Joint Pricing and Inventory Management with Regular and Expedited Supplies under Reference Price Effects

Yuan Li[1,2] *and Yu-Mei Hou*[1]

[1]Yanshan University and [2]Inner Mongolia University for The Nationalities

Abstract

Reference price effects, as an important factor affecting customers' purchase decision for a certain commodity, has an significant impact on the firms' pricing and inventory strategies. Therefore, how to determine an appropriate sales pricing and order flexibility strategy to maximize firms' profit is an essential task. A single-item, periodic-review finite horizon joint pricing and inventory system with regular and expedited supply sources under reference price effects is investigated, where demand in consecutive periods are independent as well as price and reference price sensitive random variables. The optimal pricing and ordering strategies, and the impact of reference price effects on these optimal decisions are studied. These results generalize those in Zhou and Chao (2014) in Production and Operations Management, Vol. 23, 65-80 to the reference price effects. Moreover, the operational impact of adding reference price effects is analyzed by comparing with their model. Finally, Numerical experiments characterize the qualitative properties of the optimal strategies and corresponding optimal profit values under reference price effects.

Keywords: Dynamic programming, pricing and inventory, supply flexibility, lead time, reference price effects.

1. Introduction

Reference price was first derived from the adaptation level (see Helson and Bevan [17]). Later, prospects theory (see Kahneman and Tversky [22]) and behavioral sciences (see Kalyanaram and Winer [23]) systematically elaborated on the reference price, and they indicated that customers will remember past prices with repeated transactions and develop price expectations for commodities. This expectation, captured by the reference price, acts as a benchmark against which customers compare the price of a commodity. If the current sales price is lower (higher) than the reference price, customers see it as a gain (loss), and hence are more likely (less inclined) to make the purchase. This phenomenon is usually called the reference price effects. Customers are called loss averse (loss neutral) if their demand is more (as) responsive to customers' perceived losses than (as) their perceived gains. Otherwise, they are called loss/gain seeking. Firms in many

industries, such as electronic, clothing and other tidal commodities, will fully consider this effects when making pricing strategies (see Mathies and Gudergan [26]). In practice, pricing strategy is inseparable from the consideration of inventory control decisions. Therefore, in making decisions, many firms not only consider the impact of reference price effects on pricing strategies, but also the inventory strategies simultaneously, such as Amazon, Dell and Wal-Mart (see Byrnes [2] and Feng [7]). In addition, due to the existence of customers' reference price, the flexibility of the firms' ordering strategy becomes particularly important in respond to customers' demand. Many suppliers will also provide expediting replenishment opportunities to firms. For example, the eHub system launched by Cisco in 2001 (see Grosvenor and Austin [14]). It is a trading e-marketplace that provides a platform for the planning and executing tasks across the firm's extended manufacturing supply chain. By eHub, Cisco connects with its suppliers to build up a flexible/agile supply channel, where Cisco is allowed to return the excess stock and to place expediting orders. In this way, Cisco can reduce the waste in inventory and increase the speed to response to customers' needs. From academic and practical perspectives, it is interesting and necessary to investigate the joint pricing and inventory management with supply flexibility under reference price effects.

This paper considers a coordination pricing and inventory management with regular and expedited supplies under reference price effects. The purpose of this paper is to understand how reference price effects impacts the optimal dynamic pricing and inventory replenishment policies for each supply in each period so that the total expected discounted profit is maximized. The related literature with our work for coordination pricing and inventory control involves two streams: reference price effects and supply flexibility. We review the related areas below.

The first stream of research considers the reference price effects while absence of supply flexibility. Researches on reference price effects mainly focus on pricing strategy. This aspect of research began in the 1990s. Krishnamurthi et al. [21] study the impact of reference price effects on brand selection and purchase quantity, and point out that customers have the characteristics of brand loyalty under symmetrical reference prices. Greenleaf [13] analyzes the firm's pricing strategy with reference price effects and explains how the reference price effects impacts the promotion decision of a firm during one sales period. The author shows that the firm can increase its profit by considering the reference price effects. Some recent works explore how pricing strategies should account for the reference price effects. See Kopalle et al. [20], Fibich et al. [8, 9], Popescu and Wu [28], Nasiry and Popescu [27], Chen et al. [4], Hu et al. [19] and the references therein. Arslan and Kachani [1] and Mazumdar et al. [25] provide reviews of dynamic pricing model with reference price effects. However, to our best knowledge, only a few papers have integrated the reference price effects into the pricing and inventory control model. This line of research started with Gimpl-Heersink [11], who proved the optimality of the base-stock-list-price for the single-period and two-period models when the customers are loss neutral. However, the optimality of the base-stock-list-price is more stricter for the multi-period setting. Urban [35] analyzes a single-period joint pricing and inventory model with symmetric and asymmetric reference price effects and shows that the consideration of

the reference price has a substantial impact on the firm's profitability. Zhang [37] uses a class of transformation techniques to prove the optimality of the base-stock-list-price policy, even if the single-period profit function is nonconcave. Taudes and Rudloff [33] provide an application of the two-period model from Gimpl-Heersink [11] to electronic commodities. Güler et al. [15] extend the model of Gimpl-Heersink [11] to more general concave demand functions, they deal with the non-concavity of revenue function by applying the transformation technique proposed by Zhang [37] and the inverse demand function. The optimality of the state-dependent order-up-to strategy is proved for the transformed concave revenue function model. Güler et al. [16] use the safety stock as a decision variable to characterize the steady state solution to the problem when the planning horizon is infinite. Chen et al. [5] introduce a new concave transform technique to ensure that the profit function is concave by using the preservation property of supermodularity in parameter optimization problems with the nonlattice structure proposed by Chen et al. [3], and then prove the optimality of the base-stock-list-price strategy. Other related work on this stream of research, interested readers may refer to the review by Ren and Huang [30].

The second stream of research considers the supply flexibility while absence of reference price effects. The earliest studies on this aspect can be traced back to the late 1980s and early 1990s. Henig et al. [18] consider a minimum ordering quantity contract under which the firm decides whether to order the prefixed contract amount or order more than this amount at the beginning of each period, but an incremental cost will be charged for the excess amount ordered. Tsay and Lovejoy [34] extend the replenishment decision problems to a three-stage setting where a heuristic approach is used to transform the original stochastic problem into a deterministic problem that can be solved more easily. Sethi et al. [31] study the impact of forecast quality and the flexibility level by quantity flexibility contract on the ordering decisions. Feinberg and Lewis [6] study a broader problem, where in addition to increasing inventory or disposing of it, the manager can borrow or store some inventory for one period. They show that the four-threshold policy is optimal for each period. Fu et al. [10] analyze the effect of regular and expediting replenishment with varying supply lead times from the inventory cost minimization. Zhu [39] studies the pricing and inventory strategy with returns and expediting, and shows that the optimal inventory adjustment policy follows a dual-threshold policy. Zhou and Chao [38] consider a periodic review inventory system with regular and expedited supply modes with lead time 1 and 0, respectively. They show that the optimal inventory policy is determined by two state-independent thresholds, one for each supply mode, and the optimal price follows a list-price policy. Gong et al. [12] develop a joint pricing and inventory control problem which contains a quick-response supplier with lead time 0 and a regular supplier with lead time 1 that both suffer disruption risks. Roni et al. [29] analyze a stochastic inventory model based on a hybrid inventory policy with both regular and emergency orders responding to regular and surge demands. Li et al. [24] study a quantity flexibility contract that the retailer commits an amount of quantity of newly-developed commodities, and in return the manufacturer allows the retailer to adjust the order quantities of the commitment quantities based on the inventory balance

status and the likely customer demand. A more complete literature review of this line of research is provided in recent paper by Yao and Minner [36].

