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The Study of the Optimal Joint Decision on Pricing and Inventory Policies with Dual Supplies and Reference Price

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Abstract.

The main objective of this research is to ascertain the impacts of reference price effects on the joint optimization of pricing and inventory. Using dynamic programming, we study a single-item, periodic-review finite horizon joint pricing and inventory system with dual suppliers under reference price effects. Hence, in our model, the demands in consecutive periods are designed to be independent and sensitive random variables to the sales' price and customer's reference price. The main results are that the optimal replenishment policies for the reliable and unreliable supplier are the base-stock policy and the reorder point policy, respectively, and all the parameters of the optimal policy are reference-price-dependent.

1. Introduction

Integrating decisions about pricing and inventory can significantly improve the profits of firms. Hence, it becomes a major strategy of many firms such as Dell, Amazon, FairMarket (see Feng [15]). In addition, in order to avoid the sales losses caused by the uncertainty of supply and related costs such as procurement, maintaining two supply sources that charge different unit costs and different reliabilities for one commodity is common in the procurement practice. Studies have also shown that diversification is beneficial to both the firm and its customers (see Zhou and Chao [39]). Hence, it is interesting and necessary to investigate the joint pricing and inventory decisions with dual supplies with different reliabilities.

The reference price, as the cognitive price of customers, was first derived from the adaptation level (see Helson and Bevan [22]). Later, prospects theory (see Kahneman and Tversky [25]) and behavioral sciences (see Kalyanaram and Winer [26]) systematically elaborated on the reference price, and they indicated that customers would remember past prices with repeated transactions and develop price expectations for commodities. This expectation, which is captured by the reference price, acts as a benchmark against which customers compare the price of a commodity. If the current selling price is lower (higher) than the reference price, customers see it as a gain (loss), and hence are more likely (less inclined) to make the purchase. This phenomenon is usually called the reference price effect. Customers are called loss averse (loss neutral) if their demand is more (as) responsive to customers' perceived losses than (as) their perceived gains. Otherwise, they are called loss/gain seeking. In practice, this reference price effect has an important impact on demand and therefore becomes an indispensable part of firms' decision-making.

Because of the significant effects of the reference price on customers' purchasing behaviors, reference price effects have received a great deal of attention from practitioners and researchers. Firms in many industries, such as electronics, clothing and other tidal commodities, have been aware of the reference price effects and made appropriate pricing strategies to take advantage of them in order to achieve success (see Mathies and Gudergan [28]). There is a lot of research that studies the pricing decisions with reference price effects. This line of research started in the 1990s. Krishnamurthi et al. [24] studied the impacts of reference price effects on brand selection and purchase quantity, and show that customers have the characteristics of brand loyalty under symmetrical reference prices, while it does not appear that such characteristics exist under asymmetric reference price effects. Greenleaf [20] first analyzed the firm's pricing strategy with reference price effects and explained how the reference price effects affect the promotion decision of a firm during a period. He concluded that firm's pricing decision when considering the reference price effects will increase the firm's profits. Some recent works explore how pricing strategies should account for the reference price effects. For example, see Chen et al. [8], Chenavaz et al. [9], Fibich et al. [**?**], Hu et al. [23], Nasiry and Popescu [29], Wang et al. [35] and the references therein. Arslan and Kachani [2] and Mazumdar et al. [27] provided reviews of a dynamic pricing model with reference price effects. However, the literature sparsely investigates the joint pricing and inventory strategies with consideration of the reference price effects.

To the best of our knowledge, only a few papers have integrated reference price effects into the pricing and inventory control model. This line of research started with Gimpl-Heersink [18], who proved the optimality of the base-stock-list-price for the single-period and two-period models when the customers are loss neutral. However, the optimality of the base-stock-list-price is stricter for the multiperiod setting. Urban [34] analyzed a single-period joint pricing and inventory model with symmetric and asymmetric reference price effects and shows that the consideration of the reference price has a substantial impact on the firm's profitability. Taudes and Rudloff [33] provided an application of the two-period model from Gimpl-Heersink [18] to electronic commodities. Güler et al. [21] used the safety stock as a decision variable to characterize the steady state solution to the problem when the planning horizon is infinite. Chen et al. [9] introduced a new concave transform technique to ensure that the profit function is concave by using the preservation property of supermodularity in parameter optimization problems with the nonlattice structure proposed by Chen et al. [7], and then proved the optimality of the base-stock-list-price strategy. Zhang et al. [38] studied the continuous time pricing and inventory joint decision-making problem affected by asymmetric reference price effects. Cao and Duan [3] studied the joint decision-making of continuous time pricing and inventory under the effects of stochastic reference price. Wang et al. [36] further considered the multi-period pricing and inventory decision-making model of retailers under the impact of reference price and consumers' strategic behavior. For other related works in this stream of research, interested readers may refer to the review by DeYong [11] and Ren and Huang [31]. All these studies focus on the single-supplier and the supplier is generally assumed to be reliable.

Another closely related stream of research is the multiperiod procurement models from suppliers with random yields and the coordination of pricing and inventory decisions with the absence of reference price effects. This line of work was initiated by Anupindi and Akella [1], who analyzed a multiperiod model with two unreliable suppliers and investigated the optimal inventory policy of the buyer. Recently, Federgruen and Yang [13] studied a more general problem with multiple unreliable suppliers, and show that the optimal procurement policy for each supplier is a threshold type. They also develop procedures to identify the optimal set of suppliers and the order quantity allocated to each supplier. Chen et al. [5] considered the joint pricing and inventory control problem for a system with multiple random-yield suppliers. They show that the optimal inventory replenishment policy for each supplier is in general not a reorder point policy, but rather a near reorder point policy. Zhou and Chao [36] considered an inventory system with regular and expedited supply modes with lead times of 1 and 0, respectively. They show that the optimal inventory policy is determined by two state-independent thresholds, one for each supply mode, and the optimal price follows a list-price policy. Gong et al. [19] developed a joint pricing and inventory control problem that has a quick-response supplier with a lead time of 0 and a regular supplier with a lead time of 1 that both suffer disruption risks. They show that the replenishment policy for each supplier is a reorder point policy and the optimal price is monotonic in the initial inventory level. Chao et al. [4] studied a dual-supplier inventory system in which one supplier is reliable, and the other supplier is not reliable and has a general random yield. They show that the optimal replenishment policy for the reliable supplier is a base-stock policy, the replenishment policy for the random-yield supplier is a reorder point policy, and the optimal pricing policy is a list-price policy with markdowns. Chen and Tan [6] discussed the procurement from multiple suppliers with uncertain capacities and analyze the optimal ordering policy when one of the suppliers is reliable. Their analysis shows that having a reliable supplier results in a relatively stable optimal ordering policy, despite the unreliability of the rest of the suppliers. Niu et al. [30] further studied the pricing and inventory model of dual channel retailers from the perspective of procurement interruption risk. Difrancesco et al. [12] studied the buyer's contract mechanism in the two-echelon supply chain system under random demand, supply uncertainty and interruption risk, namely risk sharing contract and repurchase contract. A more complete literature review of this line of research is provided in a recent paper by Yao and Minner [37].

Our model differs from the aforementioned two streams of research with respect to two aspects. First, our model integrates the pricing and inventory decisions with supply uncertainty, i.e., we consider two supply sources: one reliable supplier and one unreliable supplier with a random supply yield. The reliable supplier can fully deliver the firm's order in the period when it is placed with a more expensive unit ordering cost; meanwhile, the unreliable supplier can only deliver a random proportion of the firm's order quantity in the period when it is placed, but with a less expensive unit cost. Second, we consider the customers' reference price effects. Specifically, we study a single-item, periodic-review joint pricing and inventory system with reliable and random-yield suppliers under reference price effects. The demands in consecutive periods are independent and sensitive random variables to the price and reference price. In other words, the demand distribution depends on the price charged for that period and the reference price generated by customers in the same period. Unfilled demands are fully backlogged. The purpose of this paper is to find the dynamic joint optimal policies that determine the pricing and inventory replenishment in each period so that the total expected discounted profits are maximized. To the best of our knowledge, the present work is the first attempt to analyze the joint pricing and inventory control problem with reliable and random-yield suppliers under reference price effects. The guarantee of the profit-to-go function's concavity and supermodularity, which is a critical technical problem, allows us to analyze the optimal pricing and inventory strategies and the impacts of reference price effects on optimal decisions. We show that the optimal replenishment policy for the reliable supplier is a base-stock policy, the replenishment policy for the random-yield supplier is a reorder point policy, and the optimal pricing policy is a list-price policy with markdowns, and all the optimal policy parameters are reference-price-dependent. The impacts of reference price effects on the inventory replenishment strategies and the pricing decisions are also studied. We further study the operational impacts of adding reference price effects by comparing the results with the model proposed by Chao et al. [4], hereinafter referred to as CG model. All the above research extends the results of the CG model to the reference price effects.

