seen in Table 7.

Table 7: Amount of new and recovered products produced and recovered in all facilities.

Scenario	Product type	Period	New	Recovered	Nonrecovered	Inventory (new)	Inventory (recovered)
Sc. 1	1	1	6728	3131	900	611	351
		2	5839	3016	863	613	1181
		3	5553	2845	823	606	1643
	2	1	6977	2971	1071	651	0
		2	6031	2901	1047	644	454
		3	5791	2833	1022	685	860
	3	1	7083	2579	1205	654	0
		2	6106	2641	1231	627	34
		3	5879	2671	1248	663	226
Sc. 6	1	1	7064	2946	146	611	0
		2	6160	2318	151	613	0
		3	5858	2525	156	606	0
	2	1	7324	3148	140	651	0
		2	6363	2594	165	644	0
		3	6107	2573	152	685	0
	3	1	7436	3075	135	654	0
		2	6443	2423	157	627	0
		3	6200	2627	140	663	0
Sc. 11	1	1	7400	3113	139	611	0
		2	6481	2449	144	613	0
		3	6164	2668	149	606	0
	2	1	7672	3326	133	651	0
		2	6695	2741	157	644	0
		3	6423	2719	145	685	0
	3	1	7790	3249	128	654	0
		2	6780	2560	149	627	0
		3	6521	2776	133	663	0

Consequently, the outcomes illustrated above form some guidelines and frameworks for other planning modules in short-term such as master production schedule, capacity requirements planning and material requirements planning.

5.1. Sensitivity analysis

In this section, we conduct some sensitivity analyses on the uncertainty levels of uncertain parameters to investigate the performance of the proposed IPDPCLSC model. The sensitivity analyses are performed for the scenarios 2, 6, and 11. Since scenario 1 (deterministic condition) is insensitive respect to uncertainty level changes, we select scenario 2, having the lowest degree of conservatism among all scenarios, instead of scenario 1 to perform sensitivity analysis. To do so, the randomly selected nominal data are used in the numerical example and then the uncertainty level for any uncertain parameter is increased step by step from 0.01 to 0.1. Figure 4 shows the changes of objective function values (OFVs) in respect to uncertainty levels of selling prices of new products which are shipped indirectly.

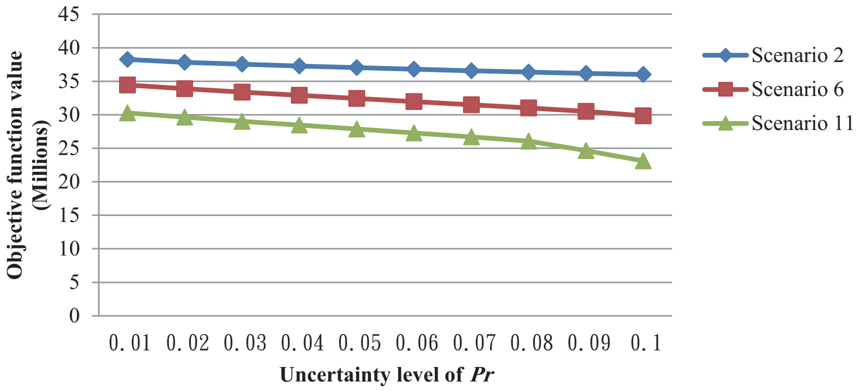

Figure 4: OFV vs. uncertainty level of Pr.

The increase in uncertainty level of selling prices of new products decreases the total profit of three scenarios. However, that of scenario 11, representing the worst-case condition, decreases with a steeper gradient compared to the scenarios 2 and 6. This observation could be explained due to higher degree of robustness of scenario 11 than others.

As illustrated in Figure 5, the total profit is decreased when the uncertainty level of selling prices of new products which are directly shipped to customer zones is increased. Meanwhile, the gradient of reduction is intensified after uncertainty level of 0.03 for scenarios 6 and 11.

Consequently, the results of conducted sensitivity analyses state that the proposed IPDPCLSC model is much sensitive to changes of uncertainty levels of selling prices of new products which are directly or indirectly sold and purchasing costs of used products. Therefore, in order to obtain the optimal aggregate production and distribution planning for the closed-loop supply chain, the uncertainty levels of these parameters should be determined, precisely. In all sensitivity analyses, the total profit related to considered scenarios are so compatible with the degree of robustness of scenarios so that those scenarios with higher degree of robustness have lower profit than others.