This study differs from the aforementioned two streams of research in two aspects. First, our model integrates pricing and inventory decision with supply flexibility, i.e., we consider two supply sources, one is expedited supply from a supplier with lead time 0, one is regular supply from the other supplier that has lead time 1. The expedited supply incurs a higher unit cost than the regular supply. Second, we consider the customers' reference price effects. Specifically, we study a single-item, periodic-review joint pricing and inventory system with regular and expedited supply sources under reference price effects. Demand in consecutive periods are independent as well as price and reference price sensitive random variables. Unfilled demands are fully backlogged. To the best of our knowledge, the present work is the first attempt to analyze the joint pricing and inventory control problem with two supply modes under reference price effects.

Our research extends the model of Zhou and Chao [38] (ZC model for short) to the reference price effects. The guarantee of the profit-to-go function's concavity and super-modularity, which is a critical technical problem, allows us to analyze the optimal pricing and inventory strategies as well as the influence of reference price effects on optimal decisions. The contributions of this paper are as follows. First, this paper establishes the optimal strategies for joint pricing and inventory control problem with two supply modes under reference price effects, the structure of the optimal policies is similar to ZC model, but all the optimal policy parameters are reference-price-dependent. More concretely, if both two supply modes are adopted, the optimal inventory replenishment policy follows a state-independent but reference-price-dependent base-stock type with two thresholds $(S_t^E(r), s_t^R(r))$, one for each supply modes, i.e., first raise the inventory level to $S_t^E(r)$ using the expedited order, and then raise the inventory position to $S_t^R(r)$ using the regular order, and to set the price at a reference-price-dependent list-price. If only the regular supply is adopted, then the optimal policy follows a state-and-reference-price-dependent base-stock policy, with the base-stock level increasing in the initial inventory level. The optimal price is state-and-reference-price-dependent and markdowns with the initial inventory level. Second, the impacts of reference price effects on regular and expedited order as well as pricing decisions are researched. Moreover, the operational impact of adding reference price effects is studied by comparing with ZC model.

The rest of this paper is organized as follows. The finite period model with stochastic dynamic programming is presented in Section 2, and the optimal policies are characterized in Section 3. Section 4 investigates the operational impacts from the perspective of adding reference price effects. Numerical results are represented in Section 5. Section 6 concludes our paper.

2. Model Description

Consider a single-item, periodic-review problem for a firm in a finite planning horizon with consecutive $T(1 \leq T \leq \infty)$ periods. The demand in period t, denoted by D_t, is

non-negative and independent random variables. Similar to Güler et al. [16], the demand $D_t(1 \leq T \leq \infty)$ is given by

$$D_t(p_t, r_t, \varepsilon_t) = d_t(p_t, r_t) + \varepsilon_t,$$

where $d_t(p_t, r_t)$ is the mean demand function who is a deterministic function of the unit selling price p_t and the reference price r_t in period t. $D_t(p_t, r_t, \varepsilon_t)$ is non-negative and follows a continuous probability distribution, ε_t is a random variable with zero mean and independent of p_t and r_t. This demand function is very general and includes the additive and multiplicative models as special cases.

The mean demand is $d_t(p_t, r_t) = \mu_t(p_t) + R_t(r_t - p_t, r_t)$, where $\mu_t(p_t) = d_t(p_t, p_t)$ is called the base demand and $R_t(r_t - p_t, r_t) = \eta^+ \max\{r_t - p_t, 0\} + \eta^- \min\{r_t - p_t, 0\}$ is called the reference price effects on demand (see Helson and Bevan [17]). The non-negative parameters η^+ and η^- measure the sensitivities of demand associated with the perceived gain and loss, respectively. Demand is classified as loss averse, loss neutral, or loss/gain seeking, depending on whether $\eta^+ \leq \eta^-$, $\eta^+ = \eta^-$, or $\eta^+ \geq \eta^-$. For more information about $R_t(r_t - p_t, r_t)$, we refer to Güler et al. [15, 16] and the references therein.

We assume that the price in each period, p_t is restricted to a bounded interval $[\underline{p}, \overline{p}]$. The reference price depends on past prices and the current price. A commonly used model for the evolution of reference price is the exponential smoothing model (Chen et al. [4, 5], Gimpl-Heersink [11], Güler et al. [15, 16]):

$$r_{t+1} = \alpha r_t + (1 - \alpha)p_t,$$

where $\alpha(0 \leq \alpha < 1)$ is the memory factor. The larger the memory factor, the longer the memory. If α is high, then customers have a long memory and past price effect is larger. If α is small, then current price has a greater effect than the past on the reference price. The initial reference price is given by $r_1 \in [\underline{p}, \overline{p}]$, and thus all r_t belong to the interval. Moreover, we introduce the following structure on the mean demand.

Assumption 1. *The mean demand $d_t(p_t, r_t)$ is concave, bounded, non-negative and continuous, strictly decreasing in p_t and increasing in r_t for $t = 1, 2, \ldots, T$.*

It is worth mentioning that the existence of the mean demand functions satisfy Assumption 1 is proved in Güler et al. [15, 16] when customers are loss neutral or loss averse and some examples are presented. Hence, this paper assumes that the customers are loss neutral or loss averse. In addition, Assumption 1 implies that $p_t(d_t, r_t)$ is concave in (d_t, r_t) (Proposition 1, Güler et al. [16]), where $p_t(d_t, r_t)$ is the inverse function of the mean demand $d_t(p_t, r_t)$ for a given r_t. Moreover, $p_t(d_t, r_t)$ is strictly decreasing in d_t and increasing in r_t for $t = 1, 2, \ldots, T$ (Proposition 1, Güler et al. [16]). Hence, determining the price is equivalent to determining the mean demand. In the discussion below, without other specification, we will focus on finding the optimal mean demand d_t for period t. Therefore, we assume that the feasible range of the mean demand in period t is $d_t \in [\underline{d}_t, \overline{d}_t]$, where $\underline{d}_t \geq 0$ and $\overline{d}_t < +\infty$.

Similar to Zhou and Chao [38], the sequence of events is as follows. First, the firm receives the regular order placed in the previous period and updates and reviews the current inventory level. Second, the expedited order is placed, if any, and then received. Third, a regular order is placed and the selling price is set. Fourth, demand is realized and excess demand is backlogged. Finally, all costs and revenue are incurred.

We summarize the notations that will be used in this paper.

$$x_t = \text{the initial inventory level before any decisions are made in period } t,$$
$$y_t^E = \text{the inventory level after placing the expedited order in period } t,$$
$$y_t^R = \text{the inventory position after placing the regular order in period } t,$$
$$c_t^E = \text{the unit ordering cost for the expedited order in period } t,$$
$$c_t^R = \text{the unit ordering cost for the regular order in period } t,$$
$$\gamma = \text{the discount factor, } 0 \leq \gamma < 1.$$

Here, $c_t^E > c_t^R$. Further, we assume that the price cannot be smaller than the ordering cost, i.e., $\underline{p} \geq c_t^E$ for every t. With the event sequence and the notations above, the order quantity from the expedited supply is $y_t^E - x$ while that from the regular supply is $y_t^R - y_t^E$. In the rest of the article, for ease of exposition, we will omit the subscript t unless confusion may arise.

