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System Availability of Inspection Constant to the Dormant Failures Considering Replacement or Repair Together with Inspection Downtime Period

*Emmanuel Hagenimana and Lixin Song**

Dalian University of Technology

Abstract

For the maintenance of the state system, the operation and inspection period to find unrevealed to the failure of the system is discussed. In the normal case, the system failure has been identified only at that moment if it is occurred after making a special test or the regular inspection to the latter, as the failure of the state system has been recognized using the inspection time. An aim of this paper is to minimize the cost to the unit time by selecting the required interval time by considering inspection and maintenance together. The availability of the state system under the inspection period where replacement or repair time with non-neglected downtime to detect the failure of the system is discussed in this article. It has seen that all corrective and preventive of maintenance system keep the system always as good as new. The numerical example for clarification to the case study has been discussed and the results showed that all procedures used in this model give the maximum limiting of the system availability similar closed in the range of 0.95.

Keywords: Inspection period, age inspection, calendar inspection, unrevealed failure, average availability, instant availability.

1. Introduction

It is known that if the failure of the system can't be found, it is advised to proceed with the application of inspection which can play a big role in finding the failure of the system. In case it doesn't work appropriately in a certain time which gives a guarantee the quality of availability of the system materials. It is better to provide all necessary equipment to protect your system devices to discover any failure which may happen. It is not compulsory to have the same consecutive interval of inspection period; it may only depend on how you want to proceed or to treat your inspection system, the more details can be checked here (Golmakani and Moakedi [7]) and (see Taghipour et al. [18]). We have observed that the inspection period is so simple and comfortable for

*corresponding author, Song Lixin.

practical than consecutive inspection which is strongly appreciated to apply to the case and it is proved to be efficient and more useful. The more application to the inspection period in industries have used inspection period with identical intervals whereas many researchers have made accent on application to the feasibility of the system material, and the finding failure interval of the system is the result of an identical inspection period checks in Jiang and Jardine [8] and (see also Martínez et al. [11]) or take a look in Golmakani and Moakedi [7].

In this research paper, we apply two different kinds of policies of inspection such as calendar based on inspection and age based on the inspection. The action plan for an activity to the inspection period at regular periodic intervals of time, it is customary for example to say each Friday which mean a periodic inspection of one day per week. The given periodic inspection, the downtime caused by the inspection together with repair or replacement period is considered as a part of the interval I. For the time of life to the inspection policy, the action plan for this inspection at the regular time of life intervals (Golmakani and Moakedi [7]), Wang et al. [22]. It is observed that the time for inspection and repair or replacement are not taken into consideration in the given interval I.

In case $T_\varepsilon = T_\Upsilon = 0$. the downtime caused by inspection with downtime caused by repair or replacement are well not considered. Therefore the periodic based with a time of life-based inspection to the policy have been taken to be identical, and both of them have the convenience and inconvenience. It is observed that the periodically based inspection is not anxious for practically compared to the time of life inspection period since it is required to keep information about the inspection done previously in (Naidu and Gopalan [12]) as well as see Sherif [17]. Hence it not appreciated by researchers since it considers all details happened just previous of renewable system material which not sufficient.

The previous research has shown that the inspection to the periodic supposes that always downtime to repair or replacement caused by inspection for the unrevealed system to be neglected but limiting of average availability of the system is taken into consideration. The more details can be seen (Cui and Xie [3]) and (Huang and Mihuang [6]), they have discussed the related issue in their research work where downtime caused by replacement or repair in this study have always been considered to be neglected. The same work on optimal to the inspection interval time to control daily failure, as well as to limiting average unavailability of the state system, has also been studied in Vaurio [21]. With periodic based inspection policy, limiting average together with instantaneous to the system have been discussed in (Sarkar and Sarkar [15]). The same analysis under to the periodic inspection extended to downtime caused by replacement or repair state system has been done in (Cui and Xie [3]).

By the similar case, it has seen that the downtime caused by replacement or repair of the system have been considered as well as downtime caused by inspection of the state system cannot be neglected. The contrary situation has been studied by different researchers such as (Ten and Ghobbar [20]) where (Jiang and Jardine [8]) have investigated the same case with Pak et al. [12]. This can be seen also in (Barroeta and

Modarres [1]) where limiting average of availability system is strongly discussed with both cases cited above have been considered to be neglected.