The remainder of this paper is organized as follows. We present the finite period model with stochastic dynamic programming in Section 2, and characterize the optimal policies in Section 3. Section 4 investigates the operational impacts from the perspective of adding reference price effects. Some numerical analysis are represented in Section 5 to verify our results. Section 6 provides some managerial insights and Section 7 concludes our paper.

2. Model Description

We consider a single-item, periodic-review problem for a firm in a finite planning horizon with $T(1 \le T \le \infty)$ periods. The firm has two suppliers, which are referred to as suppliers ℓ, $\ell = 1, 2$. Supplier 1 is reliable and can fully deliver the firm's order in the period in which it is placed with a unit ordering cost of c_1; meanwhile, supplier 2 is unreliable and can only deliver a random proportion of the firm's order in the period it is placed, but with a lower unit cost of c_2, i.e., $c_1 > c_2 > 0$. Let $\mathscr{L}_t(q_t)$ denote the random quantity the firm receives when it orders q_t from supplier 2 in period t. We assume a stochastically proportional yield model with

$$\mathscr{L}_t(q_t) = \Theta_t q_t, \quad t = 1, 2, \ldots, T,$$

where $\Theta_1, \Theta_2, \ldots, \Theta_T$ are random variables with support $[0, 1]$ and mean θ_t for period t. In the following discussion, we assume that $\Theta_1, \Theta_2, \ldots, \Theta_T$ are independent random variables.

The demand in period t, which is denoted by D_t is nonnegative and independent random variables. Similar to Güler et al. [21], the demand D_t is given by

$$D_t(p_t, r_t, \varepsilon_t) = d_t(p_t, r_t) + \varepsilon_t,$$

where $d_t(p_t, r_t)$ is the mean demand function that is a deterministic function of the unit selling price p_t and the reference price r_t in period t. $D_t(p_t, r_t, \varepsilon_t)$ is nonnegative and follows a continuous probability distribution, and ε_t is a random variable with zero mean and is independent of p_t and r_t. This demand function is very general and includes the additive and multiplicative models as special cases.

The mean demand is $d_t(p_t, r_t) = \mu_t(p_t) + R_t(r_t - p_t, r_t)$, where $\mu_t(p_t) = d_t(p_t, p_t)$ is called the base demand and $R_t(r_t - p_t, r_t) = \eta^+ \max\{r_t - p_t, 0\} + \eta^- \min\{r_t - p_t, 0\}$ is called the reference price effects on demand (see Helson and Bevan [22]). The nonnegative parameters η^+ and η^- measure the sensitivities of demand associated with the perceived gains and losses, respectively. Demand is classified as loss averse, loss neutral, or loss/gain seeking, depending on whether $\eta^+ \leq \eta^-$, $\eta^+ = \eta^-$ or $\eta^+ \geq \eta^-$. For more information about $R_t(r_t - p_t, r_t)$, we refer to Güler et al. [21] and the references therein.

We assume that the price in each period, p_t is restricted to a bounded interval $[\underline{p}, \overline{p}]$. The reference price depends on past prices and the current price. A commonly used model for the evolution of the reference price is the exponential smoothing model (Chen et al. [8]; Gimpl-Heersink [18]; Güler et al. [21]):

$$r_{t+1} = \alpha r_t + (1 - \alpha) p_t,$$

where $\alpha (0 \leq \alpha < 1)$ is the memory factor. The larger the memory factor is, the longer the memory. If α is high, then customers have a long memory and the past price effect is larger. If α is small, then the current price has a greater effect than the past on the reference price. The initial reference price is given by $r_t \in [\underline{p}, \overline{p}]$, and thus all r_t belong to the interval. Moreover, we introduce the following structure on the mean demand.

Assumption 1. *The mean demand $d_t(p_t, r_t)$ is concave, bounded, nonnegative and continuous, and it is strictly decreasing in p_t and increasing in r_t for $t = 1, 2, \ldots, T$.*

It is worth mentioning that the existence of the mean demand functions that satisfy Assumption 1 has been proved in Güler et al. [21] when customers are loss neutral or loss averse and some examples are presented. Hence, this paper assumes that the customers are loss neutral or loss averse. In addition, Assumption 1 implies that $p_t(d_t, r_t)$ is concave in (d_t, r_t) (Proposition 1, Güler et al. [21]), where $p_t(d_t, r_t)$ is the inverse function of the mean demand $d_t(p_t, r_t)$ for a given r_t. Moreover, $p_t(d_t, r_t)$ is strictly decreasing in d_t and increasing in r_t for $t = 1, 2, \ldots, T$ (Proposition 1, Güler et al. [21]). Hence, determining the price is equivalent to determining the mean demand. In the discussion below, without other specifications, we will focus on finding the optimal mean demand d_t for period t. Therefore, we assume that the feasible region of the mean demand in period t is $d_t \in [\underline{d}_t, \overline{d}_t]$, where $\underline{d}_t \geq 0$ and $\overline{d}_t < +\infty$.

We summarize the notations that will be used in this paper as follows:

x_t = the initial inventory level before any decisions are made in period t,
y_t = the inventory level after placing the order from supplier 1 in period t,
q_t = the ordering quantity from supplier 2 in period t,
h_t = the unit holding cost in period t,
b_t = the unit backorder penalty cost in period t, and
γ = the discount factor, $0 \le \gamma < 1$.

At the end of each period after demand is realized, the remaining inventory is carried over to the next period and incurs holding costs, while unsatisfied demand is backlogged and incurs shortage costs. Let $G_t(z)$ be the inventory holding/backlogging costs when the ending inventory level is z in period t. Then, the expected holding/backlogged costs can be written as

$$G_t(z) = h_t\mathrm{E}[\max\{z - \varepsilon_t, 0\}] + b_t\mathrm{E}[\max\{\varepsilon_t - z_t, 0\}],$$

and we assume that $G_t(z)$ is convex in z.

Given the initial inventory and the reference price in each period where $t = 1, 2, \ldots, T$, this problem can then be formulated as a dynamic programming and the Bellman equation for this problem is

$$\begin{aligned} V_t(x_t, r_t) = \max_{\substack{y_t \ge x_t, q_t \ge 0 \\ \underline{d}_t \le d_t \le \overline{d}_t}} &\{R_t(d_t, r_t) - c_1(y_t - x_t) - c_2\theta_t q_t - \mathrm{E}[G_t(y_t - d_t + \Theta_t q_t - \varepsilon_t] \\ &+ \gamma \mathrm{E} V_{t+1}(y_t - d_t + \Theta_t q_t - \varepsilon_t, \alpha r_t + (1-\alpha)p_t(d_t, r_t))\}. \end{aligned} \tag{2.1}$$

For convenience, we denote

$$\begin{aligned} J_t(y_t, d_t, q_t, r_t) =& R_t(d_t, r_t) - c_1 y_t - c_2\theta_1 q_t - \mathrm{E}[G_t(y_t - d_t + \Theta_t q_t - \varepsilon_t)] \\ &+ \gamma \mathrm{E} V_{t+1}(y_t - d_t + \Theta_t q_t - \varepsilon_t, \alpha r_t + (1-\alpha)p_t(d_t, r_t))\}, \end{aligned} \tag{2.2}$$

and then (2.1) becomes

$$V_t(x_t, r_t) = \max_{\substack{y_t \ge x_t, q_t \ge 0 \\ \underline{d}_t \le d_t \le \overline{d}_t}} J_t(y_t, d_t, q_t, r_t) + c_1 x_t, \tag{2.3}$$

where V_t is the profit-to-go function, J_t is the value function of period t, and E denotes the expectation operator. The terminal value is given by $V_{T+1}(x_{T+1}, r_{T+1}) = 0$.

Furthermore, we make the following assumption.

