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Solving a Multi-Objective Location-Routing Problem with Minimum Cost and Total Time Balance

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Abstract

In transportation activities, providing a workload balance for drivers and vehicles can contribute several benefits for the firm. However, few attention has been paid to the importance of workload balance in location-routing problem (LRP) research. Thus, this study intends to present a mathematical model to solve the multi-objective LRP which addresses this issue. The proposed model considers two objective functions: (1) to minimize the total cost and (2) to balance the workload in distribution activities. The main purposes of this model are to obtain the optimal location of a distribution center, number of vehicles established, and delivery routes which satisfy both of these two objectives. Furthermore, to solve the model, this study proposes the non-dominated sorting genetic algorithm-II (NSGA-II) for several problem scenarios with different numbers of customers. The experimental results show that the proposed algorithm performs well in terms of quality and computational time of the solution.

Keywords: Location-Routing Problem (LRP), multi-objective, workload balance, total time difference, NSGA-II.

1. Introduction

The location-routing problem (LRP) is an emerging area in transportation planning research. LRP covers all three decision levels in supply chain management by simultaneously planning the facilities, vehicles, and routes of a supply network. It combines two well-known planning tasks: the facility location problem (FLP) and the vehicle-routing problem (VRP), which have been frequently addressed separately (see Drexl and Schneider [6] and Prodhon and Prins [18]). Because solving FLP and VRP separately may lead to a suboptimal planning result, LRP has a critical role in transportation planning.

One important question in transportation planning is how to provide a workload balance for drivers and vehicles. There are three common workloads in transportation activities: distance traveled, working time spent, and load carried (see Sivaramkumar et al. [19]). Balancing these three elements is beneficial for a company. It provides uniformity of vehicle maintenance, improves employee satisfaction, and also can reduce the possibility of road accident, which may be caused by driver fatigue from work overload (see Morrow and Crum [14]). Nevertheless, the company still has to consider the financial aspect to obtain the minimum system-wide cost.

Unfortunately, previous studies in LRP have not sufficiently addressed the importance of workload balance. Most research merely considers balancing the distance traveled by vehicles. Sivaramkumar et al. [19] noted that this approach is only sufficient when time aspect is not considered. However, in urban transport situation, this approach is unrealistic due to several factors such as congestion, which makes the travel times of two routes with the same length may differ.

This study intends to present a mathematical model to solve the multi-objective LRP with capacitated vehicles which addresses the previously mentioned issue. The proposed model considers two objective functions: (1) to minimize the total cost associated with facility, vehicle, and distribution, and (2) to balance the workload in distribution activities, by minimizing the difference between a route with longest trip time and a route with shortest trip time. Trip time itself is calculated from travel time and service time on each customer. The main purposes of this model are to obtain the optimal location of a distribution center (DC), number of vehicles established, and delivery routes which satisfy both objectives.

LRP has nature as an NP-hard problem that as the problem size increases, getting the exact solution becomes infeasible. Therefore, this study proposes a popular metaheuristic, non-dominated sorting genetic algorithm-II (NSGA-II) (see Deb et al. [5]) to solve the aforementioned model. The computational experiments are done to several problem scenarios with different numbers of customers and depots.

The main contribution of this study is to propose a novel mathematical model which considers total time balance as an objective function to attain workload balance condition. This study also proposes the usage of NSGA-II, and compares it with two alternative metaheuristics, namely Pareto archived evolution strategy (PAES) (see Knowles and Corne [9]) and Pareto envelope-based selection algorithm (PESA) (see Corne et al. [4]) which contributes to reveal the feasibility of each technique to solve this model.

The remainder of this paper is organized in the following way. Section 2 presents the literature review of related works. Section 3 describes the problem and the proposed model. Section 4 discusses the computational experiments. Finally, Section 5 presents the conclusion and suggestions for future research. In addition, this study uses the definition of LRP as suggested by Drexl and Schneider [6], as "a mathematical optimization problem where at least the following two types of decisions must be made interdependently: (1) which facilities out of a finite or infinite set of potential ones should be used (for a certain purpose)?, (2) which vehicle routes should be built, i.e., which customer clusters should be formed and in which sequence should the customers in each clusters be visited by a vehicle from a given fleet (to perform a certain service)?".

2. Literature Review

The amount of literatures of LRP has been increasing continuously. Prodhon and Prins [18] for example, noted that at least 72 papers about LRP were published between 2007 and 2013. This number was considerably higher than the previous period which was noted by Nagy and Salhi [16].

Various studies have attempted to engage multiple objective functions into the mathematical model of LRP. Unfortunately, few authors have addressed any objectives related to workload balance. As discussed in Section 1, providing workload balance is an important task in transportation planning. This literature review only finds at least five works which tackle this issue in LRP.

There are four papers which consider minimizing the difference of distance traveled of each vehicle (route balance). Martínez-Salazar et al. [12] and developed a model to minimize distribution costs and distance differences in a multi-objective transportation location-routing problem (TLRP), which basically is an extension of two-stage LRP. Separately, Martínez-Salazar et al. [13] also proposed the same model but solved using different metaheuristics. Recently, Golmohammadi et al. [7] and Hadian et al. [8] also considered similar objectives to propose the usage of multi-objective imperialist competitive algorithm (MOICA) into LRP.

Another literature considers workload balance with different measurement. Lin and Kwok [11] attempted to balance the working time in LRP. The authors considered to balance the workload between each vehicle in case of multiple uses of vehicles. Working time, imbalance, load imbalance, and total cost are minimized. Unfortunately, Lin and Kwok [11] did not provide any information regarding the mathematical model that they used.

Balancing the working time in supply network provides more benefits for drivers and companies. This approach is commonly used in VRPTW and previously Sivaramkumar et al. [19] have proven that minimizing the difference of time traveled of each vehicle (total time balance) gives a better evenness than route balance. Based on our literature review, Lin and Kwok [11] are the only authors who already considered this approach in LRP. Thus, it is concluded that the implementation of total time balance objective in LRP is still widely expandable and this study is aimed to fill this gap.