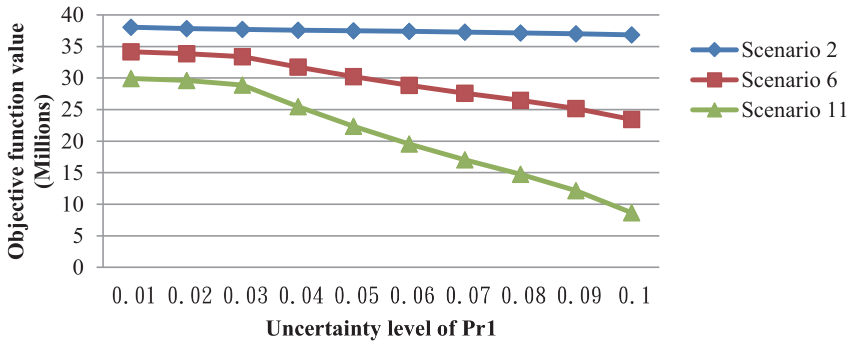

Figure 5: OFV vs. uncertainty level of Pr1.

At the following, we perform some other sensitivity analyses on the amounts of demands of new and recovered products for the deterministic, realistic, and worst-case conditions (i.e., Scenarios 1, 6, and 11). As it is demonstrated in Figure 6, the objective function values are increased in a linear form as long as the mean of new products is increased. This exhibits the direct effect of the new products' demand on the performance of the proposed IPDPCLSC model.

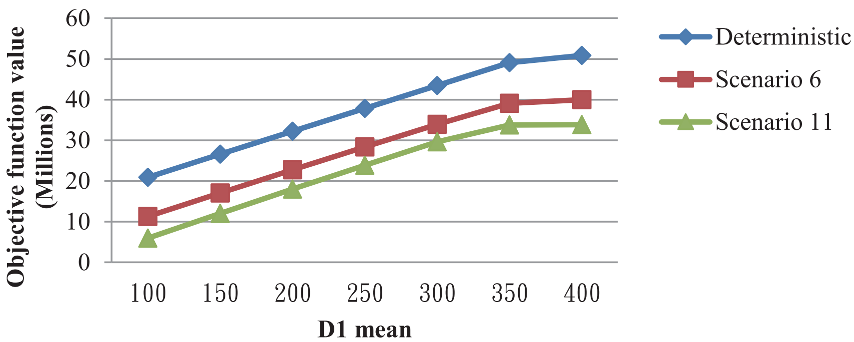

Figure 6: OFV vs. the mean of new products.

Sensitivity analysis on the demand of recovered products (See Figure 7) illustrates that total profit is increased for scenarios 6 and 11 along with increasing the mean amount of recovered products. Meanwhile, total profit for the deterministic model is increased until certain amount of recovered products (i.e., 190) and then is decreased. Another point inferred from figure 7 is reduction of profit gradient after an amount of 280 for recovered products. Therefore, it may be estimated that the scenarios 6 and 11 behave similar to deterministic model for high values of demand of recovered products.

As shown in Figures 8 and 9, increasing the amount of returns from both of customers decreases the total profit for scenarios 6 and 11. However, this procedure is not followed by the deterministic model.

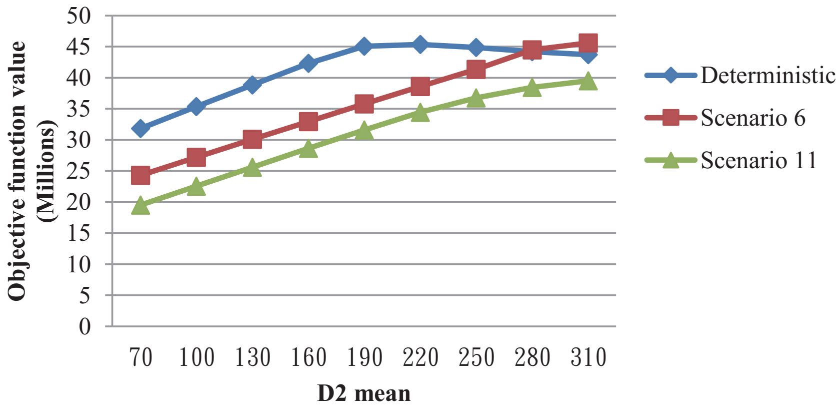

Figure 7: OFV vs. the mean of recovered products.

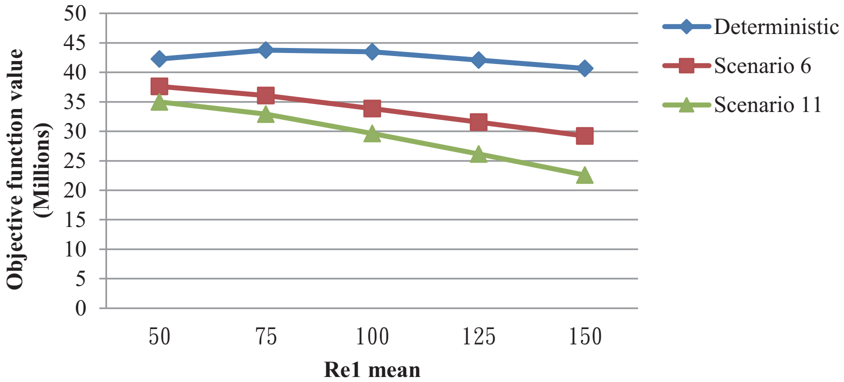

Figure 8: OFV vs. the mean of returned products from new products customers.