Assumption 2. *The inverse function $p_t(d_t, r_t)$ of the mean demand $d_t(p_t, r_t)$ is supermodular in (d_t, r_t) and the revenue function $d_t \cdot p_t(d_t, r_t)$ is joint concave in (d_t, r_t).*

Under Assumption 2, the revenue function $d_t \cdot p_t(d_t, r_t)$ is supermodular in (d_t, r_t) according to Theorem 6 in Güler et al. [21].

3. Optimal Policy and Its Analysis

In this section, we characterize the firm's optimal ordering and pricing strategies. We first need the concavity of J_t and V_t. Hence, there exists a unique optimal decision in each period for a given x_t and r_t.

Lemma 1. *For $t = 1, 2, \dots, T$, we have*

(i) $V_t(x_t, r_t)$ *is decreasing in* x_t *and increasing in* r_t*,*

(ii) $J_t(y_t, d_t.q_t, r_t)$ *is joint concave in* (y_t, d_t, q_t, r_t)*, and*

(iii) $V(x_t, r_t)$ *is joint concave in* (x_t, r_t)*.*

The conclusions in Lemma 1 are similar to the relevant results of regular replenishment literature [8] and [21], which reflects the intuition of myopic retailers, that is, if consumers have higher expectations for the reference price, this higher expectation will stimulate market demand, retailers focusing on single-period revenue will order more goods through regular replenishment to increase their inventory level. However, this single-period replenishment mode often increases the remaining inventory in the current period due to the excessive order quantity, and the backlog of these remaining inventory to the next period will produce higher inventory costs. Therefore, the importance of supply flexibility is highlighted. Supported by the concavity theory of Lemma 1, it not only demonstrates the existence of the periodic optimal solution of dynamic programming problem (2.1), but also lays a theoretical foundation for further analyzing the retailer's optimal pricing and inventory strategy under the dual replenishment mode.

For $t = 1, 2, \dots, T$, we define

$$d_t^*(r_t) = \arg \max_{\underline{d}_t \le d_t \le \bar{d}_t} \{p_t(d_t, r_t) \cdot d_t - c_1 d_t\}, \tag{3.1}$$

and we characterize the firm's optimal pricing and ordering policies in the following theorem.

Theorem 1. *For $t = 1, 2, \dots, T$, the firm's optimal policies for period t are characterized by two critical numbers $z_{t,1}^*(r_t)$ and $\xi_{t,2}^*(r_t)$, and a list price p_t^* (for an optimal average demand d_t^* as $p_t^* = p_t(d_t^*)$) as follows.*

(a) *The optimal ordering policy from supplier* 1 *is a base-stock policy with the inventory level of $z_{t,1}^*(r_t) + d_t^*(r_t)$, i.e.,*

$$y_t^*(x_t, r_t) = \max\{z_{t,1}^*(r_t) + d_t^*(r_t), x_t\}.$$

(b) The optimal ordering policy from supplier 2 is a threshold policy with the inventory level of $\xi_{t,2}^*(x_r, r_t)$, where $\xi_{t,2}^*(x_r, r_t) \ge z_{t,1}^*(r_t) + d_t^*(r_t)$, such that if $x_t < \xi_{t,2}^*(x_t, r_t)$, then the firm orders a positive quantity from supplier 2 and the ordering quantity $q_t^*(x_t, r_t)$ decreases with the starting inventory level; otherwise, the firm orders nothing from supplier 2.

(c) The optimal pricing policy is a list-price policy with markdowns. When $x_t \le z_{t,1}^*(r_t) + d_t^*(r_t)$, $d_t^*(x_t, r_t) = d_t^*(t_t)$ and $p_t^*(x_t, r_t) = p_t(d_t^*(r_t))$; and when $x_t > z_{t,1}^*(r_t) + d_t^*(r_t)$, $p_t^*(x_t, r_t)$ is a decreasing function of x_t, and $p_t^*(x_t, r_t) \le p_t(d_t^*(r_t))$.

Theorem 1 illustrates that the state space of the inventory level at the beginning of period t is divided into three intervals. If $x_t > \xi^*_{t,2}(x_r, r_t)$ then it is optimal to order nothing from either supplier. If $x_t \leq \xi^*_{t,2}(x_r, r_t)$ then it depends on whether or not $x_t \leq z^*_{t,1}(r_t)+d^*_t(r_t)$. In the first case, as the inventory level after ordering from supplier 1 is $z^*_{t,1}(r_t)+d^*_t(r_t)$ which is dependent on the reference price r_t, the ordering quantity from supplier 2 is also dependent on the reference price r_t. Consequently, in this case, there is a reference-price-dependent order-up-to level for supplier 1 and the reference-price-dependent ordering quantity for supplier 2. If, however, $z^*_{t,1}(r_t)+d^*_t(r_t) < x_t < \xi^*_{t,2}(x_t, r_t)$, then the firm only orders from supplier 2 and the order quantity is a decreasing function of x_t.

To analyze the reference price effects on the pricing and inventory policies, we need the following lemma.

Lemma 2. *For $t = 1, 2, \ldots, T$, $V_t(x_t, r_t)$ is supermodular in (x_t, r_t).*

Corollary 1. *For $t = 1, 2, \ldots, T$, we have*

(i) *$J_t(y_t, d_t, q_t, r_t)$ is supermodular in (y_t, r_t),*
(ii) *$J_t(y_t, d_t, q_t, r_t)$ is supermodular in (d_t, r_t), and*
(iii) *$J_t(y_t, d_t, q_t, r_t)$ is supermodular in (q_t, r_t).*

Based on this, we can characterize the reference price effects on the optimal inventory replenishment and pricing policies via the following theorem.

Theorem 2. *For $t = 1, 2, \ldots, T$, we have*

(i) *The optimal inventory level y^*_t from supplier 1 is increasing in r_t.*
(ii) *The optimal ordering quantity q^*_t from supplier 2 is increasing in r_t.*
(iii) *The optimal mean demand d^*_t is increasing in r_t.*
(iv) *The price p^*_t is increasing in r_t, and*
(v) *The optimal profits $V^*_t(x_r, r_t)$ are increasing in r_t.*

The implied practical significance of Theorem 2 is as follows. With the increase of consumer reference price, consumers' valuation of the goods will also increase, which will stimulate their purchase desire and increase the potential market demand. The increase of consumers' valuation will enable retailers to increase the sales. At the same time, the increase of market demand will increase the order quantity of retailers, including both supplier 1 and supplier 2. Although its cost is higher from supplier 1, retailers make up for the loss of shortage as much as possible by increasing the order quantity. In view of the positive impact of consumer reference prices on the overall market demand, retailers will still increase their orders in the face of large market demand.

4. Operational Impact of Reference Price

Since the impact of supply diversification has been discussed in Chao et al. [4], we mainly analyze the operational impacts from the perspective of the reference price effects by comparing our model with the CG model. Although the CG model considers dual suppliers, one is reliable and the other is unreliable, it doesnt take the reference price effects into consideration. To distinguish the CG model from ours, we use the superscript c to signify the notation for the CG model. The following is the main results on the impact of adding reference price effects.

Theorem 3. When $r_t > p_t$, after the reference price effects are considered, the optimal profit-to-go function and optimal policy parameters, for $t = 1, 2, \ldots, T$, satisfy

(i) $V_t^*(x_t, r_t) \geq V_t^{c^*}(x_t)$;
(ii) $y_t^*(x_t, r_t) \geq y_t^{c^*}(x_t)$;
(iii) $q_t^*(x_t, r_t) \geq q_t^{c^*}(x_t)$;
(iv) $d_t^*(x_t, r_t) \geq d_t^{c^*}(x_t)$; and
(v) $p_t^*(x_t, r_t) \geq p_t^{c^*}(x_t)$.

Otherwise, the above conclusions are opposite.

This theorem can be intuitively illustrated as follows. Part (i) states that when more consideration is given to the customers' behaviors, the firm can only do better, and thus its maximum profits will not decrease. Parts (ii), (iii), (iv) and (v) indicate that with the increase of customers' reference price, the optimal mean demand will increase and the optimal price will rise as well. In addition, considering the lead time for regular replenishment and the incremental demand under the reference price effects, the firm orders more product using expedited and regular supply to raise the inventory level to meet the customers' needs as much as possible.