In this article, we examine the instant system availability and average availability of the system underage and calendar based on the inspection period, and the scenarios A_0 and A_1 have been applied. By calendar based on the inspection time, the downtime due to inspection and replacement or repair are included in the interval, the figure 1 illustrates this case. For age-based on the inspection procedure, the schedules inspection is fixed and inspection time and the necessary replacement or repair are not included in the interval I, figure 2 demonstrates this case.

By scenarios, A_0 : the component of the system materials is restored and functioning as good as the new one at the inspection time. Similarly, by scenarios A_1, at the given inspection time, the system is detected to be functioning and then taken to the maintenance operation without considering any intervention on it.

Therefore, by Dekker [4],(Cavalcante and Lopes [2]), the following points are considered:

(i) The inspection time is executed in different constant intervals of time.

(ii) The inspection of the system materials doesn't disintegrate the working of the state system.

(iii) The inspection is always maintained as well in time it can discover the failure of the system which will be precisely accurate after operation of replacement or repair of the system.

(iv) The state system is automatically taken to the inspection time always after maintenance action of replacement or repair techniques.

The remaining of the article is established in the following: In section 2, we discuss the periodic based to the inspection procedure. In section 3, the time of lifetime based on the inspection procedure is also discussed. In every section, we consider both limiting average with instantaneous of the availability system subject to the given scenarios (A_0) and (A_1) respectively. In section 4, the results for this model have been provided. In section 5 the numerical example for clarification have been discussed in details. In section 6, Conclusions, suggestion together with further research studies have also been discussed in this section.

Remark. To improve the readability, the descriptions of the following notation have been used in this research work:

T : is the time to failure of the systems

$f(t)$: is the density function due to time failure of the systems

μ : is the mean lifetime to the system: $\mu = \int_0^x R(t)\mathrm{d}t$

$S(t)$: is the instant system availability of the system at given time t

$\overline{S}$: is the limiting average of the availability system in such that $\overline{S} = \lim \frac{1}{t} \int_0^t S(u)\mathrm{d}u$

$\widetilde{T}_1$: is the age of units time during first failure of the system

$R(t)$: is the reliability function to the systems

I : is taken as interval time,

ιI : is the time in which the first failure found

$t_\iota,\ \iota = 1, 2, \cdots m$: is the time at the point ι.

T_1^* : is the calender period during the first failure of the system

T_1 : is the calendar period during the inspections if the first failures of the system is detected

$\chi(t)$: is the evaluated system availability at time $t = 1$ or $t = 0$.

$\overline{S}$: is average to the system availability approximated by limit while $t \mapsto \infty$.

$\Phi(t)$: is noticed as system function to the reliability at any given time t.

T_Υ : is the combination of the downtime due to failure together with downtime caused by replacement time as well as inspection time.

T_ε : is the downtime when the system is functioning.

A_0 and A_1 have been taken as scenarios in this model.

2. System Availability under Calendar Based on the Inspection Procedure

By this section, we study the limiting average and the instant system availability under calendar based on the inspection procedure using both A_0 and A_1 scenarios respectively.

2.1. Formulation of the model using scenarios A_0

Using scenarios A_0 at every inspection time the renewal of system is applied. It is understandable to consider that $T_\Upsilon > I$. Therefore from the limiting average together with instant system availability of the state system we get the following.

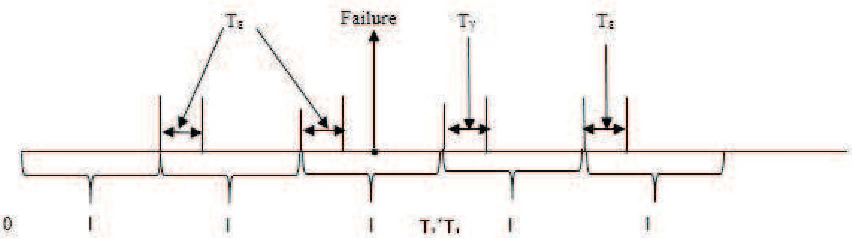

Figure 1: Representation of calendar based on the inspection time procedure.