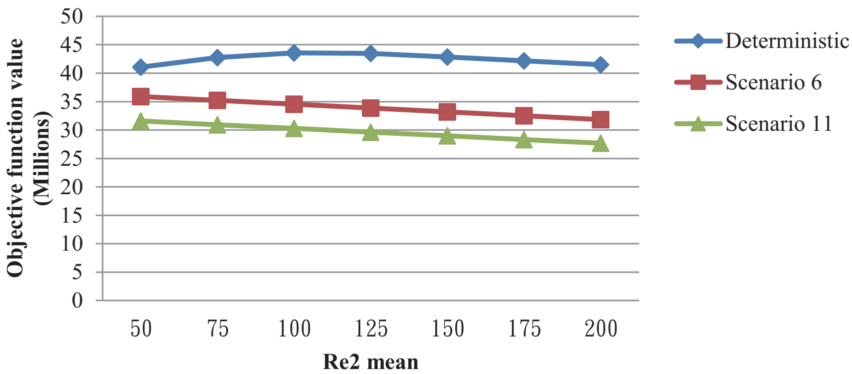

Figure 9: OFV vs. the mean of returned products from recovered products.

6. Concluding Remarks

This paper presents a novel robust optimization model based on polyhedral uncertainty set for integrated production and distribution planning in multi-echelon, multi-products closed-loop supply chains over a mid-term multi-periods horizon. The proposed model is able to consider different transportation modes, direct or indirect shipments, and several customer zones for different products beside the traditional features considered for tackling such problems in the literature. Computational results were provided by using a numerical example and its related scenarios to discuss different features of the proposed robust optimization model to handle the uncertainty of parameters. According to the achieved results, using robust optimization method would assure the feasibility of the model under uncertainty. Also, the DM could select the best approach through the provided sensitivity analysis. In our view, the realistic approach (scenario 6) will provide suitable solutions with reasonable costs of handling uncertainty.

It should be noted that when there is not reliable and historical data for making probability distribution of uncertain parameters, we cannot use stochastic programming methods. In this case, robust optimization method based on different uncertainty sets could be efficiently used. Since many real world problems usually have the limitation of data availability, the robust optimization method is a suitable tool to deal with the uncertainty of such problems.

Some managerial implications, which are inferred from our experiments and could be applied in real world cases, are as follows:

- The proposed model is able to provide robust medium-term aggregate plans in closed-loop supply chains involving different types of customers. The need for such a model may be observed in high-tech electronics, copiers and printers industries.
- Although the polyhedral uncertainty set is used to model the uncertainty of parameters in a realistic viewpoint, the amount of adjustable parameter that controls the size of uncertainty has a direct impact on the conservatism level of the model so that for its large values the polyhedral set leads to worst-case solutions. Therefore, this parameter should be determined according to managerial preferences. Specifying different scenarios, illustrated in previous Section, can help senior management to select the most suitable scenario for adjustable parameter Γ.
- Another important parameter affecting the performance of the robust model is the uncertainty level of each uncertain parameter and therefore should be determined according to decision maker preferences and some historical data.
- Unlike the deterministic model, the proposed robust model assures feasibility of the model for any realization of uncertain parameters varying within the polyhedral uncertainty set.
- Although the proposed model is suitable for planning in a centralized closed-loop supply chain, it could be applied for planning in decentralized ones through applying the conceptual framework proposed in [34].

The following research directions could be covered by researchers and practitioners in the future. Considering another objective for example maximization of responsiveness or minimization of environmental issues in the context of multi-objective programming could be addressed. The proposed model has a general structure which could be implemented in real cases of industries with required modifications. The problem could be studied under assumption of box uncertainty sets for uncertain parameters and evaluated its performance with the proposed model. Also, developing a heuristic or metaheuristic algorithms to solve the proposed IPDPCLSC model for large and real cases is an interesting future research. The proposed model could be decomposed into production and distribution problems and solved with Benders decomposition algorithm for very large sizes. By this way, global optimum solutions are achieved in reasonable time. Another interesting future research is evaluating the performance of the proposed model with possibilistic programming approaches through applying the models in real case production-distribution problems.

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