5. Numerical Analysis

In this section, we present several numerical experiments to illustrate the impacts of reference price on the optimal policy parameters, including the optimal price p_t^*, the optimal inventory level y_t^* after placing the order from supplier 1, the ordering quantity q_t^* from supplier 2 and the optimal profit V_t^*. Besides, we analyze the operational impacts on firm's profit by adding reference price effects via comparing with CG model. All experiments below are performed in MATLAB R2014b on a laptop with an Intel(R) Core (TM) i5-7200U central processing unit CPU (2.50 GHz, 2.70GHz) and 8.0 GB of RAM running 64-bit Windows 10 Enterprise.

Consider a system with planning horizon $T = 4$. We perform the numerical simulations with the following basic parameter values: $c_1 = 18$, $c_2 = 15$, $\eta^+ = 0.3$, $\eta^- = 0.5$, $\gamma = 0.95$. The mean demand function is given by $d(p_t, r_t) = 200 - 2p_t + 1.5 \max\{r_t - p_t, 0\} + 2.5 \min\{p_t - r_t, 0\}$, the inventory holding or backlogged cost is $G_t(z) = h_t \mathrm{E}[\max\{z - \varepsilon_t, 0\}] + b_t \mathrm{E}[\max\{\varepsilon_t - z, 0\}]$ with $h_t = 2$, $b_t = 20$ and $\varepsilon_t \sim$ Uniform $[-1, 1]$.

We first analyze the impact of the memory factor α on the optimal price p_t^*, the optimal inventory level y_t^* after placing the order from supplier 1, the optimal ordering quantity q_t^* from supplier 2 and the optimal profit V_t^*. We report the results for $\alpha = 0.3$, 0.5, 0.7, 0.9. The corresponding results are shown in Figure 1 to Figure 4. It is shown from Figures 1-4 that the optimal price p_t^*, the optimal inventory level y_t^* after placing the order from supplier 1, the optimal ordering quantity q_t^* from supplier 2 and the optimal profit V_t^* are increasing in the reference price r, which is consistent with Theorem 2. Furthermore, the optimal price p_t^*, the optimal inventory level y_t^* after placing the order from supplier 1, the optimal ordering quantity q_t^* from supplier 2 and the optimal profit V_t^* are decreasing in the memory factor α. This indicates that the memory factor α has a negative impact on these optimal variables. Figures 1-4 suggest that with the increase of memory factor α, i.e., the customers' ability to remember past prices becomes weaker, they adapt to the new price at a lower rate and less loyalty, then the firm should decrease its sales price while reducing its order quantity from both supplier 1 and 2. This will inevitably affect the firm's profits.

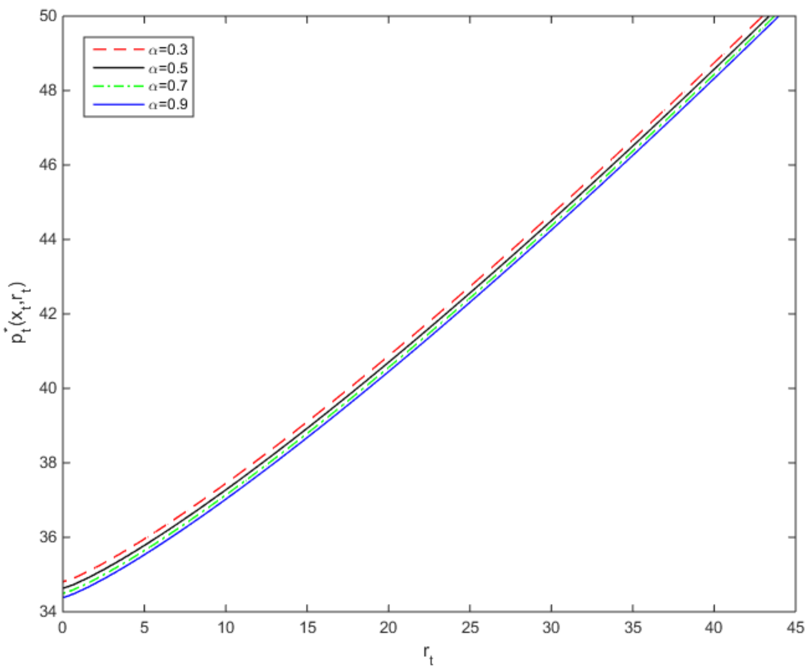

Figure 1: The impact of memory factor α on the optimal price p_t^*.

Next, we analyze the impact of the reference price effect coefficient η on the optimal price p_t^*, the optimal inventory level y_t^* after placing the order from supplier 1, the optimal ordering quantity q_t^* from supplier 2 and the optimal profit V_t^*. We report the results for $\eta = 0.3$, 0.5, 0.7, 0.9. The corresponding results are shown in Figure 5 to Figure 8. It is shown from Figures 5-8 that the optimal price p_t^*, the optimal inventory level y_t^* after placing the order from supplier 1, the optimal ordering quantity q_t^* from supplier 2 and the optimal profit V_t^* are decreasing in reference price effect coefficient η when $r_t < p_t$ while increasing in reference price effect coefficient η when $r_t > p_t$, which is consistent with Theorem 3. This indicates that the reference price effect coefficient η has a negative

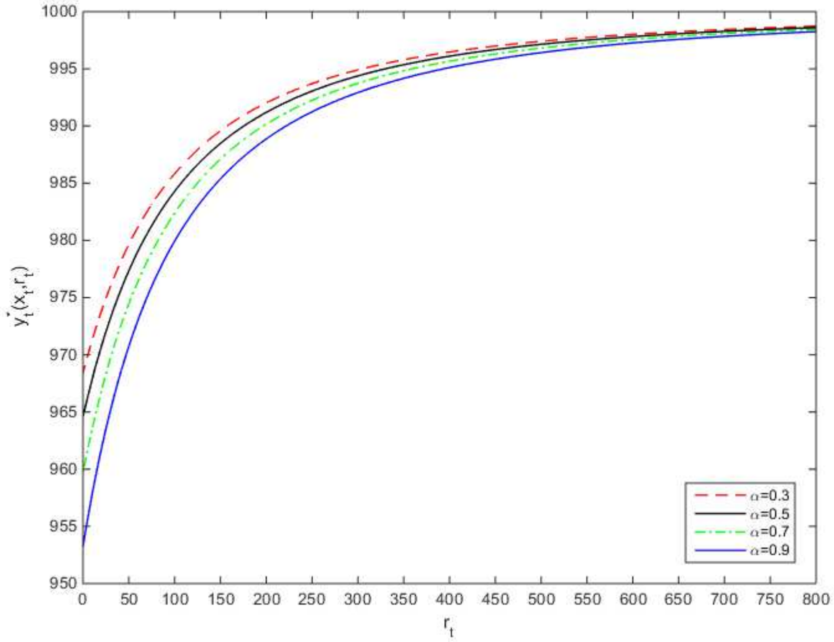

Figure 2: The impact of memory factor α on the optimal inventory level y_t^* after placing the order from supplier 1.

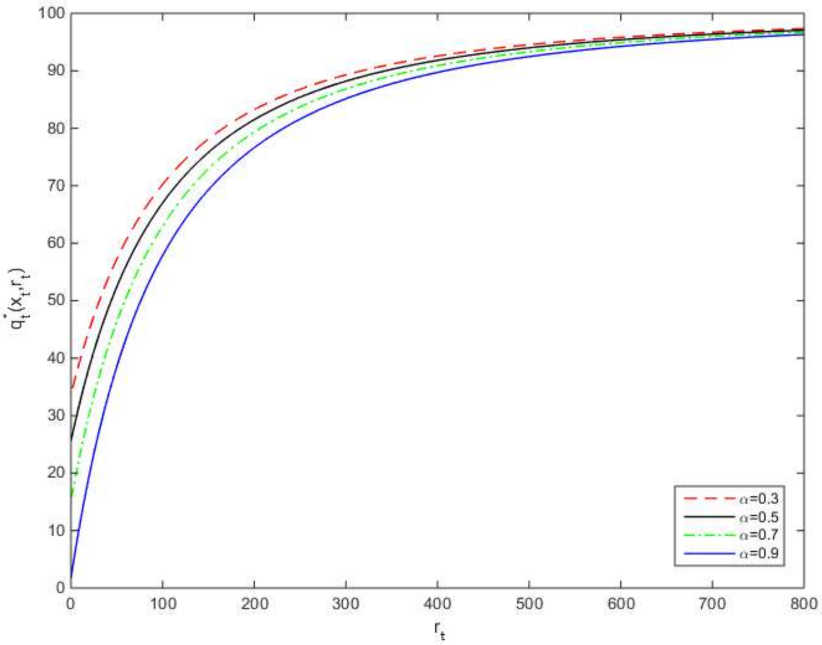

Figure 3: The impact of memory factor α on the optimal ordering quantity q_t^* from supplier 2.