Theorem 1. *Assume that due to scenarios A_0, the instant system availability under calendar based to the inspection procedure is obtained respectively by the following:*

$$S(t) = \begin{cases} \Phi(t) & if\, t \leq I \\ 0 & if\, \iota I < t < I + T_\varepsilon \\ \Phi(t - \iota I - T_\varepsilon)S(\iota I) & if\, \iota I + T_\varepsilon \leq t < \iota I + T_\Upsilon \\ \Phi(t - \iota I - T_\varepsilon)S(\iota I) & \\ +\Phi(t - \iota I - T_\Upsilon)[1 - S(\iota I)] & if\, \iota I + T_\Upsilon < t < (\iota + 1)I \end{cases} \tag{2.1}$$

with $\iota = 1, 2, \ldots.$
A limiting to the average of the state system availability have been given by the following:

$$\overline{S} = \frac{1}{I}(\varphi \int_0^{I-T_\varepsilon} \Phi(\nu)\mathrm{d}\nu + (1 - \varphi) \int_0^{I-T_\varepsilon} \Phi(\nu)\mathrm{d}\nu) \tag{2.2}$$

with

$$\varphi = \lim_{\iota \to \infty} S(\iota I) \frac{\Phi(I - T_F)}{1 - \Phi(I - T_\varepsilon) + \Phi(I - T_\Upsilon)}.$$

Proof. $S(t) = \Phi(t)$ since $t \leq I$, and $S(t) = 0$, since $\iota I < t < \iota I + T_\varepsilon$ As $\iota I + T_\varepsilon \leq t < \iota I + T_\Upsilon$, the following is obtained,

$$\begin{aligned} S(t) &= P(\chi(t) = 1) \\ &\quad P(\chi(t) = 1 \mid \chi(\iota I) = 1)P(\chi(\iota I) = 1) + P(\chi(t) = 1 \mid \chi(\iota I) = 0)P(\chi(\iota I) = 0) \\ &= \Phi(t - \iota I - T_\varepsilon)S(\iota I) + 0.[1 - S(\iota I)] \\ &= \Phi(t - \iota I - T_\varepsilon)S(\iota I), \end{aligned}$$

If the state system is functioning at give time $\iota I(\chi(\iota I) = 1)$ it have renewal at give time $\iota I + T_\varepsilon$, then the given state system is constitute failed at $\iota I(\chi(\iota I) = 0)$ requires T_Υ time to be repaired or replaced.

And the state system become down in the time interval $\iota I, \iota I + T_\Upsilon$ means that $\chi(t) = 0$ for $\iota I < t < \iota I + T_\Upsilon$.

Since $\iota I + T_\Upsilon \leq t \leq (\iota + 1)I$, we get,

$$S(t) = P(\chi(t) = 1 \mid \chi(\iota I) = 1)P(\chi(\iota I) = 1) + P(\chi(t) = 1 \mid \chi(\iota I) = 0)P(\chi(\iota I) = 0)$$

$$= \Phi(t - \iota I - T_\varepsilon)S(\iota I) + \Phi(t - \iota I - T_F)[1 - S(\iota I)],$$

As the state system is functioning at given time ιI it have renewal at given time $\iota I + T_\varepsilon$, then the given state system is failed at $\iota I + T_\Upsilon$. The proof of the given relation (2.1) above is completed.

Let $t = (\iota + 1)I$ and apply the relation (2.1), we obtain

$$S((\iota + 1)I) = [\Phi(I - T_\varepsilon) - \Phi(I - T_\Upsilon)]S(\iota I) + \Phi(I - T_\Upsilon), \iota = 1, 2, \ldots \tag{2.3}$$

which is known as the first order non homogeneous differential equation by the theorem 1.13 can be seen Elaydi [5], $\lim_{\iota\to\infty}$, exists, if $0 < \Phi(I - T_\varepsilon) - \Phi(I - T_\varepsilon < 1$. Let $\iota \to \infty$, to the given equation (2.3) we get.

$$\varphi = \lim_{\iota\to\infty} S(\iota I) = \frac{\Phi(I - T_\Upsilon)}{1 - \Phi(I - T_\varepsilon) + \Phi(I - T_\Upsilon)}$$