At the end of each period after demand is realized, the remaining inventory is carried over to the next period and incurs holding cost, while unsatisfied demand is backlogged and incurs shortage cost. Let $H_t(z)$ be the inventory holding/backlogging cost when the ending inventory level is z in period t, then the expected holding/backlogged cost can be written as:

$$G_t(y, d) = \mathtt{E}[H_t(y - d - \varepsilon_t)],$$

where y is the initial inventory level after expedited ordering at period t. We assume that $G_t(y, d)$ is convex in y and d.

Given the initial inventory x and reference price r in each period $t = 1, 2, \ldots, T$. Then, this problem can be formulated as a dynamic programming and the Bellman equation for this problem is:

$$v_t(x, r) = \max_{\substack{y^R \geq y^E \geq x \\ \underline{d}_t \leq d \leq \overline{d}_t}} \left\{ d \cdot p(d, r) - c_t^E y^E - c_t^R (y^R - y^E) - G_t(y^E, d) \right.$$

$$\left. + \gamma \mathtt{E}[v_{t+1}(y^R - d - \varepsilon_t, \alpha r + (1 - \alpha)p(d, r))] \right\} + c_t^E x, \tag{2.1}$$

where v_t is the profit-to-go function and $G(y^E, d) = \mathtt{E}[H_t(y^E - d - \varepsilon_t)]$, $\mathtt{E}$ denotes the expectation operator. The terminal value is given by $v_{T+1}(x, r) = c_{T+1}^E x$.

To facilitate the analysis, let $V_t(x, r) = v_t(x, r) - c_{T+1}^E x$, and so $V_{T+1}(x, r) = 0$. Therefore, the Bellman equation (2.1) becomes

$$V_t(x, r) = \max_{\substack{y^R \geq y^E \geq x \\ \underline{d}_t \leq d \leq \overline{d}_t}} J_t(y^E, y^R, d, r), \tag{2.2}$$

where
$$J_t(y^E, y^R, d, r) = d[p(d,r) - \gamma c_{t+1}^E] - (c_t^E - c_t^R)y^E + (\gamma c_{t+1}^E - c_t^R)y^R - G_t(y^E, d)$$
$$+ \gamma E[V_{t+1}(y^R - d - \varepsilon_t, \alpha r + (1 - \alpha)p(d,r))]. \tag{2.3}$$

Furthermore, we make the following assumption.

Assumption 2. *The inverse function $p_t(d,r)$ of the mean demand $d_t(p,r)$ is supermodular in (d,r), and the revenue function $d \cdot p_t(d,r)$ is joint concave in (d,r).*

Under Assumption 2, the revenue function $d \cdot p_t(p,r)$ is supermodular in (d,r) by Theorem 6 in Güler et al. [16].

3. Optimal Operational Strategies and Its Analysis

In this section, we first characterize the optimal ordering for regular and expedited along with pricing strategies, and then analyze the impact of reference price effects on the optimal pricing and inventory strategies.

3.1. Optimal pricing and inventory strategies

This subsection characterizes the optimal ordering for regular and expedited along with pricing strategies. We first need the concavity of J_t and V_t, which is shown in the following lemma, and hence there exists a unique optimal decision in each period for a given x and r. For the smoothness of the paper, all the proofs of main results in this section are available in the Appendix.

Lemma 1. *For $t = 1, 2, \ldots, T$, we have*
  (i) *$V_t(x,r)$ is decreasing in x and increasing in r.*
 (ii) *$J_t(y^E, y^R, d, r)$ is joint concave in (y^E, y^R, d, r).*
(iii) *$V_t(x,r)$ is joint concave in (x,r).*

For any y^E and y^R, define
$$d_t(y^E, y^R, r) = \arg \max_{\underline{d}_t \leq d \leq \overline{d}_t} J_t(y^E, y^R, d, r),$$

then the optimal price is given by
$$p_t(y^E, y^R, r) = p_t(d_t(y^E, y^R, r), r). \tag{3.1}$$

In addition, denote
$$(s_t^E(r), s_t^R(r)) = \arg \max_{y^R \geq y^E} J_t(y^E, y^R, d_t(y^E, y^R, r), r), \tag{3.2}$$

$$S_t^R(x,r) = \arg \max_{y^R} J_t(x, y^R, d_t(x, y^R, r), r). \tag{3.3}$$

Let
$$\overline{p}_t(r) = \overline{p}_t(s_t^E(r), s_t^R(r), r) = p_t(d_t(s_t^E(r), s_t^R(r), r), r), \tag{3.4}$$

$$P_t(x,r) = p_t(x, S_t^R(x,r), r) = p_t(d_t(x, S_t^R(x,r), r), r). \qquad (3.5)$$

It is obvious that $s_t^R(r) = S_t^R(s_t^E(r), r)$ and $\overline{p}_t(r) = P_t(s_t^E(r), r)$. With these optimal parameters, the optimal pricing and inventory strategies are analyzed below.

The following lemma illustrates the submodularity and supermodularity of $J_t(y^E, y^R, d, r)$ in (y^E, p) and (y^E, y^R), respectively.

Lemma 2. *For $t = 1, 2, \ldots, T$, we have*
 (i) $J_t(y^E, y^R, d, r)$ *is submodular in (y^E, p).*
 (ii) $J_t(y^E, y^R, d(y^E, y^R, r), r)$ *is supermodular in (y^E, y^R).*

According to Lemma 2 above and Theorem 2.2.8 in Simchi-Levi et al. [32], we can obtain the following theorem which characterizes the monotonicity of the optimal parameters defined in Eqs. $(3.1)-(3.5)$.

Theorem 1. *For $t = 1, 2, \ldots, T$, we have*
 (i) $p_t(y^E, y^R, r)$ *is decreasing in y^E.*
 (ii) $p_t(y^E, y^R, r)$ *is decreasing in y^R.*
 (iii) $S_t^R(x, r)$ *is increasing in x and $P_t(x, r)$ is decreasing in x.*

On the basis of the analysis above, we can characterize the optimal inventory replenishment and pricing policies for each period via the theorem below.