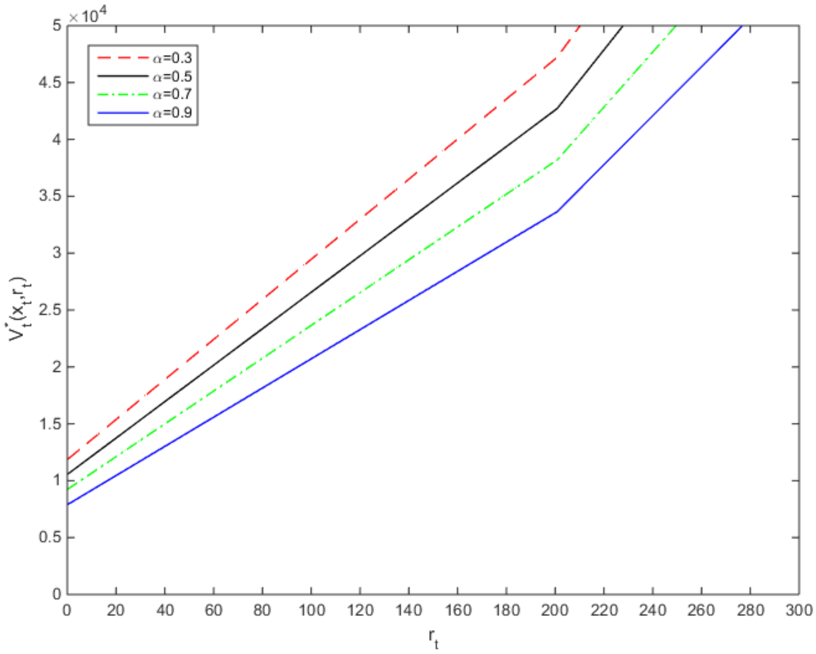

Figure 4: The impact of memory factor α on the the optimal profit V_t^*.

impact on these optimal strategy parameters when $r_t < p_t$ while has a positive impact on these optimal strategy parameters when $r_t > p_t$. Figures 5-8 suggest that when $r_t < p_t$, with the increase of reference price effect coefficient η, i.e., consumers are more sensitive to the difference between the reference price and the actual sales price, so they are more reluctant to buy, then the firm should decrease its sales price while reducing its order quantity from both supplier 1 and 2. This will inevitably affect the firm's profits. When $r_t > p_t$, the opposite is true, i.e., consumers will think that they have earned it and will be more willing to buy. The firm should increase its sales price while increasing its order quantity from both supplier 1 and 2 to gain more profits.

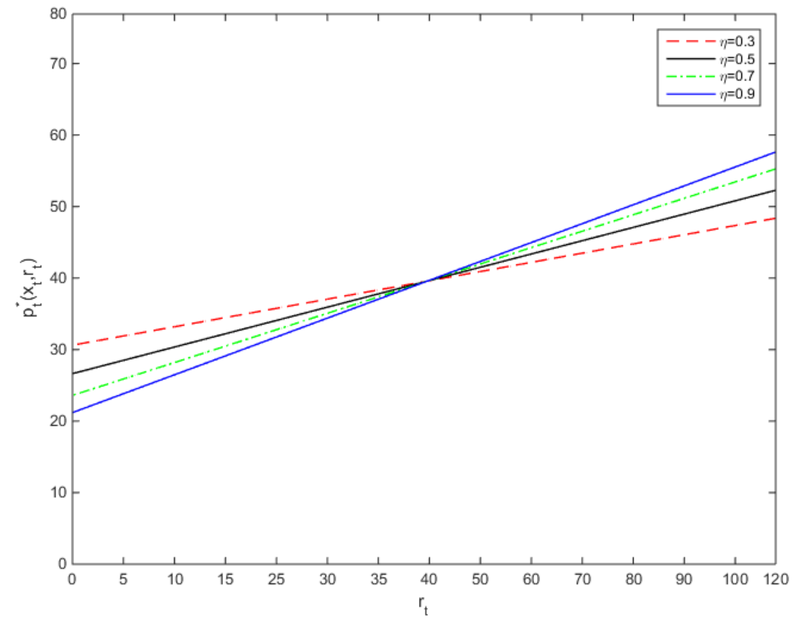

Figure 5: The impact of reference price effect coefficient η on the optimal price p_t^*.

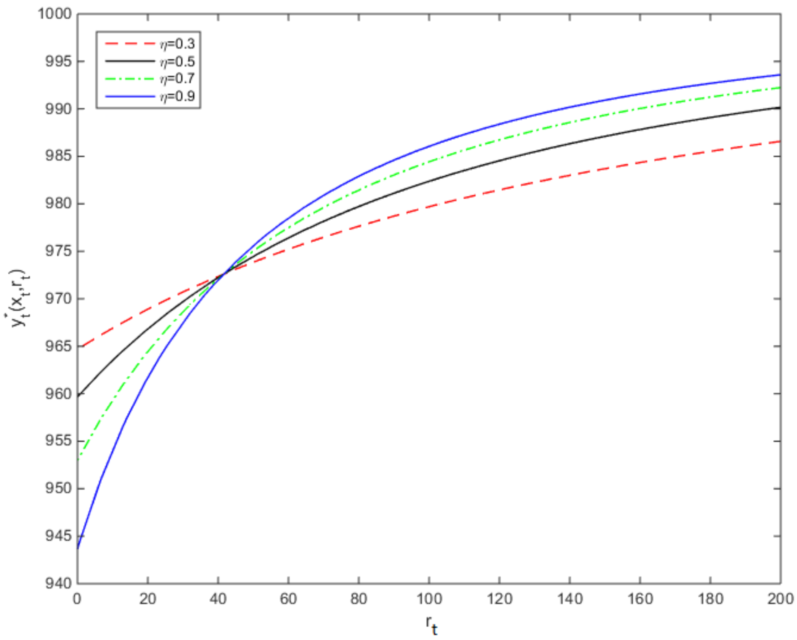

Figure 6: The impact of reference price effect coefficient η on the optimal inventory level y_t^* after placing the order from supplier 1.

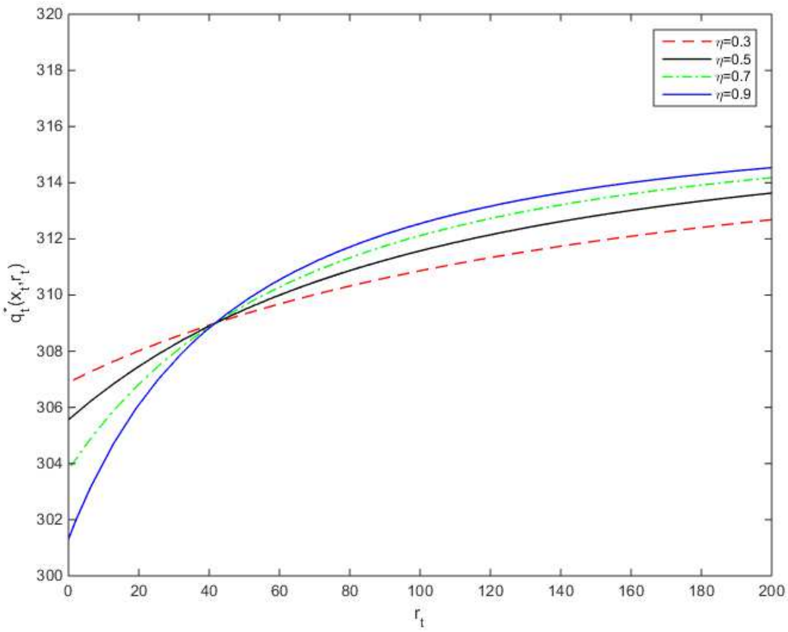

Figure 7: The impact of reference price effect coefficient η on the optimal ordering quantity q_t^* from supplier 2.