Therefore, the given above relation may also obtain by applying the ergodicity due to Markov chain with state space of the system to the unrevealed system together with the revealed one combined with the transition to the probability to the state system, which can be seen in (Shang [16] and Kharoufeh et al. [9]). Therefore the given relation above could be obtained from ergodicity to the Markov Chain and state space to the failure found interval time or never founded together with transition to the probability system by the given inspection of interval time ιI to the next state system in inspection interval time $(\iota + 1)I$. Since $t \in (0, J)$ and applied in relation (2.1), we get

$$S(t + \iota I) = \begin{cases} 0 & \text{if } 0 < t < T_\varepsilon \\ \Phi(t - T_\varepsilon)S(\iota I) & \text{if } T_\varepsilon \le t < T_\epsilon \\ \Phi(t - T_\varepsilon)S(\iota I) + \Phi(t - T_\Upsilon)[1 - S(\iota I)] & \text{if } T_\Upsilon \le t \le I \end{cases}$$

Let $\iota \to \infty$, we get

$$S(t + \iota I) = \begin{cases} 0 & \text{if } 0 < t < T_\varepsilon \\ \phi(t - T_\varepsilon) & \text{if } T_\varepsilon \le t < T_\Upsilon \\ \phi(t - T_\varepsilon) + \Phi(t - T_\Upsilon)[1 - \phi)] & \text{if } T_\Upsilon \le t < I \end{cases}$$

Therefore, the limiting average to the system availability is given by

$$\begin{aligned} \overline{S} &= \frac{1}{I}(\int_{T_\varepsilon}^{T_\Upsilon} \varphi\Phi(t - T_\varepsilon)\mathrm{d}t + \int_{T_\Upsilon}^{I} [\varphi\Phi(t - T_\varepsilon) + (1 - \varphi)\Phi(t - T_\Upsilon)]\mathrm{d}\nu) \\ &= \frac{1}{I}(\varphi \int_{T_\varepsilon}^{I} \Phi(t - T_\varepsilon)\mathrm{d}t + (1 - \varphi) \int_{T_\Upsilon}^{I} \Phi(t - T_\Upsilon)\mathrm{d}t) \\ &= \frac{1}{I}(\varphi \int_{0}^{I - T_\varepsilon} \Phi(t)\mathrm{d}t + (1 - \varphi) \int_{0}^{I - T_\Upsilon} \Phi(t)\mathrm{d}t). \end{aligned}$$

The proof completed.

2.2. Formulation of the model using scenarios A_1

Using scenarios A_1 at every inspection time the renewal of system is applied. It is understandable to consider that $T_\Upsilon > I$. Therefore, from the limiting average together with instant system availability of the state system we get the following.

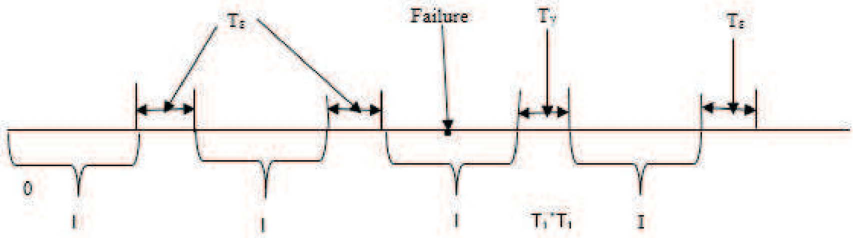

Figure 2: Representation of age based on the inspection time procedure.

Theorem 2. *Assume that due to scenarios A_1 the instant system availability under calendar based on the inspection procedure is obtained respectively by the following:*

$$S(t) = \begin{cases} \Phi(t) & if\, t \leq I \\ 0 & if\, \iota I < t < \iota I + T_\varepsilon \\ \Phi(t - \iota T_\varepsilon) + \sum_{\iota=1}^{\iota-1} \xi(t - \iota I - T_\Upsilon) p_\iota & if\, \iota I + T_\varepsilon < t < \iota I + T_\Upsilon \\ \Phi(t - \iota T_\varepsilon) + \sum_{\iota=1}^{\iota} \xi(t - \iota I - T_\Upsilon) p_\iota & if\, \iota I + T_\Upsilon \leq t \leq (\iota + 1) I \end{cases} \tag{2.4}$$