Theorem 2. *For $t = 1, 2, \ldots, T$, the optimal ordering and pricing policies are characterized as follows*
 (i) *If $x \le s_t^E(r)$, then $(y_t^{E*}, y_t^{R*}) = (s_t^E(r), s_t^R(r))$ and $p_t^*(r) = \overline{p}_t(r)$.*
 (ii) *If $s_t^E(r) < x < S_t^R(x, r)$, then $(y_t^{E*}, y_t^{R*}) = (x, S_t^R(x, r))$ and $p_t^*(x, r) = P_t(x, r)$.*
 (iii) *If $x > S_t^R(x, r)$, then $(y_t^{E*}, y_t^{R*}) = (x, x)$ and $p_t^*(x, r) = p_t(x, x, r)$.*

This theorem shows that, when initial inventory level $x \le s_t^E(r)$, the optimal inventory replenishment policy follows a state-independent but reference-price -dependent base-stock type with two thresholds $(s_t^E(r), s_t^R(r))$, one for expedited order and the other for regular order. Specifically, if the initial inventory level x at the beginning of period t is less than $s_t^E(r)$, it is optimal to order up to $s_t^E(r)$ using the expedited order, then use the regular order to raise the inventory position to $s_t^R(r)$, and to set the price at $\overline{p}_t(r)$ which is a reference-price-dependent list-price. If the initial inventory level x is greater than $s_t^E(r)$, then only the regular order is used to bring the inventory position to $\max\{x, S_t^R(x, r)\}$ and set the price at $p_t(\max\{x, S_t^R(x, r)\})$, which follows a state-and-reference-price-dependent price and markdowns with the initial inventory level x.

To facilitate the characterization of optimal policy parameters, we let $\tilde{y}^E = y^E - d$, $\tilde{y}^R = y^R - d$, then the optimal equation (2.1) becomes

$$v_t(x, r) = \max_{\substack{y^R \ge y^E \ge \underline{x} - d \\ \underline{d}_t \le d \le \overline{d}_t}} \left\{ [d \cdot p(d, r) - c_t^E d] - (c_t^E - c_t^R)\tilde{y}^E - G_t(\tilde{y}^E - \varepsilon_t) - c_t^R \tilde{y}^R \right.$$

$$+\gamma \mathbb{E}[v_{t+1}(\tilde{y}^R - \varepsilon_t, \alpha r + (1-\alpha)p(d,r))]\Big\} + c_t^E x,$$

where $G_t(\tilde{y}^E - \varepsilon_t) = \mathbb{E}[H_t(\tilde{y}^E - \varepsilon_t)]$.

Similar to Zhou and Chao [38], the optimal policy parameters $s_t^E(r)$, $s_t^E(r)$ and $S_t^R(x,r)$ are given by

$$s_t^E(r) = \tilde{y}_t^1(r) + \tilde{d}_t(r), \ \ s_t^R(r) = \tilde{y}_t^2(r) + \tilde{d}_t(r), \ \text{and} \ S_t^R(x,r) = \tilde{y}_t^2(r) + \tilde{d}_t(x,r),$$

where $\tilde{y}_t^2(r)$ is the maximizer of

$$\Phi_t(\tilde{y}^R) = \max_{\underline{d}_t \le d \le \overline{d}_t} \{-c_t^R \tilde{y}^R + \gamma \mathbb{E}[v_{t+1}(\tilde{y}^R - \varepsilon_t, \alpha r + (1-\alpha)p(d,r))]\}.$$

$\tilde{y}_t^1(r)$ is the maximizer of

$$W_t(\tilde{y}^E) = -(c_t^E - c_t^R)\tilde{y}^E - G_t(\tilde{y}^E - \varepsilon_t) + \Phi_t(\tilde{y}^E \vee \tilde{y}_t^2(r)),$$

where "$\vee$" is the maximun operator, i.e., $x \vee y = \max\{x,y\}$ for any real numbers x and y. Furthermore,

$$\tilde{d}_t(r) = \arg \max_{\underline{d}_t \le d \le \overline{d}_t} [d \cdot p(d,r) - c_t^E d],$$

$$\tilde{d}_t(x,r) = \arg \max_{\underline{d}_t \le d \le \overline{d}_t} \{[d \cdot p(d,r) - c_t^E d] + W_t((x-d) \vee \tilde{y}_t^1(r))\}.$$

3.2. The impact of reference price effects on optimal strategies

In this subsection, we analyze the impact of the reference price on the optimal pricing and inventory policies. To do this, we need the following lemma.

Lemma 3. *For $t = 1, 2, \ldots, T$, $V_t(x,r)$ is supermodular in (x,r).*

Corollary 1. *For $t = 1, 2, \ldots, T$, we have*
  (i) $J_t(y^E, y^R, d, r)$ *is supermodular in* (y^E, r).
 (ii) $J_t(y^E, y^R, d, r)$ *is supermodular in* (y^R, r).
(iii) $J_t(y^E, y^R, d, r)$ *is supermodular in* (d, r).

Based on this, we can characterize the reference price effects on optimal policy parameters via the following theorem.

Theorem 3. *For $t = 1, 2, \ldots, T$, we have*
  (i) *The optimal inventory level after expedited order y_t^{E*} is increasing in r_t.*
 (ii) *The optimal inventory level after regular order y_t^{R*} is increasing in r_t.*
(iii) *The optimal mean demand is d_t^* increasing in r_t.*
 (iv) *The optimal price is p_t^* increasing in r_t.*
  (v) *The optimal profit is $V_t^*(x,r)$ increasing in r_t.*

4. Operational Impact of Reference Price Effects

Since the impact of supply diversification has been discussed in Zhou and Chao [38]. This section mainly analyze the operational impact from the perspective of reference price effects by comparing our model with ZC model. Although the ZC model considers both regular and expedited supply modes, it doesn't take the reference price effects into consideration. To distinguish the ZC model from ours, we use the superscript f to signify the notation for the ZC model. The following is the main results on the impact of adding reference price effects.

Theorem 4. *After the reference price effects is considered, the optimal profit-to-go function and optimal policy parameters satisfy, for $t = 1, 2, \ldots, T$,*

(i) $V_t^*(x, r) \geq V_t^{f^*}(x)$.

(ii) $y_t^{E^*}(r) \geq y_t^{E^{f^*}}$.

(iii) $y_t^{R^*}(r) \geq y_t^{R^{f^*}}$.

(iv) $d_t^*(x, r) \geq d_t^{f^*}(x)$.

(v) $p_t^*(x, r) \geq p_t^{f^*}(x)$.

Proof. This follows directly from Theorem 3 that the ZC model is a special case of our model, i.e., $r_t = 0$ for all $t = 1, 2, \ldots, T$. □

This theorem can be intuitively illustrated as follows. Part (i) states that when more consideration of the customers' behavior, the firm can only do better, thus its maximum profit does not go down. Part (ii), (iii), (iv) and (v) indicate that with the increase of customers' reference price, the optimal mean demand will increase and the optimal price will rise as well. In addition, considering the lead time for regular replenishment and the incremental demand under reference price effects, the firm orders more by expedited and regular supply to raise the inventory level so as to meet the customers' needs as much as possible.

5. Numerical Experiments

In this section, we present several numerical experiments to illustrate the impact of reference price on the optimal policy parameters (the optimal inventory level for expedited supply, the optimal inventory position for regular supply, and the optimal price). Besides, we analyze the operational impacts on firm's profit by adding reference price effects via comparing with ZC model. All experiments below are performed in MATLAB R2014b on a laptop with an Intel(R) Core (TM) i5-7200U central processing unit CPU (2.50 GHz, 2.70GHz) and 8.0 GB of RAM running 64-bit Windows 10 Enterprise.