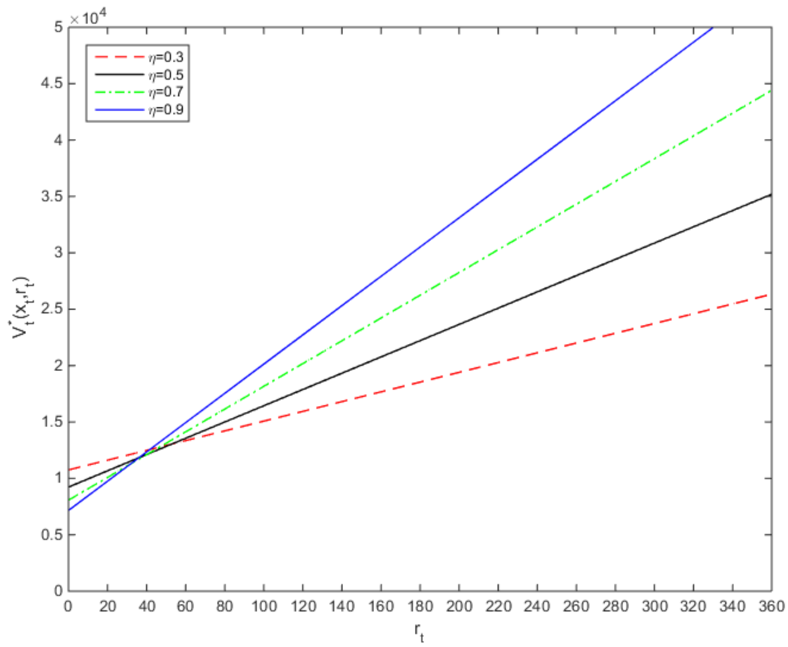

Figure 8: The impact of reference price effect coefficient η on the the optimal profit V_t^*.

6. Managerial Insights

In this section, we provide some insights for management practice, which can be adopted by firms to formulate their pricing and inventory strategies with a reliable supplier and an unreliable supplier under the reference price effects.

(1) When the reference price effects is considered, customers' ability to remember past prices has a significant effect on managing the optimal pricing and inventory decisions. As memory factor α increases, customers adapt to the new price information at a lower rate and become less loyal to the commodity. At this time, the firm should apply a low sales price to achieve a positive reference price effects. At the same time, the inventory level for both supplier 1 and supplier 2 should also be reduced so as to reduce the holding cost caused by demand uncertainty.

(2) The reference price effect coefficient η has a negative impact on optimal price, optimal inventory level and profits when $r_t < p_t$ while has a positive impact on these optimal strategy parameters when $r_t > p_t$. When $r_t < p_t$, with the increase of reference price effect coefficient η, i.e., consumers are more sensitive to the difference between the reference price and the actual sales price, so they are more reluctant to buy, then the firm should apply a low sales price while reducing its order quantity from both supplier 1 and 2. When $r_t > p_t$, the opposite is true, i.e., consumers will think that they have earned it and will be more willing to buy. The firm should apply a high sales price while increasing its order quantity from both supplier 1 and 2 to gain more profits.

Through the discussion of this paper, it can be seen that the consumers' reference price effects have great impacts on the pricing and inventory strategies of retailers. The main task of firms is to increase the consumers' reference price, improve the consumers' valuation of the commodities, and then improve their satisfaction and loyalty to the commodities, which in turn increases the revenue.

7. Conclusions

Our research complements the existing research stream in coordinating pricing and inventory replenishment decisions from two aspects. On the one hand, we consider inventory planning decisions for dual supply sources, i.e., one reliable and one unreliable supplier with random supply yields. On the other hand, we consider the impact of the customers' behaviors (i.e., customers' reference price) on the joint pricing and inventory replenishment decision. We study a single-item, periodic-review joint pricing and inventory system with reliable and random-yield suppliers under reference price effects. The demands in consecutive periods are independent and sensitive random variables to the price and reference price. Unfilled demands are fully backlogged. We show that the optimal replenishment policy for the reliable supplier is a base-stock policy, the replenishment policy for the random yield supplier is a reorder point policy, and the optimal pricing policy is a list-price policy with markdowns, and all the optimal policy parameters are reference-price-dependent. The impacts of the reference price effects on the inventory replenishment strategies and the pricing decisions are also studied. We further study the operational impacts of adding reference price effects by comparing our model with the CG model. All the above research extends the results of the CG model to the reference price effects.

Though this paper has identified the effects of the reference price on dynamic pricing and ordering decisions with random yields, there are still some shortcomings that can be investigated in the future. First, this paper analyzes the pricing and order flexibility decisions of a single firm under reference price effects, and does not assess the influence of reference price effects on suppliers. An interesting future research topic is to examine the pricing and inventory decisions for suppliers and to design an appropriate coordination mechanism so that a win-win outcome for both parties can be obtained. Second, in our study, the customers' reference price can be observed by firms. However, the information on the customers' reference price is difficult to get in reality. Thus, demand learning can be incorporated into formulating pricing and inventory strategies in the presence of the reference price effects.

Acknowledgements

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Appendix

Proof of Lemma 1. (i) The monotonicity of $V_t(x_t, r_t)$ in x_t is because $J_t(y_t, d_t, q_t, r_t)$ is independent of x_t and the feasible set $\{(y_t, d_t, q_t, r_t) \mid y_t \geq x_t, \underline{d}_t \leq d_t \leq \overline{d}_t, q_t \geq 0\}$ shrinks as x_t increases. The monotonicity of $V_t(x_t, r_t)$ in r_t is similar to that of Theorem 1 in Güler et al. [21].

Next, we prove (ii) and (iii) by induction. Starting from $t = T$ it is obvious that $V_{T+1}(x_{T+1}, r_{T+1}) = 0$ is concave, and then (iii) is true. For (ii), since $G_T(\cdot)$ is a convex function and $y_T - d_T + \Theta_T q_T$ is a linear combination of y_T, d_T, q_T for any fixed Θ_T, then $G_T(y_T - d_T + \Theta_T q_T - \varepsilon_T)$ is joint convex in (y_T, d_T, q_T) for any fixed Θ_T. Thus, each term in (2.2) is concave and its concavity is preserved by maximization. Therefore, (ii) holds for $t = T$. Next, suppose that (ii) and (iii) are valid for $t = k + 1$. Each term in (2.2) is concave except for $V_{k+1}(y_k - d_k + \Theta_k q_k - \varepsilon_k, \alpha r_k + (1-\alpha)p_k(d_k, r_k))$ and we thus need the concavity of $V_{k+1}(y_k - d_k + \Theta_k q_k - \varepsilon_k, \alpha r_k + (1-\alpha)p_k(d_k, r_k))$ in (y_k, d_k, q_k, r_k). By defining $\tilde{\tau}(y_k, d_k, q_k, \varepsilon_k)$ and $\tilde{r}(d_k, r_k)$ as

$$\tilde{\tau}(y_k, d_k, q_k, \varepsilon_k) = y_k - d_k + \Theta_k q_k - \varepsilon_k, \quad \tilde{r}(d_k, r_k) = \alpha r_k + (1-\alpha)p_k(d_k, r_k),$$

the following holds for any pair (y_k^1, y_k^2), (d_k^1, d_k^2), (q_k^1, q_k^2) and (r_k^1, r_k^2):

$$\tilde{\tau}\Big(\frac{y_k^1 + y_k^2}{2}, \frac{d_k^1 + d_k^2}{2}, \frac{q_k^1 + q_k^2}{2}, \varepsilon_k\Big) = \frac{y_k^1 + y_k^2}{2} - \frac{d_k^1 + d_k^2}{2} + \Theta_k \frac{q_k^1 + q_k^2}{2} - \varepsilon_k$$

$$= \frac{1}{2}(y_k^1 - d_k^1 + \Theta_k q_k^1 - \varepsilon_k) + \frac{1}{2}(y_k^2 - d_k^2 + \Theta_k q_k^2 - \varepsilon_k),$$
$$\tilde{r}\Big(\frac{d_k^1 + d_k^2}{2}, \frac{r_k^1 + r_k^2}{2}\Big) \geq \frac{1}{2}\tilde{r}(d_k^1, r_k^1) + \frac{1}{2}\tilde{r}(d_k^2, r_k^2).$$