3. Methodology

This study proposes a multi-objective LRP model with capacitated vehicles to address the workload balance issue. In order to solve this model, this study proposes to use a popular metaheuristic, namely NSGA-II.

Several characteristics and assumptions are considered in the proposed model:

(1) The capacity of each DC is unlimited.

(2) Only one DC will be opened.

(3) Homogeneous vehicle.

(4) The amount of demand for each customer, the distance, and travel time between nodes are assumed to be deterministic.

(5) The travel times between nodes are not symmetrical.

(6) The service time in each customer is assumed to be deterministic and uniform.

3.1. Mathematical model

The problem described above can be formulated as the following mathematical program:

Sets and parameters

I	Number of potential DC locations.
J	Number of customers.
K	Number of vehicles established.
L	Number of all nodes $(I \cup J)$.
G_i	Fixed cost of opening DC i.
F	Fixed cost of establishing a vehicle.
P	Variable cost per distance unit.
Q	Capacity of vehicle.
d_j	Demand of customer j.
c_{ij}	Distance from i to j.
s	Service time.
t_{ij}	Travel time from i to j.
TT_k	Total trip time at route k.

Decision variables

$$X_i = \begin{cases} 1, & \text{if DC } i \text{ is opened,} \\ 0, & \text{otherwise.} \end{cases}$$

$$Y_{ijk} = \begin{cases} 1, & \text{if a vehicle goes from } i \text{ to } j \text{ on route } k, \\ 0, & \text{otherwise.} \end{cases}$$

$$Z_ij = \begin{cases} 1, & \text{if the demand of customer } j \text{ is served by DC } i, \\ 0, & \text{otherwise.} \end{cases}$$

U_{lk} = auxiliary variable for sub-tour elimination constraints on route k.

Objective Functions

$$\min\ f_1 = \sum_{i \in I} G_i X_i + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} Y_{ijk} F + \sum_{i \in L} \sum_{j \in L} \sum_{k \in K} Y_{ijk} c_{ij} P, \tag{3.1}$$

$$\min\ f_2 = (TT)_{\max} - (TT)_{\min}. \tag{3.2}$$

<u>*Constraints*</u>

$$\sum_{i\in I} X_i = 1 \tag{3.3}$$

$$\sum_{i\in L}\sum_{k\in K} Y_{ijk} = 1 \qquad \forall\, j \in J \tag{3.4}$$

$$\sum_{i\in I}\sum_{j\in J} Y_{ijk} \leq 1 \qquad \forall\, k \in K \tag{3.5}$$

$$\sum_{j\in L} Y_{jik} - \sum_{j\in L} Y_{ijk} = 0 \qquad \forall\, i \in L,\ \forall\, k \in K \tag{3.6}$$

$$\sum_{l\in J} Y_{ilk} - \sum_{l\in L} Y_{ljk} \leq 1 + Z_{ljk} \qquad \forall\, i \in I,\ \forall\, j \in J,\ \forall\, k \in K,\ \forall\, l \neq j \tag{3.7}$$

$$U_{lk} - U_{jk} + JY_{ljk} \leq J - 1 \qquad \forall\, l, j \in L,\ \forall\, k \in K \tag{3.8}$$

$$0 \leq U_{lk} \leq J - 1 \qquad \forall\, l \in J,\ \forall\, k \in K \tag{3.9}$$

$$\sum_{i\in I}\sum_{j\in J} Y_{ijk} d_j \leq Q \qquad \forall\, k \in K \tag{3.10}$$

$$TT_k = \sum_{i\in L}\sum_{j\in L} Y_{ijk}\tilde{t}_{ij} + \sum_{i\in L}\sum_{j\in J} Y_{ijk} s \qquad \forall\, k \in K \tag{3.11}$$

$$(TT)_{\max} = \max[TT_k] \qquad \forall\, k \in K \tag{3.12}$$

$$(TT)_{\min} = \min[TT_k] \qquad \forall\, k \in K \tag{3.13}$$

This model considers two objective functions. The objective function (3.1) is an economical objective which minimizes the total cost of the network, including the cost of establishing DC, the cost of establishing vehicles, and the cost of routing from DC to customers. The objective function (3.2) minimizes the difference between a route with longest trip time and a route with shortest trip time. The latter objective is called as total time balance objective. Furthermore, trip time is calculated from travel time and service time on each customer.

Several constraints are also considered. Constraint (3.3) ensures that only one DC is opened from all potential locations. Constraint (3.4) ensures that each customer will only be visited once and only by one vehicle. Constraint (3.5) guarantees that each vehicle is only routed from one DC. Constraint (3.6) is flow conversation constraint which assures that each vehicle will depart from a customer node after entering it. The relationship between DC and customers is guaranteed by Constraint (3.7), while Constraints (3.8) and (3.9) are the sub-tour elimination constraints. Constraint (3.10) limits the vehicle in terms of capacity. Constraint (3.11) defines the total trip time of each vehicle route. Lastly, constraints (3.12) and (3.13) define the longest and shortest route in terms of total trip time.

3.2. Solution representation

The solution representation in this study is shown in Figure 1. It contains a string of numbers which consists of one bit for the selected DC location, J bits for a permutation of customers, and a bit for the amount of vehicle used. The initial solution is generated randomly to select one of the potential DC location and a permutation of customer nodes. After that, the amount of vehicle is calculated and the vehicle routes are recorded.