Consider a system with planning horizon $T = 4$. We perform experimental analysis with the following stationary parameter values: $c^E = 18$, $c^R = 15$, $\eta^+ = 1.5$, $\eta^- = 2.5$, $\gamma = 0.95$. The mean demand function is given by $d(p, r) = 200 - 2p + 1.5 \max\{r - p, 0\} + 2.5 \min\{p - r, 0\}$, the inventory holding or backlogged cost is $H(x) = h \max\{x, 0\} +$

$b \max\{-x, 0\}$ with $h = 2$, $b = 20$ and $x \in [0, 100]$. $\varepsilon_t \sim \text{Uniform}[-1, 1]$. We report the results for $\alpha = 0.35, 0.55, 0.75$.

We first analyze the impact of changes in reference price r on the optimal pricing and inventory strategies. It is shown from Figure $1-4$ that the optimal inventory level for expedited supply y^{E*}, the optimal inventory position for regular supply y^{R*}, the optimal price p^* and the optimal profit V^* are increasing in the reference price r, which is consistent with Theorem 3. Table $1-4$ list the operational impact from the perspective of reference price effects by comparing our model with ZC model, which is consistent with Theorem 4. Moreover, from Figure 1 and Table 1, we can see that the slope of y^{E*} increases with the increase of reference price r (when $r > 25$). This indicates that with the increase of customers' reference price, customers valuation of commodities will increase, the firm will raise its inventory level for expedited supply higher to meet the increasing demand. Figure 2 and Table 2 for y^{R*} presents the similar features. Figure 4 and Table 4 illustrate that considering the reference price effects will bring substantial profits to the firm. Therefore, reference price has a positive effect on the optimal inventory level for expedited supply y^{E*}, the optimal inventory position for regular supply y^{R*}, the optimal price p^* and the optimal profit V^*.

Next, we examine how the reference price parameter α (i.e., memory α) factor affects the firm's optimal pricing and inventory strategies. Figures $1-3$ and Tables $1-3$ suggest that with the increase of α, i.e., the customers' ability to remember past prices becomes weaker. This means that customers adapt to the new price information at a lower rate and less loyalty, then the firm should decrease its sales price while reducing both the inventory level for expedited supply and the inventory position for regular supply. This demonstrates that the memory factor α has a negative impact on the optimal price and inventory decisions. Figure 4 and Table 4 show that the optimal profit will decrease as α increase, i.e., the memory factor α also has a negative impact on profit.
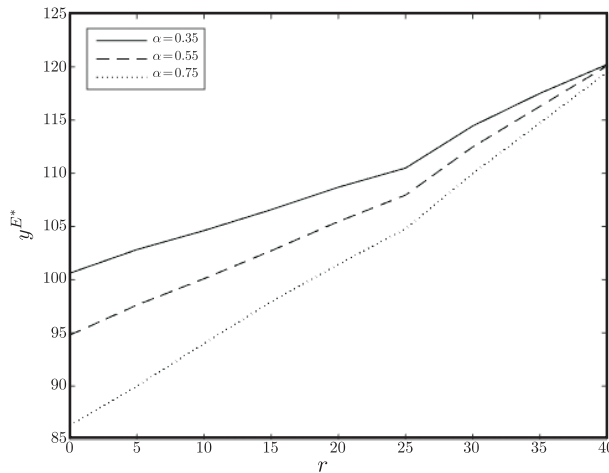

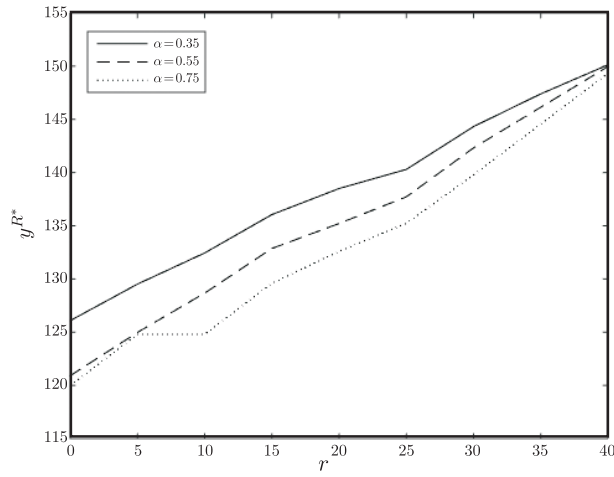Figure 1: The impact of reference price r on optimal inventory level for expedited supply.

Figure 2: The impact of reference price r on optimal inventory position for regular supply.
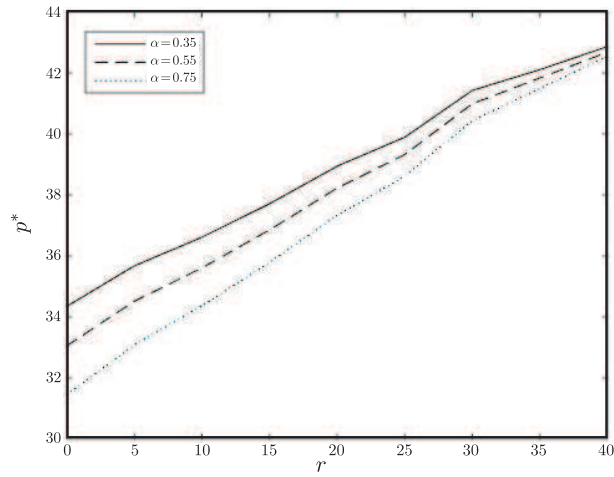

Figure 3: The impact of reference price r on optimal price.

Table 1: The comparison of optimal inventory level for expedited supply between ours and ZC model.

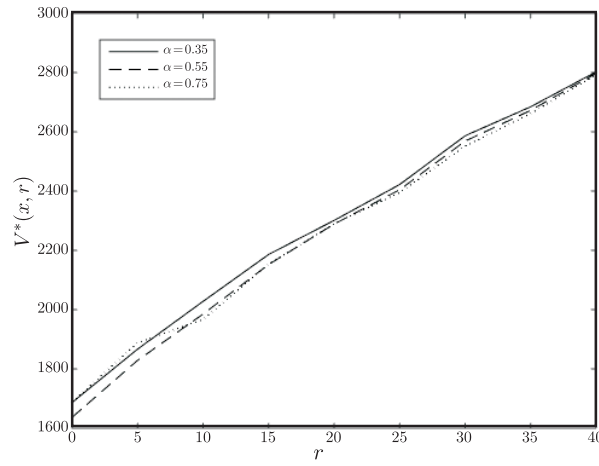| α	ZC model	y^{E^*} Our model					
		$r=10$	$r=15$	$r=20$	$r=25$	$r=30$	$r=35$
0.35	76.388	104.610	106.580	108.690	110.490	114.450	117.510
0.55	76.388	100.900	102.720	105.460	107.950	112.470	116.280
0.75	76.388	94.006	97.928	101.44	104.79	109.98	114.73

Figure 4: The impact of reference price r on optimal profit.

Table 2: The comparison of optimal inventory position for regular supply between ours and ZC model.

α	ZC model	y^{R^*}					
		Our model					
		$r = 10$	$r = 15$	$r = 20$	$r = 25$	$r = 30$	$r = 35$
0.35	101.140	132.440	136.050	138.500	140.300	144.320	147.400
0.55	101.140	128.670	132.860	135.210	137.720	142.320	146.140
0.75	101.140	124.800	129.590	132.600	135.230	139.790	144.560

Table 3: The comparison of optimal price between ours and ZC model.