Since $p_k(d_k, r_k)$ is concave in (d_k, r_k) according to Assumption 1. Thus, we obtain

$$\begin{aligned}
&V_{k+1}\Big[\tilde{\tau}\Big(\frac{y_k^1 + y_k^2}{2}, \frac{d_k^1 + d_k^2}{2}, \frac{q_k^1 + q_k^2}{2}, \varepsilon_k\Big), \tilde{r}\Big(\frac{d_k^1 + d_k^2}{2}, \frac{r_k^1 + r_k^2}{2}\Big)\Big] \\
&\geq V_{k+1}\Big[\tilde{\tau}\Big(\frac{y_k^1 + y_k^2}{2}, \frac{d_k^1 + d_k^2}{2}, \frac{q_k^1 + q_k^2}{2}, \varepsilon_k\Big), \frac{1}{2}\tilde{r}(d_k^1, r_k^1) + \frac{1}{2}\tilde{r}(d_k^2, r_k^2)\Big] \\
&= V_{k+1}\Big[\frac{1}{2}\tilde{\tau}(y_k^1 - d_k^1 + \Theta_k q_k^1 - \varepsilon_k) + \frac{1}{2}\tilde{\tau}(y_k^2 - d_k^2 + \Theta_k q_k^2 - \varepsilon_k), \frac{1}{2}\tilde{r}(d_k^1, r_k^1) + \frac{1}{2}\tilde{r}(d_k^2, r_k^2)\Big] \\
&\geq \frac{1}{2}V_{k+1}\Big[\tilde{\tau}\Big(y_k^1, d_k^1, q_k^1, \varepsilon_k), \tilde{r}(d_k^1, r_k^1)\Big] + \frac{1}{2}V_{k+1}\Big[\tilde{\tau}\Big(y_k^2, d_k^2, q_k^2, \varepsilon_k), \tilde{r}(d_k^2, r_k^2)\Big],
\end{aligned}$$

where the first and second inequalities follow from (i) and the induction assumption, respectively. We get the concavity of $V_{k+1}(y_k - d_k + \Theta_k q_k - \varepsilon_k, \alpha r_k + (1-\alpha)p_k(d_k, r_k))$ in (y_t, d_t, q_t, r_t). Then, $J_k(y_k, d_k, q_k, r_k)$ is joint concave in (y_k, d_k, q_k, r_k). Therefore, $V_k(x_k, r_k)$ is joint concave in (x_k, r_k). We complete the proof. □

Proof of Theorem 1. We first prove the results on $y_t^*(x_t, r_t)$ and $p_t^*(x_t, r_t)$. For convenience, we define

$$\begin{aligned}
J_t(z_t, q_t, p_t, r_t) = &- c_2\theta_t q_t - \mathrm{E}[G_t(z_t + \Theta_t q_t - \varepsilon_t)] \\
&+ \gamma \mathrm{E}[V_{t+1}(z_t + \Theta_t q_t - \varepsilon_t, \alpha r_t + (1-\alpha))p_t],
\end{aligned}$$

where $z_t = y_t - d_t$. The convexity of $G_t(\cdot)$ follows from Lemma 1. This, along with that the support of Θ_t is $[0, 1]$, it can be easily verified that $J_t(z_t, q_t, p_t, r_t)$ is joint convex and submodular in (z_t, q_t). In addition, the optimal equation (2.1) can be rewritten as

$$\begin{aligned}
V_t(x_t, r_t) = &\max_{\underline{d}_t \leq d_t \leq \overline{d}_t} \Big\{R_t(d_t, r_t) - c_1 d_t + \max_{z_t \geq x_t - d_t} \Big\{ - c_1 z_t + \max_{q_t \geq 0, \underline{p}_t \leq p_t \leq \overline{p}_t} J_t(z_t, q_t, p_t, r_t)\Big\}\Big\} \\
&+ c_1 x_t. && \text{(A.1)}
\end{aligned}$$

Then, it follows that $-c_1 z_t + \max_{q_t \geq 0, \underline{p}_t \leq p_t \leq \overline{p}_t} J_t(z_t, q_t, p_t, r_t)$ is concave in z_t, $\max_{z_t \geq x_t - d_t} \Big\{ - c_1 z_t + \max_{q_t \geq 0, \underline{p}_t \leq p_t \leq \overline{p}_t} J_t(z_t, q_t, p_t, r_t)\Big\}$ is concave in $(x_t - d_t)$ and the maximand in (3.1) is supermodular in (x_t, d_t). Then, it follows from (A.1) and Theorem 2.2.8 in Simchi-Levi et al. [32] that $d_t^*(x_t, r_t)$ is increasing in x_t. In addition, we define

$$z_{t,1}^*(r_t) = \arg\max_{z_t} \Big\{ - c_1 z_t + \max_{q_t \geq 0, \underline{p}_t \leq p_t \leq \overline{p}_t} J_t(z_t, q_t, p_t, r_t)\Big\}.$$

Then, $\max_{z_t \geq x_t - d_t} \Big\{ - c_1 z_t + \max_{q_t \geq 0, \underline{p}_t \leq p_t \leq \overline{p}_t} J_t(z_t, q_t, p_t, r_t)\Big\}$ is increasing in d_t and equals $-c_1 z_{t,1}^*(r_t) + \max_{q_t \geq 0, \underline{p}_t \leq p_t \leq \overline{p}_t} J_t(z_{t,1}^*(r_t), q_t, p_t, r_t)$ when $d_t \geq x_t - z_{t,1}^*(r_t)$. Hence, according to the definition of $d_t^*(r_t)$ in (3.1), it is easy to verify that $d_t^*(x_t, r_t) = d_t^*(r_t)$ and

$p_t^*(r_t) = p_t(d_t^*(r_t))$ when $x_t \le z_{t,1}^*(x_t) + d_t^*(r_t)$ and $q_t^*(x_t, r_t) = \max_{z_t \ge x_t - d_t} \Big\{ -c_1 z_t + \max_{q_t \ge 0, \underline{p}_t \le p_t \le \overline{p}_t} J_t(z_t, q_t, p_t, r_t) \Big\}$ from (A.1). Meanwhile, when $x_t > z_{t,1}^*(r_t) + d_t^*(r_t)$, $d_t^*(r_t) \le d_t^*(x_t, r_t) \le x_t - z_{t,1}^*(r_t)$ and so $y_t^*(x_t, r_t) = x_t$. Therefore, $p_t^*(x_r, r_t) = p_t^*(r_t)$ when $x_t \le z_{t,1}^*(r_t) + d_t^*(r_t)$; and when $x_t > z_{t,1}^*(r_t) + d_t^*(r_t)$, since $d_t^*(x_t, r_t)$ is increasing in x_t, $p_1^*(x_t, r_t) = p_t(d_t^*(x_t, r_t))$ is a decreasing function of x_t, where $p_t^*(x_t, r_t) \le p_t^*(r_t)$.

We next prove that $q_t^*(x_t, r_t)$ is decreasing in x_t. We define

$$\begin{aligned} J_t(w_t, q_t, d_t, p_t, r_t) = & - c_2 \theta_t q_t - \mathrm{E}[G_t(w_t - d_t - \varepsilon_t)] \\ & + \gamma \mathrm{E}[V_{t+1}(w_t - d_t - \varepsilon_t, \alpha r_t + (1-\alpha) p_t], \end{aligned}$$

where $w_t = y_t + \Theta_t q_t$. The convexity of $G_t(\cdot)$ follows from Lemma 1, and we note that the support of Θ_t is $[0,1]$. Thus, it can be easily verified that $J_t(w_t, q_t, d_t, p_t, r_t)$ is joint convex and submodular in (w_t, q_t). In addition, the optimal equation (2.1) can be rewritten as

$$\begin{aligned} V_t(x_t, r_t) = & \max_{\underline{d}_t \le d_t \le \overline{d}_t} \Big\{ R_t(d_t, r_t) - c_1 d_t + \max_{w_t \ge x_t + \Theta_t q_t} \Big\{ -c_1 z_t + \max_{q_t \ge 0, \underline{p}_t \le p_t \le \overline{p}_t} J_t(w_t, q_t, d_t, p_t, r_t) \Big\} \Big\} \\ & + c_1 x. \end{aligned} \tag{A.2}$$

Then, it follows from the concavity of $-c_1 z_t + \max_{q_t \ge 0, \underline{p}_t \le p_t \le \overline{p}_t} J_t(z_t, q_t, p_t, r_t)$ in w_t, then $\max_{z_t \ge x_t - d_t} \Big\{ -c_1 z_t + \max_{q_t \ge 0, \underline{p}_t \le p_t \le \overline{p}_t} J_t(z_t, q_t, p_t, r_t) \Big\}$ is concave in $(x_t + \Theta_t q_t)$, and the maximand in in (A.2) is submodular in (x_t, d_t). Then, it follows from (A.1) and Theorem 2.2.8 in Simchi-Levi et al. [32] that $q_t^*(x_t, r_t)$ is decreasing in x_t.