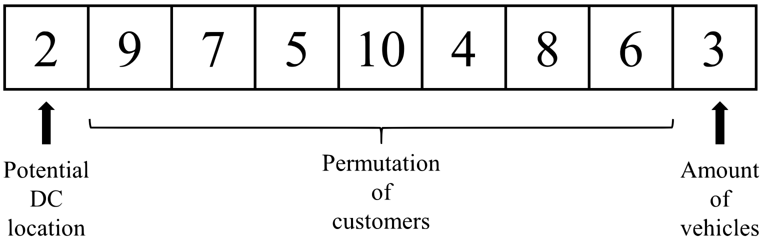

Figure 1: Solution representation.

3.3. NSGA-II.

NSGA-II generates a set of Pareto optimal solutions which is suitable for dealing with a trade-off situation. This method creates offspring based on crossover and mutation mechanism to avoid being trapped in local solution. A binary tournament selection is implemented to create a mating pool for parents, then the algorithm combines them using a single-point crossover technique. Subsequently, a combination of randomized mutation for DC selection and swap mutation technique for customer nodes are implemented. The general procedures of NSGA-II are described in Table 1, while Figure 2 and Figure 3 respectively present the mechanisms of the single-point crossover and the mutation techniques.

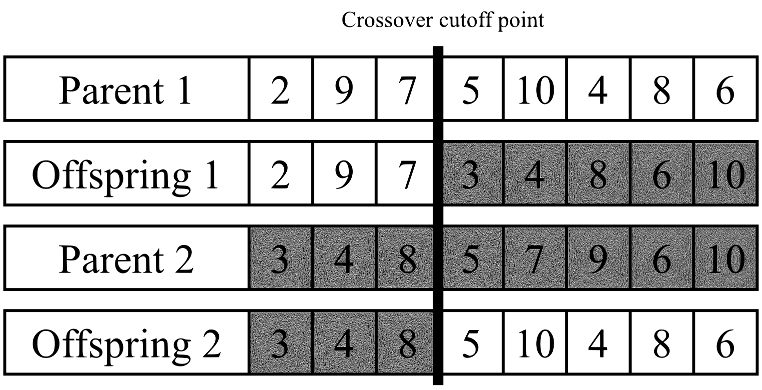

Figure 2: Single-point crossover.

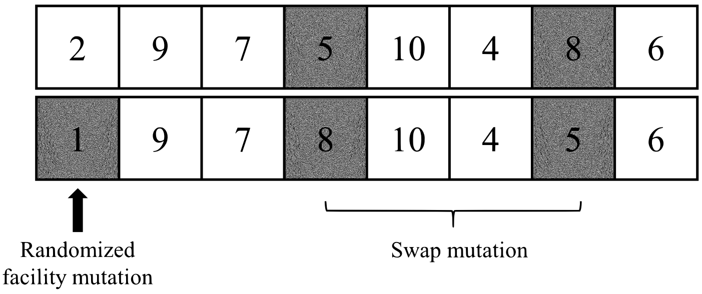

Figure 3: Swap mutation.

Table 1: General Procedures of NSGA-II.

Step 1: Inputs
(1) Population size, N (2) Number of generations, g (3) Probability of crossover, P_c (4) Probability of mutation, P_m
Step 2: Initialization
(1) Initialize population P with size N (2) Generate the initial solution P_0
Step 3: Evaluation
(1) Calculate fitness value for each chromosome. (2) Fast non-dominated sorting, assign rank for P_0 based on Pareto dominance.
Step 4: Generate Child Population
(1) Binary tournament selection. (2) Crossover and mutation, generate offspring Q_t.
Step 5: Main loop

While stopping criterion ($t > g$) is not met, do :
    (1) Calculate fitness value for each chromosome.
    (2) Merge the current population and offspring, $R_t = P_t \cup Q_t$.
    (3) Fast non-dominated sorting to assign rank for R_t.
    (4) Crowding distance assignment for R_t.
    (5) Sort the R_t based on the frontier rank and crowding distance $(F_i, \prec_n)$.
    (6) $P_{t+1} = \varnothing$, $i = 1$
      While $P_{t+1} + F_i \leq N$, do :
        (7) $P_{t+1} = P_{t+1} \cup F_i$
        (8) $i = i + 1$
    (9) Binary tournament selection.
    (10) Crossover and mutation, generate Q_t.

Return Q_g as the final solution.

4. Computational Experiments

In order to evaluate the effectiveness of the proposed algorithm, this study implements NSGA-II into three classes of data sets: small, medium, and large. These instances are modified from three well-known instances of capacitated LRP (see Christofides and Eilon [2], Perl [17], and Tuzun and Burke [20]) to incorporate the travel time measurement between nodes. The details of each data set are shown in Table 2. The computational experiments are executed on a PC with an Intel Core I7-3770 CPU (3.40 GHz) and 16 GB memory. All algorithms are coded in MATLAB R2018b.

4.1. Parameter settings

The quality of a metaheuristic algorithm is highly influenced by the value the parameters. Thus, it is important to find the optimal settings for each algorithm. Firstly, four parameters need to be calibrated in NSGA-II: population size (N), number of generations (g), probability of crossover (P_c), and probability of mutation (P_m). Secondly, three parameters are considered in PAES: number of generations (g), maximum size of archive (*Refpop*), and the number of grids (*Grids*). Lastly, there are four parameters in PESA: number of generations (g), size of internal population (P_I), probability of crossover (P_c), and the number of grids (*Grids*).

This study deploys a three-level Taguchi procedure and utilizes L9 orthogonal array design with smaller-the-better response. This approach is previously used by Mousavi et al. [15] to optimize and compare the performance of metaheuristics. Table 3 shows the levels of each parameter from previous pilot study and the optimal settings for each algorithm.

Table 2: Data sets.