for,

$$\xi(t) = \begin{cases} \Phi(t) & if\, t \leq I - T_\Upsilon \\ 0 & if\, \iota I - T_\Upsilon < t < \iota I + T_\varepsilon - T_\Upsilon \\ \Phi(t - \iota I - T_\varepsilon) + \sum_{\iota=1}^{\iota-1} \xi(t - \iota I) q_\iota & if\, \iota I + T_\varepsilon - T_\Upsilon < t < \iota I \\ \Phi(t - \iota I - T_\varepsilon) + \sum_{\iota=1}^{\iota} \xi(t - \iota I) q_\iota & if\, \iota I < t < (\iota + 1) I - T_\Upsilon \end{cases} \tag{2.5}$$

$p_\iota = \Phi(\tau_{\iota-1}) - \Phi(\tau_\iota)$, $q_\iota = \Phi(t_{\iota-1}) - \Phi(t_\iota,\ \iota = 1, 2, \ldots.$

$$\tau_i = \begin{cases} 0 & if\, i = 0 \\ (I - T_\varepsilon) i + T_\varepsilon & if\, i = 1, 2, \ldots.. \end{cases}$$

$$t_\iota = \begin{cases} 0 & if\, \iota = 0 \\ (I - T_\varepsilon)\iota + T_\varepsilon - T_\Upsilon & if\, \iota = 1, 2, \ldots. \end{cases}$$

Proof. $S(t) = \Phi(t)$ when $t \leq I$, and $S(t) = 0$ when $\iota I < t < \iota I + T_\varepsilon$ If the state system is not functioning in the given interval time $[T_1^*, T_1 + T_\Upsilon]$, we obtain

$$\begin{aligned} S(t) &= P(\chi(t) = 1) \\ &= P(\chi(t) = 1, t < T_1^*) + P(\chi(t) = 1, T_1^* \leq t < T_1 + T_\Upsilon) + P(\chi(t) = 1, T_1 + T_\Upsilon \leq t) \end{aligned}$$

$$= P(\chi(t) = 1 \mid T_1^* > t)P(T_1^* > t) + P(\chi(t) = 1, T \leq t - T_\Upsilon. \tag{2.6}$$

Let the first failure to state system be at $(\iota + 1)I$, then the state system is noticed to be always up in the given interval time of $[\iota I + T_\varepsilon, (\iota + 1)I]$. Therefore the right hand side of first term in the relation (2.6) is expressed by the following

$$P(\chi(t) = 1 \mid T_1^* > t)P(T_1^* > t).P(T_1^* > t) = P(\widetilde{T}_1 > t - \iota T_\varepsilon) = \Phi(t - \iota T_\varepsilon)$$

for $\iota I + T_\varepsilon \leq t \leq (\iota + 1)I)$.

It is noticed that, all inspections to the calendar based procedure are executed at time $\iota I, \iota = 1, 2, \ldots$.

Assume that the first failure of the system is obtained at time ιI, and the renewal system at time $\iota I + T_\Upsilon$. The renewal cycle began from the time $\iota I + T_\Upsilon$ which can be inspected at time $(1 + \iota)I$, where the given inspection interval time for the first to the renewal cycle of the system is denoted as $I - T_\Upsilon$ taken into account to the renewal time $\iota I + T_\Upsilon$. From this, all renewal cycles have been determined by the first inspection interval of the system, $I - T_\Upsilon$ considering to the renewal time.

Given $\xi(t)$ the system availability at time t, under calendar-based on the inspection procedure in the initial inspection time interval $I - T_\Upsilon$. By taking into account to the renewal of the inspected time interval or subsequent inspection intervals I until the failure of the state system to be found. We get the following

$$P(\chi(t) = 1 \mid T_1 = iI) = \xi(t - iI - T_\Upsilon).$$

For $\iota I + T_\varepsilon < t < \iota I + T_\Upsilon$, the right hand side for the second term to the relation (2.6) is expressed in the following:

$$\begin{aligned}
&P(\chi(t) = 1, T_1 \leq t - T_\Upsilon) \\
&\quad = \sum_{\iota=1}^{\iota-1} P(\chi(t) = 1 \mid T_1 = \iota I)P(T_1 = iI) \\
&\quad = \xi(t - I - T_\Upsilon)P(0 < \widetilde{T}_1 < I) + \sum_{\iota=2}^{\iota-1} \xi(t - \iota I - T_\Upsilon)P((\iota-1)I + T_\varepsilon - T^* \leq \iota I) \\
&\quad = \xi(t - I - T_\Upsilon)P(0 < \widetilde{T}_1 < I) + \sum_{\iota=2}^{\iota-1} \xi(t - \iota I - T_\Upsilon)P((\iota-1)(I - T_\varepsilon) < \widetilde{T}_1 < i(I - T_\varepsilon) + T_\varepsilon) \\
&\quad = \sum_{\iota=1}^{\iota-1} B(t - iI - T_\Upsilon)p_\iota.
\end{aligned}$$

In the case $\iota I + T_\Upsilon < t < (\iota + 1)I$, the right hand side for the second term to the relation (3.3) is expressed by following

$$P(\chi(t) = 1, T_1 \leq t - T_\Upsilon) = \sum_{\iota=1}^{\iota} \xi(t - \iota I - T_\Upsilon p_\iota)$$

The proof of the equation (3.4) is completed.

Using the same procedure applied in above relation of $S(t)$ and $\xi(t)$, we get it from the derivation of the given relation above in (3.5). As can be seen, by the previous results, the primary inspection to the interval time of the renewal cycle has $I - T_\Upsilon$ taking account into the renewal time, where the first cycle time starts at time 0 inspection of interval time is expected at I.

Hence, replacement or repair process is not considered as renewal cycle process. Therefore if the first cycle is removed, all type of the renewal cycles develop the renewal process at time I. Consequently, the limiting average of the system availability. Hence the limiting average of system availability to the renewal process contained all of the renewal cycles. It is noticed that the system availability is expected to the equal to the cycle of

$$\mu_0 = \int_0^\infty \Phi(t)\mathrm{d}t,$$

of a mean lifetime to the system availability. Therefore the limiting average of the system availability is given by,

$$\begin{aligned}\overline{S} &= \frac{\int_0^\infty \Phi(t)\mathrm{d}t}{\sum_{\iota=1}^\infty [I - T_\Upsilon + T_\Upsilon)P(T_1 = \iota I - T_\Upsilon)} \\ &= \frac{\int_0^\infty \Phi(t)\mathrm{d}t}{I\sum_{\iota=1}^\infty \iota q_\iota}\end{aligned}$$

The proof have been completed.

It have seen that

$$S(t) = \xi(t) \quad \text{as} \quad T_\varepsilon = T_\Upsilon = 0.$$

The more details of this model case is derived bye some researchers which can be seen in Kharoufeh et al. [9] together with Sarkar and Sarkar [15] where they have taken the similar case into consideration.

3. Availability System and Age Based on the Inspection Procedure

From above, we have seen that,the system availability if calendar based on the inspection procedure is discussed. In this part, we discuss the instant system availability together with limiting availability of the state system using age based on the inspection procedure with scenarios A_0 and A_1.

3.1. Formulation of the model using the scenarios A_0

Using scenarios A_0, the state system has a renewal at every inspection time, the instant system availability to the state system, we obtain the results as follow.

Theorem 3. *By using the scenarios A_0, the instant system availability to the state system together period based on the inspection procedure can be obtained by the following:*

$$S(t) = \begin{cases} \Phi(t) & if\, t \leq I \\ 0 & if\, I < t < I + T_\varepsilon \\ \Phi(I)\Phi(t - I - T_\varepsilon & if\, I + T_\varepsilon \leq t < I + T_\Upsilon \\ \Phi(I)S(t - I - T_\varepsilon) + [1 - \Phi(I)]S(t - I - T_\Upsilon) & if\, t \geq I + T_\Upsilon. \end{cases} \tag{3.1}$$

The limiting availability to the state system is obtained by

$$\overline{S} = \frac{\int_0^I \Phi(t)\mathrm{d}t}{I + T_\varepsilon \Phi(I) + T_\Upsilon[1 - \Phi(I)]}. \tag{3.2}$$