α	ZC model	p^*					
		Our model					
		$r = 10$	$r = 15$	$r = 20$	$r = 25$	$r = 30$	$r = 35$
0.35	28.967	36.615	37.712	38.943	39.892	41.429	42.110
0.55	28.967	35.610	36.854	38.226	39.328	40.991	41.847
0.75	28.967	34.366	35.790	37.332	38.625	40.437	41.492

Table 4: The comparison of optimal profit between ours and ZC model.

α	ZC model	V^*					
		Our model					
		$r = 10$	$r = 15$	$r = 20$	$r = 25$	$r = 30$	$r = 35$
0.35	1510.7	2027.8	2186.0	2301.0	2422.0	2586.5	2684.5
0.55	1510.7	1984.9	2152.0	2288.5	2404.4	2586.7	2672.3
0.75	1510.7	1966.8	2153.3	2289.7	2394.4	2551.5	2661.2

6. Conclusions

Our research complements the existing research stream in coordinating pricing and inventory replenishment decisions from two aspects. On the one hand, we consider inventory planning decisions for supply diversification, i.e., regular and expedited supply modes. On the other hand, we consider the impact of the customers' behavior (i.e. customers' reference price) on pricing and inventory replenishment decisions. We study the optimal pricing and ordering flexibility strategies under reference price effects. The operational impact shows that considering the supply flexibility and reference price effects simultaneously will bring substantial profit. The above research generalized the results of Zhou and Chao [38].

Our research also provides some inspiration for management practice, which can be adopted by firms to formulate its pricing and inventory strategies with the reference price effects.

(1) When the reference price effects is considered, customers' ability to remember past prices has a significant effect on managing the optimal pricing and inventory decisions. As memory factor α increases, customers adapt to the new price information at a lower rate and become less loyal to the commodity. Hence, the firms should reduce the sales price to achieve a positive reference price effects, thereby stimulating demand. At the same time, the inventory level for expedited supply and the optimal inventory position for regular supply should also be reduced so as to reduce the holding cost caused by demand uncertainty.

(2) The reference price has a positive effect on the optimal inventory level for expedited supply, the optimal inventory position for regular supply, the optimal price and the optimal profit. Firms should make good use of this positive effect of reference price to raise their profit.

Though this paper has identified the effects of reference price on dynamic pricing and ordering flexibility decisions, there still some shortcomings that can be investigated in the future. First, this paper analyzes the pricing and order flexibility decisions of a single firm under reference price effects, and unaware of the influence of reference price effects on suppliers. An interesting future research topic is to examine the pricing and inventory decisions for suppliers, and to design an appropriate coordination mechanism so that a win-win outcome for both parties can be obtained. Second, in our study, the customers' reference price can be observed by firms. However, the information on customers' reference price is difficult to get in reality. Thus, demand learning can be incorporated into formulating pricing and inventory strategy in the presence of the reference price effects. Third, with the rapid development of information technology centered on the mobile Internet, customers' purchase patterns are also diversified. In this case, how to study the reference price of customers on firms' pricing and inventory decisions is also one of the interesting and meaningful research directions in the future.

Acknowledgements

Appendix

Proof of Lemma 1 (i). The monotonicity of $V_t(x,r)$ in x is straightforward because $J_t(y^E, y^R, d, r)$ is independent of x and the feasible set $\{(y^E, y^R, d) \mid y^R \geq y^E \geq x, \underline{d}_t \leq d \leq \overline{d}_t\}$ shrinks as x increases. The monotonicity of $V_t(x,r)$ in r is similar to that of Theorem 1 in Güler et al. [15].

Next, we prove (ii) and (iii) by induction. Starting from $t = T$, it is obvious that $V_{T+1}(x,r) = 0$ is concave, then (iii) is true. For (ii), each term in Eq. (2.3) is concave and concavity is preserved by maximization, so (ii) holds for $t = T$. Suppose that (ii) and (iii) are valid for $t = k + 1$. Each terms in Eq. (2.3) is concave except for $V_{k+1}(y^R - d - \varepsilon_k, \alpha r + (1-\alpha)p(d, r))$, we thus need the concavity of $V_{k+1}(y^R - d - \varepsilon_k, \alpha r + (1-\alpha)p(d, r))$ in (y^R, d, r). By defining $\tilde{\tau}(y^R, d, \varepsilon_k)$ and $\tilde{r}(d, r)$ as:

$$\tilde{\tau}(y^R, d, \varepsilon_k) = y^R - d - \varepsilon_k, \qquad \tilde{r}(d, r) = \alpha r + (1 - \alpha)p(d, r).$$

. Since $p_k(d_k, r_k)$ is concave in (d_k, r_k) by Assumption 1, then the following holds for any pair (y_1^R, y_2^R), (d_1, d_2) and (r_1, r_2):

$$\tilde{\tau}\left(\frac{y_1^R + y_2^R}{2}, \frac{d_1 + d_2}{2}, \varepsilon_k\right) = \frac{y_1^R + y_2^R}{2} + \frac{d_1 + d_2}{2} - \varepsilon_k = \frac{1}{2}(y_1^R + d_1 - \varepsilon_k) + \frac{1}{2}(y_2^R + d_2 - \varepsilon_k),$$

$$\tilde{r}\left(\frac{d_1 + d_2}{2}, \frac{r_1 + r_2}{2}\right) \geq \frac{1}{2}\tilde{r}(d_1, r_1) + \frac{1}{2}\tilde{r}(d_2, r_2).$$

Thus we obtain

$$V_{k+1}\left(\tilde{\tau}\left(\frac{y_1^R + y_2^R}{2}, \frac{d_1 + d_2}{2}, \varepsilon_k\right), \tilde{r}\left(\frac{d_1 + d_2}{2}, \frac{r_1 + r_2}{2}\right)\right)$$

$$\geq V_{k+1}\left(\tilde{\tau}\left(\frac{y_1^R + y_2^R}{2}, \frac{d_1 + d_2}{2}, \varepsilon_k\right), \frac{1}{2}\tilde{r}(d_1, r_1) + \frac{1}{2}\tilde{r}(d_2, r_2)\right)$$

$$= V_{k+1}\left(\frac{1}{2}\tilde{\tau}(y_1^R + d_1 + \varepsilon_k) + \frac{1}{2}\tilde{\tau}(y_1^R + d_1 + \varepsilon_k), \frac{1}{2}\tilde{r}(d_1, r_1) + \frac{1}{2}\tilde{r}(d_2, r_2)\right)$$

$$\geq \frac{1}{2}V_{k+1}\left(\tilde{\tau}(y_1^R + d_1 + \varepsilon_k), \tilde{r}(d_1, r_1)\right) + \frac{1}{2}V_{k+1}\left(\tilde{\tau}(y_2^R, d_2, \varepsilon_k), \tilde{r}(d_2, r_2)\right)$$

where the first and second inequality follows from (i) and the induction assumption, respectively. We thus get the concavity of in $V_{k+1}(y^R - d - \varepsilon_k, \alpha r + (1 - \alpha)p(d, r))$ in (y^R, d, r). Then $J_k(y^E, y^R, d, r)$ is joint concave in (y^E, y^R, d, r). Therefore, $V_k(x, r)$ is joint concave in (x, r). □