Instances	Class	Customer - DC	Travel Time
Mod-Perl83-A	Small	6 - 2	Asymmetric
Mod-Perl83-B	Small	8 - 2	Asymmetric
Mod-Perl83-C	Small	10 - 2	Asymmetric
Mod-Perl83-D	Small	12 - 2	Asymmetric
Mod-Christofides69	Medium	50 - 5	Asymmetric
Mod-P111112	Large	100 - 10	Asymmetric

4.2. Small data set

For small data set, this study compares the performance of NSGA-II with the exact solution results. The exact solutions are obtained using LINGO 11.0 solver. Four different instances are considered, namely Mod-Perl83-A, Mod-Perl83-B, Mod-Perl83-C, and Mod-Perl83-D (see Perl [17]). Each method is executed only once for each instance and the computational results are shown in Table 4 and Table 5. However, since NSGA-II

Table 3: Parameter ranges for each algorithm.

Algorithm	Parameters	Range	Levels			Optimal Value
			1	2	3	
NSGA-II	g	100-300	100	200	300	100
	N	100-200	100	150	200	100
	P_c	0.7-0.9	0.7	0.8	0.9	0.8
	P_m	0.1-0.3	0.1	0.2	0.3	0.3
PAES	g	100-200	100	150	200	100
	Refpop	5-15	5	10	15	10
	Grids	3-9	3	6	9	6
PESA	g	100-300	100	200	300	300
	P_I	100-200	100	150	200	100
	P_c	0.7-0.9	0.7	0.8	0.9	0.9
	Grids	3-9	3	6	9	3

returns multiple solutions which are non-dominating each other, the comparison is done only to the average value of Pareto solutions.

The results show that NSGA-II is competitive for solving small-sized problem. For Mod-Perl83-A instance, LINGO 11.0 is able to obtain the global optimum point, while NSGA-II provides Pareto solutions near the global optimum point. Furthermore, the results also show the limitation of the exact solution. As the size of the problem increases, LINGO 11.0 can only obtain a local optimum point after two hours running time. Meanwhile, NSGA-II is able to acquire competitive values in a relatively short time.

4.3. Medium and large data sets

In this stage, this study compares NSGA-II with two alternative algorithms for the medium and large data sets. This approach is implemented because LINGO 11.0 is not able to find solutions in both data sets. The goal of this stage is to see whether our proposed algorithm is competitive for solving the model in medium and large-sized problems.

The performance of a multi-objective optimization algorithm is indicated by four key aspects. Li and Yao [10] detailed those aspects as cardinality, spread, convergence, and uniformity of the solutions. Thus, this study compares NSGA-II, PAES and PESA in terms of five quality indicators: (1) error ratio (ER) (see Mousavi et al. [15]) to capture the ratio of Pareto solutions to the population, (2) modified maximum spread (MS') (see Adra and Fleming [1]) to capture the diversity of solutions, (3) mean-ideal distance (MID) (see Mousavi et al. [15]) to measure the convergence of solutions into ideal point (0,0), (4) Spacing metric (SM) (see Collette and Siarry [3]) to measure the uniformity between solutions, and (5) CPU time (CPU) required to run the algorithm.

An algorithm performs better if it has a lower ER, higher MS', lower MID, lower SM, and lower CPU.

To provide a better comparison, all algorithms are executed with ten replications for each data set. Furthermore, the results are analyzed using one-way analysis of variance (ANOVA) with a 95% confidence level and Tukey post hoc test. The statistical tests are executed using Minitab 18.0 software.

Table 4: Computational results of LINGO 11.0 for the small data set.

Data Set	$CPU(s)$	Objective Value		Note
		Total Cost	Time Difference	
Mod-Perl83-A	290	234.473	0.008	Global Optimum
Mod-Perl83-B	7200	299.712	1.396	Local Optimum
Mod-Perl83-C	7200	366.751	1.436	Local Optimum
Mod-Perl83-D	7200	369.352	0.029	Local Optimum

Table 5: Computational results of NSGA-II for the small data set.

Data Set	$CPU(s)$	Objective Value		Note
		Total Cost	Time Difference	
Mod-Perl83-A	12.078	236.075	0.001	Average of Pareto solutions
Mod-Perl83-B	11.291	303.002	1.337	Average of Pareto solutions
Mod-Perl83-C	10.644	368.464	2.787	Average of Pareto solutions
Mod-Perl83-D	12.040	370.474	0.005	Average of Pareto solutions

4.3.1. Medium data set

The medium-sized data is named Mod-Christofides69 (see Christofides and Eilon [2]) which consists of 50 nodes of customers and five potential DC locations. The experiment demonstrates that NSGA-II is competitive to solve medium-sized data set. The computational results of NSGA-II, PAES, and PESA for medium-sized data sets are shown in Table 6, Table 7, and Table 8, respectively. Five one-way ANOVA tests are deployed to see whether there is a significant performance difference. Table 9 shows the results of one-way ANOVA tests at a 95% confidence level, it indicates that there are significant differences between NSGA-II, PAES, and PESA in all indicators.

Thus, we plot the boxplot and apply Tukey post hoc test for further analysis. Figures 4-8 show the boxplots of each indicator and Table 10 presents the result of Tukey test. The boxplots and Tukey test prove that NSGA-II is superior in three quality indicators: ER, MS', and MID. This concludes that NSGA-II produces better amount of solutions per population with better spread and convergence, which implies that the decision maker can consider more and better solutions to solve the model with NSGA-II. However, it is interesting to note that NSGA-II achieves the worst solutions uniformity. Moreover, Figure 9 presents the best Pareto solutions of each algorithm in the medium data set.

Table 6: Computational results of NSGA-II for the medium data set.

Iterations	ER	MS'	MID	SM	$CPU(s)$
1	0.90	15.372	24.357	0.029	22.127
2	0.90	39.117	23.328	0.087	23.219
3	0.96	25.027	23.416	0.038	23.905
4	0.91	18.910	23.067	0.068	23.758
5	0.90	37.972	23.316	0.104	23.359
6	0.94	26.375	23.175	0.071	22.494
7	0.87	24.564	23.165	0.116	22.823
8	0.92	31.667	23.342	0.070	22.120
9	0.92	25.735	23.115	0.050	25.531
10	0.88	39.809	23.348	0.108	24.502
Average	0.91	28.455	23.363	0.074	23.384
St. dev.	0.027	8.461	0.368	0.030	1.084

Table 7: Computational results of PAES for the medium data set.