Proof. clearly, $S(t) = \Phi(t)$, if $t \leq I$, and $S(t) = 0$, since $I < t < I + T_\varepsilon$ for $I + T_\varepsilon \leq t < I + T_\Upsilon$, we get

$$\begin{aligned} S(t) &= P(\chi(t) = 1) \\ &= P(\chi(I) = 1)P(\chi(t) = 1 \mid \chi(I) = 1) + P(\chi(I) = 0)P(\chi(t) = 1 \mid \chi(I) = 0) \\ &= \Phi(I)\Phi(t - I - T_\varepsilon) + (1 - \Phi(I)) \cdot 0 \\ &= \Phi(I)\Phi(t - I - T_\varepsilon). \end{aligned}$$

If a state of system is tested to be working at the given time I, ($i.e\chi(I) = 1$) has renewed at time $I + T_\varepsilon$, Similarly the state system have been seen failed at time I $\chi(I) = 0$ requires T_Υ time to be replaced or repaired respectively.

Therefore, the state system is not working at some given intervals of time $(I, I+T_\Upsilon)$ means that $\chi(t) = 0$, for $I < t < I + T_\Upsilon$.

When $t > I + T_\Upsilon$, the following is obtained

$$\begin{aligned} S(t) &= P(\chi(I) = 1)P(\chi(t) = 1 \mid \chi(I) = 1) + P(\chi(I) = 0)P(\chi(t) = 1 \mid \chi(I) = 0) \\ &= \Phi(I)S(t - I - T_\varepsilon) + (1 - \Phi(I))S(t - I - T_\Upsilon), \end{aligned}$$

If a state systems are tested to be working at the given time I and it must be renewed in the given time $I + T_\varepsilon$. In the similarly way, the state of the system have been seen failed at time I which must have a renewal at the given time $I + T_\Upsilon$.

Hence the proof for the relation (3.1) is completed.

As the state system must be renewed at every inspection time severally of the period or the state system is not taken to be functioning, average to the availability system has been achieved using a renewal process properties as the following.

$$\overline{S} = \frac{\int_0^I \Phi(t)\mathrm{d}t}{I - T_\varepsilon \Phi(I) + T_\Upsilon[1 - \Phi(I)]}$$

Hence the proof is completed.

3.2. Formulation of the model using the scenarios A_1

Using scenarios A_0, the state system has a renewal at every inspection time. The instant system availability to the state system, we obtain the results as follow.

Theorem 4. *By using the scenarios A_0, the instant system availability together time based on the inspection procedure is obtained by the following:*

$$S(t) = \begin{cases} \sum_{\iota=1}^{m(t)} S(t - t_j - T_\Upsilon)p_j & if\, t > n(t)(I + T_\varepsilon) + I \\ \Phi(t - n(t)T_\varepsilon) + \sum_{\iota=1}^{m(t)} S(t - t_j - T_\Upsilon)p_j & if\, t \le n(t)(I + T_\varepsilon) + I. \end{cases} \tag{3.3}$$

Where

$$m(t) = [\frac{(t - T_\Upsilon + T_\varepsilon)}{(I + T_\varepsilon)}], n(t) = [\frac{t}{(I + T_\varepsilon)}], t_\iota = (\iota - 1)(I + T_\varepsilon) + I, p_\iota = \Phi((\iota - 1)I) - \Phi(\iota I).$$

A limiting average to the system availability system gives as follows,

$$\overline{S} = \frac{\int_0^\infty \Phi(t)\mathrm{d}t}{I + T_\varepsilon \sum_{i=1}^\infty \iota p_\iota + T_\Upsilon - T_\varepsilon} = \frac{\mu}{(I + T_\varepsilon) \sum_{i=0}^\infty \Phi(\iota I) + T_\Upsilon - T_\varepsilon}. \tag{3.4}$$

Proof. If the state system is taken down in the interval time of $[T_1^*, T_1 + T_\Upsilon]$, we get

$$\begin{aligned} S(t) &= P(\chi(t) = 1) \\ &= P(\chi(t) = 1, t < T_1^* + P(\chi(t) = 1, T_1^* \le t < T_1 + T_\Upsilon + P(\chi(t) = 1, T_1 + T_\Upsilon \le t \\ &= P(\chi(t) = 1 \mid T_1^* > t)P(T_1^* > t) + P(\chi(t) = 1, T_1 \le t - T_\Upsilon). \end{aligned} \tag{3.5}$$

Using the definition to the $n(t)$, we obtain

$$[n(t)(I + T_\varepsilon) \le t \le n(t) + 1)(I + T_\varepsilon) + I, n(t) + 1)(I + T_\varepsilon)]$$

while it is considered as an inspection period.