Proof of Lemma 2. The proof is similar to that of Lemma 2 in Zhou and Chao [38].□

Proof of Theorem 1. (i) and (iii) is the direct consequence of Lemma 2.
(ii) Let

$$y^R(y^E, r) = \arg \max_{y^R \geq y^E} J_t(y^E, y^R, d(y^E, y^R), r),$$

then y^R is increasing in y^E follows from Lemma 2 (ii) and Theorem 2.2.8 in Simchi-Levi et al. [32]. This together with (i) yields the result.   □

Proof of Theorem 2. The result follows from the concavity of $J_t(y^E, y^R, d, r)$ and Eqs. (3.1)-(3.5).   □

Proof of Lemma 3. We prove this lemma by induction. Starting from $t = T$, it is obvious that $V_{T+1}(x, r) = 0$ is supermodular in (x, r). Thus $J_T(y^E, y^R, d, r)$ is supermodular in (x, r). Thus $J_T(y^E, y^R, d, r)$ is supermodular in (x, r) since the first four terms in $J_T(y^E, y^R, d, r)$ are independent of x. Following the maximization preserves supermodularity yields the supermodularity of $V_T(x, r)$ in (x, r).

Assume that the result holds for $t = k + 1$. Next, we need to show that the result is still true for $t = k$. Since $J_k(y^E, y^R, d, r)$ is independent of x, we only need to proof the supermodularity of $J_k(y^E, y^R, d, r)$ in (y^E, r), (y^R, r) and (d, r), which is equivalent to the supermodularity of $J_k(y^E, y^R, d, r)$ in (y^E, x, r), (y^R, x, r) and (d, x, r).

Firstly, we proof the supermodularity of $J_k(y^E, y^R, d, r)$ in (y^E, r). The terms in $J_k(y^E, y^R, d, r)$ either depends on y^E or r or is a constant with respect to y^E and r except for $-G(y^E, d)$, so it suffices to show the submodularity of $G(y^E, d)$ in (y^E, r).

For any pair (y_1^E, y_2^E) and (r_1, r_2) with $y_1^E > y_2^E$ and $r_1 > r_2$. Let

$$\tau_1 = y_1^E - d(p, r_1) - \varepsilon_k, \qquad \tau_2 = y_1^E - d(p, r_2) - \varepsilon_k,$$
$$\tau_3 = y_2^E - d(p, r_1) - \varepsilon_k, \text{ and } \tau_4 = y_2^E - d(p, r_2) - \varepsilon_k,$$

By the monotonicity of the mean demand function d_k, we have $\tau_3 < \tau_4$. Thus, by the concavity of H_k, we have

$$H_k(\tau_1) - H_k(\tau_3) = H_k(\tau_3 + (y_1^E - y_2^E)) - H_k(\tau_3)$$
$$\leq H_k(\tau_4 + (y_1^E - y_2^E)) - H_k(\tau_4)$$
$$= H_k(\tau_2) - H_k(\tau_4),$$

this implies that $H_k(y_1^E - d(p, r) - \varepsilon_k) - H_k(y_2^E - d(p, r) - \varepsilon_k)$ is decreasing in r. Hence, $G(y^E, d)$ is submodular in (y^E, r), then $-G(y^E, d)$ is supermodular in (y^E, r). This gives the supermodularity of $J_k(y^E, y^R, d, r)$ in (y^E, r).

Secondly, we proof the supermodularity of $J_k(y^E, y^R, d, r)$ in (y^R, r). The terms in $J_k(y^E, y^R, d, r)$ either depends on y^R or r or is a constant with respect to y^R and r except for $V_{k+1}(y^R - d - \varepsilon_t, \alpha r + (1 - \alpha)p(d, r))$, so it suffices to show the supermodularity of $V_{k+1}(y^R - d - \varepsilon_t, \alpha r + (1 - \alpha)p(d, r))$ in (y^R, r).

Consider arbitrary pair (y_1^R, y_2^R) and (r_1, r_2) with $y_1^R > y_2^R$ and $r_1 > r_2$. Fix ε_k, let

$$(\tau_1, \xi_1) = (y_1^R - d(p, r_1) - \varepsilon_k, \xi_1), \qquad (\tau_2, \xi_2) = (y_1^R - d(p, r_2) - \varepsilon_k, \xi_2),$$

$$(\tau_3, \xi_1) = (y_2^R - d(p, r_1) - \varepsilon_k, \xi_1), \text{ and } (\tau_4, \xi_2) = (y_2^R - d(p, r_2) - \varepsilon_k, \xi_2),$$

where $\xi_1 = \alpha r_1 + (1 - \alpha)p(d, r_1)$, $\xi_2 = \alpha r_2 + (1 - \alpha)p(d, r_2)$. Then we obviously have $\xi_1 > \xi_2$, $\tau_3 < \tau_4$. Thus, we get

$$\begin{aligned}
V_{k+1}(\tau_1, \xi_1) - V_{k+1}(\tau_3, \xi_1) &= V_{k+1}(\tau_3 + (y_1^R - y_2^R), \xi_1) - V_{k+1}(\tau_3, \xi_1) \\
&\geq V_{k+1}(\tau_4 + (y_1^R - y_2^R), \xi_1) - V_{k+1}(\tau_4, \xi_1) \\
&= V_{k+1}(\tau_2, \xi_1) - V_{k+1}(\tau_4, \xi_1) \\
&\geq V_{k+1}(\tau_2, \xi_2) - V_{k+1}(\tau_4, \xi_2),
\end{aligned}$$

where the first inequality follows from the concavity of V_{k+1}, and the last inequality follows from the supermodularity of $V_{k+1}(\tau, \xi)$ in (τ, ξ) by induction assumption, which implies that $V_{k+1}(y_1^R - d(p, r) - \varepsilon_k, \xi) - V_{k+1}(y_2^R - d(p, r) - \varepsilon_k, \xi)$ is increasing in r. We thus get the supermodularity of $V_{k+1}(y^R - d - \varepsilon_k, \alpha r + (1 - \alpha)p(d, r))$ in (y^R, r). Consequently, $J_k(y^E, y^R, d, r)$ is supermoduar in (y^R, r).

Thirdly, the supermodularity of $J_k(y^E, y^R, d, r)$ in (d, r) is similar to that of Theorem 6 in Güler et al. [16].

In summary, $J_k(y^E, y^R, d, r)$ is supermodular in (x, r). So $V_k(x, r)$ is supermodular in (x, r). This completes the proof. $\square$

Proof of Theorem 3. (i), (ii) and (iii) are the direct consequence of Corollary 1, while (iv) is the direct consequence of (iii) and Assumption 1. (v) has been proved in Lemma 1. $\square$

References

[1] Arslan, H. and Kachani, S. (2011). *Dynamic pricing under consumer reference-price effects*, In: Cochran J., ed. Wiley Encyclopedia of Operations Research and Management Science, John Wiley & Sons, Inc., Hoboken, USA.

[2] Byrnes, J. (2003). *Dell manages profitability, not inventory*, Harvard Business School Working Knowledge, http: //hbswk.hbs.edu/archive/3497.html.

[3] Chen, X., Hu, P. and He, S. (2013). *Technical note-preservation of supermodularity in parametric optimization problems with nonlattice structures*, Operations Research, Vol.61, 1166-1173.