Iterations	ER	MS'	MID	SM	$CPU(s)$
1	0.95	15.371	23.913	0.038	4.218
2	0.96	18.338	23.740	0.033	3.445
3	0.95	20.063	23.968	0.024	4.339
4	0.96	14.427	24.044	0.025	3.017
5	0.97	7.468	23.819	0.020	3.042
6	0.94	9.799	24.087	0.025	3.916
7	0.97	3.637	23.969	0.022	4.051
8	0.97	16.683	24.029	0.020	3.099
9	0.95	23.610	23.682	0.041	4.528
10	0.95	15.603	24.182	0.020	4.478
Average	0.96	14.500	23.943	0.027	3.813
St. dev.	0.011	6.009	0.157	0.008	0.609

4.3.2. Large data set

The data considered in the last experiment is named Mod-P111112 (see Tuzun and Burke [20]) which consists of 100 nodes of customers and ten potential DC locations. The experiments demonstrate that NSGA-II is competitive to solve large-sized data set.

Table 8: Computational results of PESA for the medium data set.

Iterations	ER	MS'	MID	SM	$CPU(s)$
1	0.92	17.953	23.649	0.075	57.227
2	0.97	6.240	23.483	0.031	61.294
3	0.94	9.224	23.411	0.035	58.645
4	0.94	29.451	23.800	0.050	59.591
5	0.97	1.503	23.443	0.020	59.979
6	0.96	8.882	23.679	0.030	57.671
7	0.95	3.436	23.268	0.051	60.503
8	0.94	11.819	23.365	0.032	59.617
9	0.95	12.081	23.386	0.028	59.005
10	0.97	5.161	23.663	0.039	58.379
Average	0.95	10.575	23.515	0.039	59.191
St. dev.	0.017	8.169	0.172	0.016	1.259

Table 9: ANOVA results for the medium data set.

Indicators	F-Value	P-Value
ER	17.85	0
MS'	15.19	0
MID	14.33	0
SM	15.17	0
CPU	7553.78	0

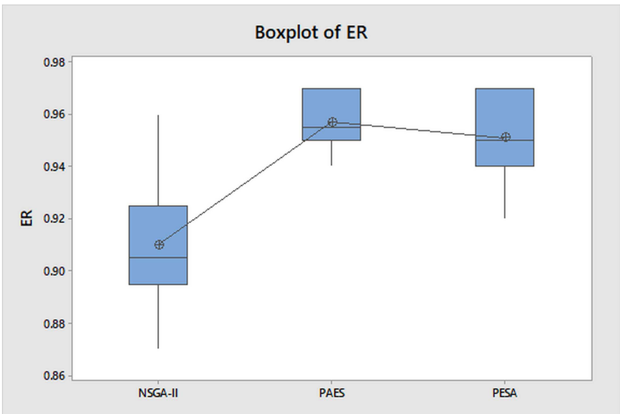

Figure 4: The boxplot of ER for the medium data set.

The computational results of NSGA-II, PAES, and PESA for large-sized data sets are shown in Table 11, Table 12, and Table 13. Moreover, Table 14 shows the results of

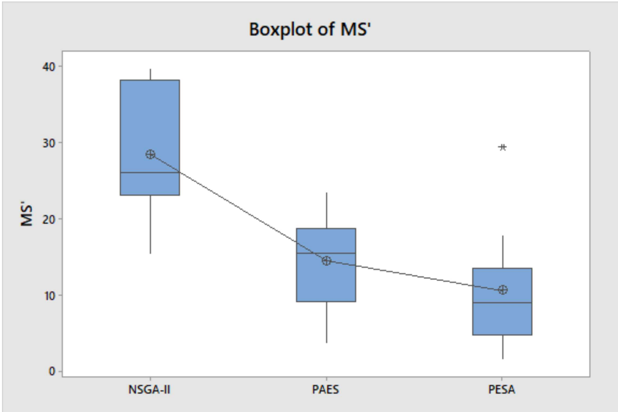

Figure 5: The boxplot of MS' for the medium data set.

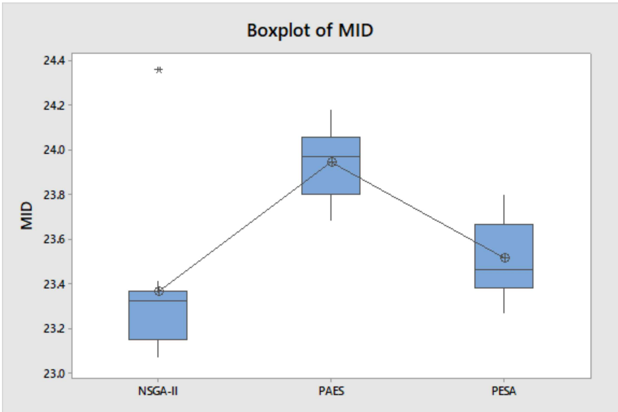

Figure 6: The boxplot of MID for the medium data set.

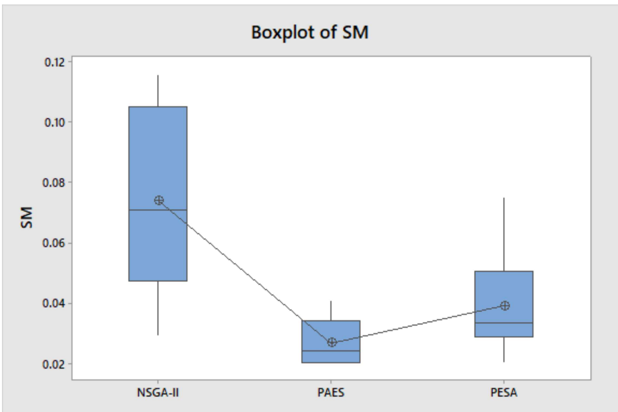

Figure 7: The boxplot of SM for the medium data set.

one-way ANOVA tests at a 95% confidence level, it indicates that there are significant differences between NSGA-II, PAES, and PESA in all indicators.