Finally, we define $\xi_{t,2}^*(x_t, r_t) = \inf\{x_t \mid x_t \ge z_{t,1}^*(r_t) + d_t^*(r_t), q_t^*(x_t, r_t) = 0\}$. Then, we clearly have $\xi_{t,2}^*(x_t, r_t) \ge z_{t,1}^*(r_t) + d_t^*(r_t)$, and $q_t^*(x_t, r_t) = 0$ when $x_t \ge \xi_{t,2}^*(x_t, r_t)$. The proof is complete. □

Proof of Lemma 2. We prove this lemma by induction. Starting from $t = T$, it is obvious that $V_{T+1}(x_{T+1}, r_{T+1}) = 0$ is supermodular in (x_T, r_T). Thus, $J_t(y_T, d_T, q_T, r_T)$ is supermodular in (x_T, r_T) since the first four terms in $J_t(y_T, d_T, q_T, r_T)$ are independent of x_T. Then, the maximization that preserves the supermodularity yields the supermodularity of $V_T(x_T, r_T)$ in (x_T, r_T).

Assume that the result holds for $t = k + 1$. Next, we need to show that the result is still valid for $t = k$. Since $J_k(y_k, d_k, q_k, r_k)$ is independent of x_k, we only need to prove the supermodularity of $J_k(y_k, d_k, q_k, r_k)$ in (y_k, r_k), (d_k, r_k) and (q_k, r_k), which is equivalent to the supermodularity of $J_k(y_k, d_k, q_k, r_k)$ in (y_k, x_k, r_k), (d_k, x_k, r_k) and (q_k, x_k, r_k).

(i) We prove the supermodularity of $J_k(y_k, d_k, q_k, r_k)$ in (y_k, r_k). The terms in $J_k(y_k, d_k, q_k, r_k)$ either depend on y_k or r_k or are constants with respect to y_k and r_k except for the last two terms. Therefore, it suffices to show the submodularity of $G_k(y_k - d_k + \Theta_k q_k - \varepsilon_k)$ in (y_k, r_k) and the supermodularity of $V_{k+1}(y_k - d_k + \Theta_k q_k - \varepsilon_k, \alpha r_k + (1-\alpha) p_k)$ in (y_k, r_k).

First, we prove the submodularity of $G_k(y_k - d_k + \Theta_k q_k - \varepsilon_k)$ in (y_k, r_k). For any pair (y_k^1, y_k^2) and (r_k^1, r_k^2) with $y_k^1 > y_k^2$ and $r_k^1 > r_k^2$, let

$$\tau_1 = y_k^1 - d_k(p_k, r_k^1) + \Theta_k q_k - \varepsilon_k, \qquad \tau_2 = y_k^1 - d_k(p_k, r_k^2) + \Theta_k q_k - \varepsilon_k,$$
$$\tau_3 = y_k^2 - d_k(p_k, r_k^1) + \Theta_k q_k - \varepsilon_k, \qquad \text{and } \tau_4 = y_k^2 - d_k(p_k, r_k^2) + \Theta_k q_k - \varepsilon_k.$$

According to the monotonicity of the mean demand function d_k, we have $\tau_3 < \tau_4$. Thus, according to the concavity of G_k, we have

$$\begin{aligned} G_k(\tau_1) - G_k(\tau_3) &= G_k(\tau_3 + (y_1^E - y_2^E)) - G_k(\tau_3) \\ &\le G_k(\tau_4 + (y_1^E - y_2^E)) - G_k(\tau_4) \\ &= G_k(\tau_2) - G_k(\tau_4), \end{aligned}$$

which implies that $G_k(y_k^1 - d_k(p_k, r_k) - \varepsilon_k) - G_k(y_k^2 - d_k(p_k, r_k) - \varepsilon_k)$ is decreasing in r_k. Therefore, $G_k(y_k - d_k + \Theta_k q_k - \varepsilon_k)$ is submodular in (y_k, r_k), and then $-G_k(y_k - d_k + \Theta_k q_k - \varepsilon_k)$ is supermodular in (y_k, r_k). This proves the supermodularity of $J_k(y_k, d_k, q_k, r_k)$ in (y_k, r_k).

Second, we prove the supermodularity of $V_{k+1}(y_k - d_k + \Theta_k q_k - \varepsilon_k, \alpha r_k + (1-\alpha)p_k)$ in (y_k, r_k).

Consider the arbitrary pair (y_k^1, y_k^2) and (r_k^1, r_k^2) with $y_k^1 > y_k^2$ and $r_k^1 > r_k^2$. We fix ε_k and let

$$(\tau_1, \xi_1) = (y_k^1 - d_k(p_k, r_k^1) - \varepsilon_k, \xi_1), \qquad (\tau_2, \xi_2) = (y_k^1 - d_k(p_k, r_k^2) - \varepsilon_k, \xi_2),$$
$$(\tau_3, \xi_1) = (y_k^2 - d_k(p_k, r_k^1) - \varepsilon_k, \xi_1), \qquad \text{and } (\tau_4, \xi_2) = (y_k^2 - d_k(p_k, r_k^2) - \varepsilon_k, \xi_2),$$

where $\xi_1 = \alpha r_k^1 + (1-\alpha)p_k(d_k, r_k^1)$ and $\xi_2 = \alpha r_k^1 + (1-\alpha)p_k(d_k, r_k^2)$. Then, we obviously have $\xi_1 > \xi_2$ and $\tau_2 < \tau_4$. Thus, we have

$$\begin{aligned} V_{k+1}(\tau_1, \xi_1) - V_{k+1}(\tau_3, \xi_1) &= V_{k+1}(\tau_3 + (y_k^1 - y_k^2), \xi_1) - V_{k+1}(\tau_3, \xi_1) \\ &\ge V_{k+1}(\tau_4 + (y_k^1 - y_k^2), \xi_1) - V_{k+1}(\tau_4, \xi_1) \\ &= V_{k+1}(\tau_2, \xi_1) - V_{k+1}(\tau_4, \xi_1) \\ &\ge V_{k+1}(\tau_2, \xi_2) - V_{k+1}(\tau_4, \xi_2), \end{aligned}$$

where the first inequality follows from the concavity of V_{k+1} and the second inequality follows from the supermodularity of $V_{k+1}(\tau, \xi)$ in (τ, ξ) according to the induction assumption. This implies that $V_{k+1}(y_k^1 - d_k(p_k, r_k) + \Theta_k q_k - \varepsilon_k, \xi) - V_{k+1}(y_k^2 - d_k(p_k, r_k) + \Theta_k q_k - \varepsilon_k, \xi)$ is increasing in r_k. We thus get the supermodularity of $V_{k+1}(y_k - d_k + \Theta_k q_k - \varepsilon_k, \alpha r_k + (1-\alpha)p_k)$ in (y_k, r_k). Consequently, $J_k(y_k, d_k, q_k, r_k)$ is supermodular in (y_k, r_k).

Third, the supermodularity of $J_k(y_k, d_k, q_k, r_k)$ in (d_k, r_k) is similar to that of Theorem 6 in Güler et al. [21].

(ii) The supermodularity of $J_k(y_k, d_k, q_k, r_k)$ in (q_k, r_k) is similar to the supermodularity of $J_k(y_k, d_k, q_k, r_k)$ in (y_k, r_k). Thus, we omit it here.

In summary, $J_k(y_t, d_t, q_t, r_t)$ is supermodular in (x_t, r_t). Hence, $V_k(x_k, r_k)$ is supermodular in (x_t, r_t). This completes the proof. □

Proof of Theorem 2. (i), (ii) and (iii) are the direct consequences of Corollary 1, while (iv) is the direct consequence of Assumption 1. (v) has been proved in Lemma 1(i). □

Proof of Theorem 3. This follows directly from Theorem 2 that the CG model is a special case of our model, i.e., $r_t = p_t$ for all $t = 1, 2, \ldots, T$. □

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