[4] Chen, X., Hu, P. and Hu, Z. Y. (2017). *Efficient algorithms for the dynamic pricing problem with reference price effect*, Management Science, Vol.63, 4389-4408.

[5] Chen, X., Hu, P., Shum, S. and Zhang, Y. (2016). *Dynamic stochastic inventory management with reference price effects*, Operations Research, Vol.64, 1529-1536.

[6] Feinberg, E. A. and Lewis, M. E. (2005). *Optimality of four-threshold policies in inventory systems with customer returns and borrowing/storage options*, Probability in the Engineering and Informational Sciences, Vol.19, 45-71.

[7] Feng, Q. (2010). *Integrating dynamic pricing and replenishment decisions under supply capacity uncertainty*, Management Science, Vol.56, 2154-2172.

[8] Fibich, G., Gavious, A. and Lowengart, O. (2003). *Explicit solutions of optimization models and differential games with nonsmooth (asymmetric) reference-price effects*, Operations Research, Vol.51, 721-734.

[9] Fibich, G., Gavious, A. and Lowengart, O. (2005). *The dynamics of price elasticity of demand in the presence of reference price effects*, Journal of the Academy of Marketing Science, Vol.33, 66-78.

[10] Fu, K., Hsu, V. N. and Lee, C. Y. (2009). *Note-optimal component acquisition for a single-product, single-demand assemble-to-order problem with expediting*, Manufacturing & Service Operations Management, Vol.11, 229-236.

[11] Gimpl-Heersink, L. (2008). *Joint pricing and inventory control under reference price effects*, Ph.D. Thesis, Vienna University of Economics and Business Administration, Vienna, Austria.

[12] Gong, X. T., Chao, X. and Zheng, S. H. (2014). *Dynamic pricing and inventory management with dual suppliers of different lead times and disruption risks*, Production and Operations Management, Vol.23, 2058-2074.

[13] Greenleaf, E. A. (1995). *The impact of reference price effects on the profitability of price promotions*, Marketing Science, Vol.14, 82-104.

[14] Grosvenor, F., Austin, T. A. (2001). *Ciscos eHub initiative*, Supply Chain Management Review, 28-35.

[15] Güler, M. G., Bilgiç, T. and Güllü, R. (2014). *Joint inventory and pricing decisions with reference effects*, IIE Transactions, Vol.46, 330-343.

[16] Güler, M. G., Bilgiç, T. and Güllü, R. (2015). *Joint pricing and inventory control for additive demand models with reference effects*, Annals of Operations Research, Vol.226, 255-276.

[17] Helson, H. and Bevan, W. (1964). *An investigation of variables in judgement of relative area*, Journal of Experimental Psychology, Vol.67, 335-341.

[18] Henig, M., Gerchak, Y., Ernst, R. and Pyke, D. F. (1997). *An inventory model embedded in designing a supply contract*, Management Science, Vol.43, 184-189.

[19] Hu, Z., Chen, X. and Hu, P. (2016). *Technical note-dynamic pricing with gain-seeking reference price effects*, Operations Research, Vol.64, 150-157.

[20] Kopalle, P. K., Rao, A. G. and Assunção, J. L. (1996). *Asymmetric reference price effects and dynamic pricing policies*, Marketing Science, Vol.15, 60-85.

[21] Krishnamurthi, L., Mazumdar, T. and Raj, S. P. (1992). *Asymmetric response to price in consumer brand choice and purchase quantity decisions*, Journal of Consumer Research, Vol.19, 387-400.

[22] Kahneman, D. and Tversky, A. (1979). *Prospect theory: An analysis of decision under risk*, Econometrica, Vol.47, 263-292.

[23] Kalyanaram, G. and Winer, R. S. (1995). *Empirical generalizations from reference price research*, Marketing Science, Vol.14, 161-169.

[24] Li, X., Lian, Z., Choong, K. K. and Liu, X. (2016). *A quantity-flexibility contract with coordination*, International Journal of Production Economics, Vol.179, 273-284.

[25] Mazumdar, T., Raj, S. P. and Sinha, I. (2005). *Reference price research: Review and propositions*, Journal of Marketing, Vol.69, 84-102.

[26] Mathies, C. and Gudergan, S. (2007). *Revenue management and customer centric marketing-how do they influence travellers' choices?* Journal of Revenue and Pricing Management, Vol.6, 331-346.

[27] Nasiry, J. and Popescu, I. (2011). *Dynamic pricing with loss-averse consumers and peak-end anchoring*, Operations Research, Vol.59, 1361-1368.

[28] Popescu, I. and Wu, Y. (2007). *Dynamic pricing strategies with reference effects*, Operations Research, Vol.55, 413-429.

[29] Roni, M. S., Jin, M. Z. and Eksioglu, S. D. (2015). *A hybrid inventory management system responding to regular demand and surge demand*, Omega, Vol.52, 190-200.

[30] Ren, H. and Huang, T. L. (2018). *Modeling customer bounded rationality in operations management: A review and research opportunities*, Computers & Operations Research, Vol.91, 48-58.

[31] Sethi, S., Yan, H. and Zhang, H. (2004). *Quantity flexible contracts: Optimal decisions and information revisions*, Decision Science, Vol.35, 691-712.

[32] Simchi-Levi, D., Chen, X. and Bramel, J. (2014). The Logic of Logistics: Theory, Algorithms, and Applications for Logistics Management, 3rd ed, Springer, New York, USA.

[33] Taudes, A. and Rudloff, C. (2012). *Integrating inventory control and a price change in the presence of reference price effects: a two-period model*, Mathematical Methods of Operations Research, Vol.75, 1-37.

[34] Tsay, A. A. and Lovejoy, W. S. (1999). *Quantity flexibility contracts and supply chain performance*, Manufacturing & Service Operations Management, Vol.1, 89-111.

[35] Urban, T. (2008). *Coordinating pricing and inventory decisions under reference price effects*, International Journal of Manufacturing Technology and Management, Vol.13, 78-94.

[36] Yao, M. and Minner, S. (2017). *Review of multi-supplier models in supply chain management: An update*, Working Paper, Technische Universität München, Germany.

[37] Zhang, Y. (2010). *Essays on robust optimization, integrated inventory and pricing, and reference price effect*, Ph.D. Thesis, University of Illinois at Urbana-Champaign, Urbana, Illinois, USA.

[38] Zhou, S. X. and Chao, X. (2014). *Dynamic pricing and inventory management with regular and expedited supplies*, Production and Operations Management, Vol.23, 65-80.

[39] Zhu, S. X. (2012). *Joint pricing and inventory replenishment decisions with returns and expediting*, European Journal of Operational Research, Vol.216, 105-112.

School of Economics and Management, Yanshan University, Qinhuangdao, Hebei province, P.R.China.

College of Mathematics, Inner Mongolia University for The Nationalities, Tongliao, The inner mongolia autonomous region, P.R.China.

E-mail: imunliyuan@163.com

Major area(s): Supply chain and logistics management, behavior operation management, variational inequalities and complementarity problems.

School of Economics and Management, Yanshan University, Qinhuangdao, Hebei province, P.R.China. (Corresponding author)

E-mail: hym_1220@163.com

Major area(s): Supply chain and logistics management, medical, health and pension management.