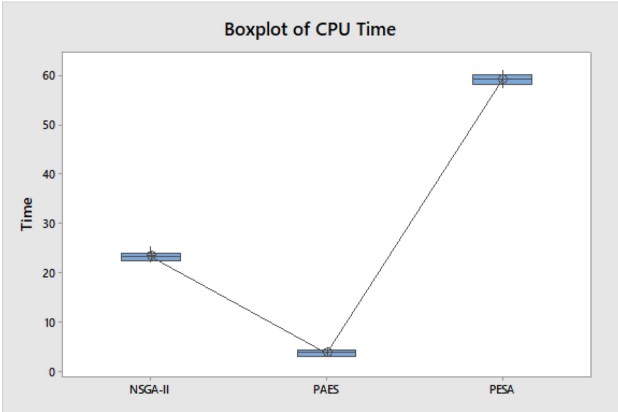

Figure 8: The boxplot of CPU time for the medium data set.

Table 10: Tukey post hoc test for the medium data set.

Indicators	Factors	N	Mean	Grouping		
ER	PESA	10	0.957	A		
	PAES	10	0.951	A		
	NSGA-II	10	0.91		B	
MS'	NSGA-II	10	28.45	A		
	PAES	10	15		B	
	PESA	10	10.57		B	
MID	PAES	10	23.943	A		
	PESA	10	23.515		B	
	NSGA-II	10	23.363		B	
SM	NSGA-II	10	0.0741	A		
	PESA	10	0.0391		B	
	PAES	10	0.0269		B	
CPU	PESA	10	59.191	A		
	NSGA-II	10	23.384		B	
	PAES	10	3.813			C

Note: Two factors with different grouping letters indicate that they are significantly different.

Furthermore, the boxplot and Tukey post hoc test are deployed. Figures 10, 11, 12, 13, and 14 show the boxplots of each indicator and Table 15 presents the result of Tukey test. The boxplots and Tukey test prove that NSGA-II is also superior in three quality indicators: ER, MS', and MID. This means that NSGA-II produces better amount of solutions per population with better solutions' diversity and convergence, which implies that the decision maker can consider more and better solutions to solve the model with NSGA-II. Also, it is interesting to note that PAES obtains competitive solutions in a

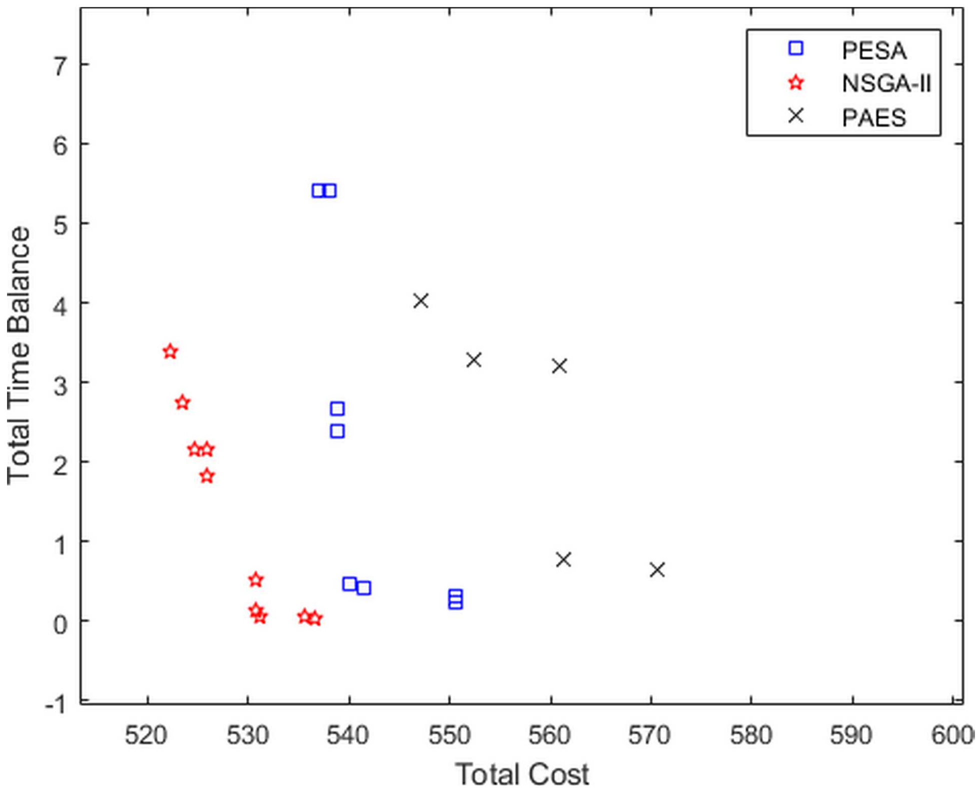

Figure 9: The best solutions from ten iterations for the medium data set.

relatively short time even for a large data set. Moreover, Figure 15 presents the best Pareto solutions of each algorithm in the large data set.

Table 11: Performance of NSGA-II with large data set.

Iterations	ER	MS'	MID	SM	$CPU(s)$
1	0.86	121.720	24.514	0.135	57.423
2	0.86	76.670	24.332	0.058	58.045
3	0.90	72.562	24.634	0.078	57.163
4	0.85	89.125	25.200	0.135	56.852
5	0.88	102.718	24.565	0.098	56.393
6	0.86	72.855	24.391	0.062	60.987
7	0.88	47.534	24.375	0.093	57.628
8	0.90	33.600	24.073	0.075	61.236
9	0.88	101.157	24.443	0.048	56.188
10	0.87	46.014	24.227	0.100	59.551
Average	0.87	76.395	24.475	0.088	58.147
St. dev.	0.017	28.160	0.302	0.030	1.824

Table 12: Performance of PAES with large data set.

Iterations	ER	MS'	MID	SM	$CPU(s)$
1	0.90	52.222	26.958	0.041	5.789
2	0.93	37.143	26.627	0.053	4.297
3	0.94	61.188	26.338	0.038	5.510
4	0.91	32.898	27.017	0.071	5.854
5	0.91	36.499	26.661	0.052	4.917
6	0.92	43.813	25.884	0.053	5.778
7	0.90	56.302	26.775	0.050	5.806
8	0.91	36.527	26.510	0.040	5.763
9	0.90	22.369	26.106	0.053	6.182
10	0.93	51.084	26.904	0.056	5.972
Average	0.92	43.005	26.578	0.051	5.587
St. dev.	0.014	12.041	0.374	0.010	0.563

Table 13: Performance of PESA with large data set.

Iterations	ER	MS'	MID	SM	$CPU(s)$
1	0.96	11.010	24.905	0.031	186.697
2	0.98	14.944	25.180	0.020	182.584
3	0.98	6.020	25.969	0.020	195.976
4	0.96	1.507	24.924	0.037	186.178
5	0.96	32.132	25.281	0.039	184.581
6	0.95	27.896	25.351	0.037	191.800
7	0.96	88.000	25.482	0.038	173.004
8	0.94	60.546	25.469	0.021	188.063
9	0.97	11.232	25.444	0.020	189.250
10	0.93	53.040	25.832	0.032	175.204
Average	0.96	30.633	25.384	0.030	185.334
St. dev.	0.016	28.204	0.343	0.008	7.017

Table 14: ANOVA results for the large data set.

Indicators	F-Value	P-Value
ER	71.96	0
MS'	9.70	0.001
MID	95.673	0
SM	25.27	0
CPU	4844.99	0

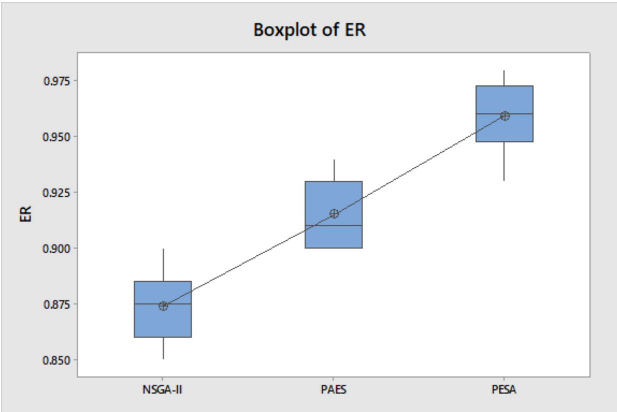

Figure 10: The boxplot of ER for the large data set.

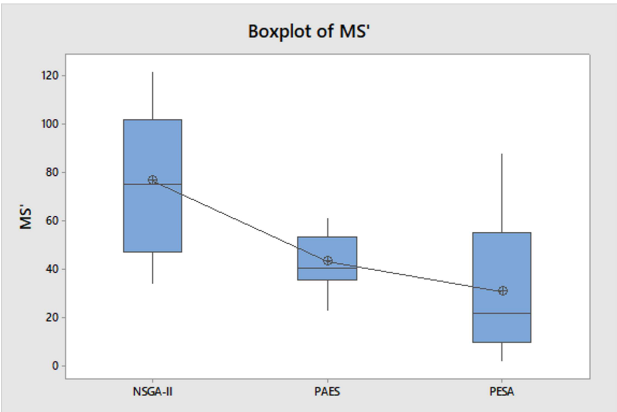

Figure 11: The boxplot of MS' for the large data set.

5. Conclusions

This study intends to present a mathematical model to solve the multi-objective LRP with capacitated vehicles which addresses workload balance issue. The proposed model considers two objective functions: (1) to minimize the total cost associated with facility, vehicle, and distribution, and (2) to balance the workload in distribution activities,

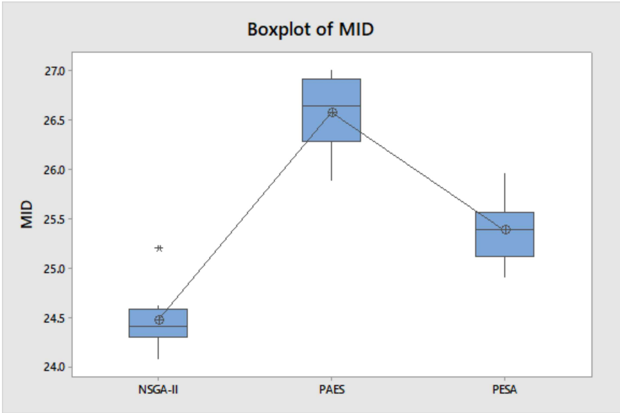

Figure 12: The boxplot of MID for the large data set.

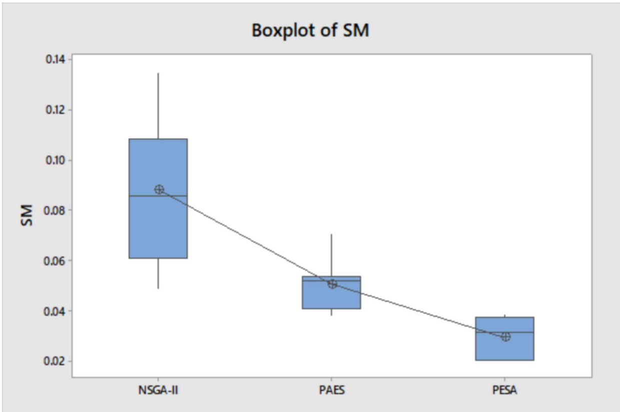

Figure 13: The boxplot of SM for the large data set.

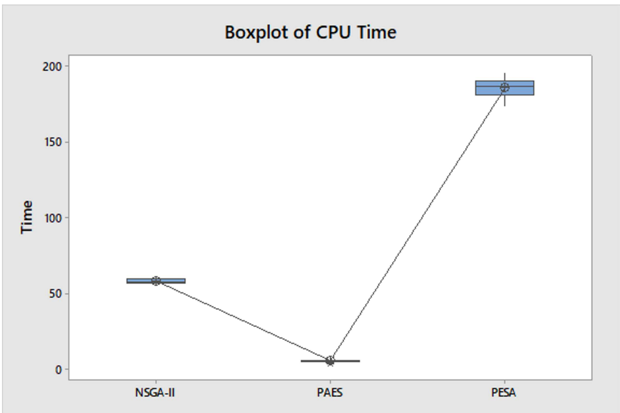

Figure 14: The boxplot of CPU time for the large data set.

by minimizing the difference between a route with longest trip time and a route with shortest trip time. The main purposes of this model are to obtain the optimal location

Table 15: Tukey post hoc test for the large data set.

Indicators	Factors	N	Mean	Grouping		
ER	PESA	10	0.959	A		
	PAES	10	0.915		B	
	NSGA-II	10	0.874			C
MS'	NSGA-II	10	76.400	A		
	PAES	10	43		B	
	PESA	10	30.630		B	
MID	PAES	10	26.578	A		
	PESA	10	25.384		B	
	NSGA-II	10	24.475			C
SM	NSGA-II	10	0.088	A		
	PESA	10	0.051		B	
	PAES	10	0.030			C
CPU	PESA	10	185.330	A		
	NSGA-II	10	58.147		B	
	PAES	10	5.587			C

Note: Two factors with different grouping letters indicate that they are significantly different.

of a DC, number of vehicles established, and delivery routes which satisfy both of these two objectives. Furthermore, the details of our proposed model are described in Section 3.1.

This study also proposes NSGA-II to solve the proposed model. In order to evaluate the effectiveness of the proposed algorithm, this study compares it with LINGO 11.0 solver for small-sized data sets and two alternative metaheuristics, namely PAES and PESA for medium-sized and large-sized data sets. The experimental results show that the proposed algorithm performs well in terms of solution quality and computational time. For small data sets, NSGA-II is able to obtain competitive solutions in relatively low computational time, compared to the exact solution from LINGO 11.0. For medium and large data sets, NSGA-II is proven to be superior to PAES and PESA in terms of the cardinality, spread, and convergence of the solutions. This result implies that NSGA-II is able to provide the decision maker with more solutions that have better diversity and solution quality, with respect to the cost minimization and total time balance objectives.

Lastly, this study provides several future research directions. Some larger cases of LRP with capacitated DCs can be considered, so the relationship between the cost of opening DC and vehicles can be further explained. Also, the time window for each customer is an interesting aspect to be analyzed. Furthermore, the uncertainty aspects of transportation activity also can be modeled using stochastic or fuzzy approach to capture the volatility of travel time.

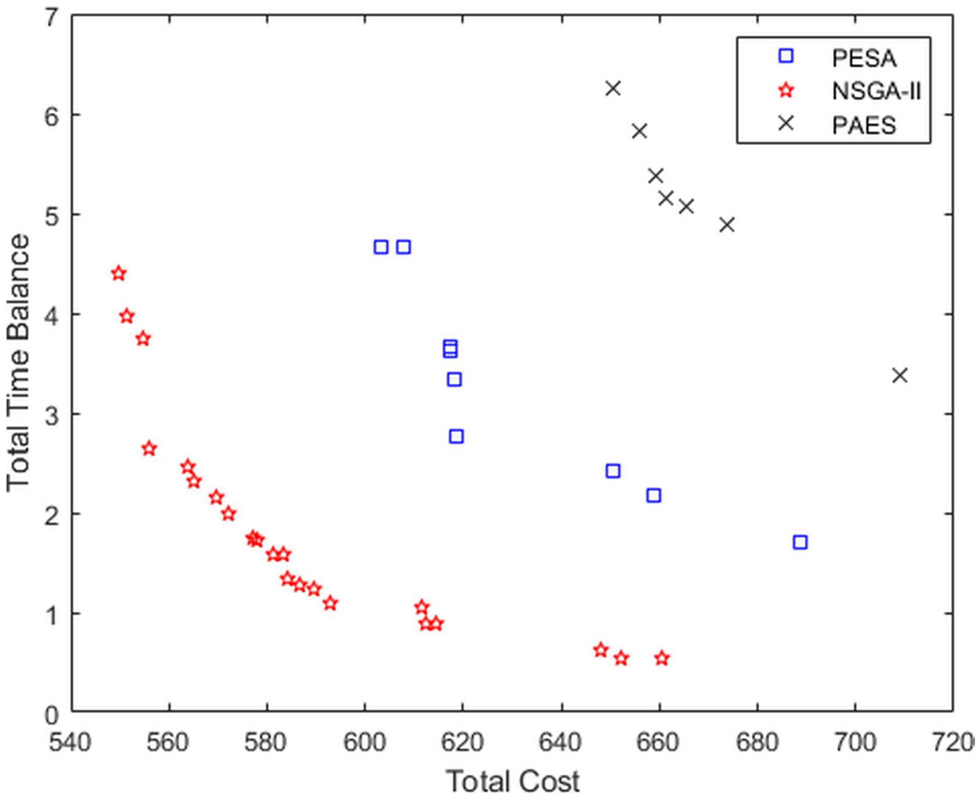

Figure 15: The best solutions from ten iterations for the large data set.

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