Hence, by the first term in the right hand side to the relation (3.3), we obtain the following

$$\begin{aligned} P(X(t) = 1 \mid T_1^* > t)P(T_1^* > t) \\ &= \begin{cases} 0.P(T_1^* > t) \text{ if } t > n(t)(I + T_\varepsilon) + I \\ 1.P(T_1^* > t) \text{ if } t \le n(t)(I + T_\varepsilon) + I. \end{cases} \\ &= \begin{cases} 0 & \text{if } t > n(t)(I + T_\varepsilon) + I \\ P(\widetilde{T}_1 > t - n(t)T_\varepsilon) & \text{if } t < n(t)(I + T_\varepsilon) + I. \end{cases} \\ &= \begin{cases} 0 & \text{if } t > n(t)(I + T_\varepsilon) + I \\ \Phi(t - n(t)T_\varepsilon) & \text{if } t < n(t)(I + T_\varepsilon) + I. \end{cases} \end{aligned}$$

We denote that the considered inspection up to T_1 have been executed at time $t_\iota, \iota = 1, 2, \ldots, m(t)$.

Assume an initial failed have been determined at time t_ι, the state system can be renewed at time $t_\iota + T_\Upsilon$. and displace the given first cycle of the state system at time $[0, t_j + T_\Upsilon]$ with taking account to the time $t_1 + T_\Upsilon$, since the original position is fixed at time $t = 0$. Therefore we get

$$P(\chi(t) = 1 \mid T_1 = t_\iota) = S(t - t_\iota - T_\Upsilon),$$

since

$$t > t_\iota + T_\Upsilon.$$

From the above expression, we have the equivalent to the relation (3.3) which is determined as the following

$$\begin{aligned} P(\chi(t) = 1, T_1 \le t - T_\Upsilon) \\ &= \sum_{\iota=1}^{m(t)} P(\chi(t) = 1 \mid T_I = t_\iota) P(T_1 = t_\iota) \\ &= \sum_{\iota=1}^{m(t)} S(t - t_\iota - T_\Upsilon) P(t_\iota - I < T_1^* \le t_\iota) \\ &= \sum_{\iota=1}^{m(t)} S(t - t_\iota - T_\Upsilon P(\iota - 1)I < \widetilde{T}_I \le \iota I \\ &= \sum_{\iota=1}^{m(t)} S(t - t_\iota - T_\Upsilon) P_\iota. \end{aligned}$$

The proof of relation (3.3) is completed.

It is considered that, the replacement or repair time to the failure of the state system take the state system as new as the original one. And the limiting average of the system availability have been obtained using the renewal process property by the following:

$$\begin{aligned} \overline{S} &= \frac{\int_0^\infty \Phi(t)\mathrm{d}t}{\sum_{\iota=1}^\infty [I_\iota + T_\Upsilon + P(T_1 = t_\iota)]} \\ &= \frac{\int_0^\infty \Phi(t)\mathrm{d}t}{\sum_{\iota=1}^\infty (t_\iota + T_\Upsilon) p_\iota} \\ &= \frac{\int_0^\infty \Phi(t)\mathrm{d}t}{(I + T_\varepsilon) \sum_{\iota=1}^\infty (\iota p_\iota) + T_\Upsilon - T_\varepsilon}. \end{aligned}$$

Hence the proof is completed.

By considering both scenarios A_0 and A_1, the considered model can be reduced to the same one discussed by some researchers such as Cui and Xie [3] if $T_\varepsilon = 0$. Their given article provide only instantaneous system availability system. Therefore by combination of such instant system availability with limiting average of the system availability have been discussed.

Table 1: Optimal failure finding interval and renewal in every inspection period of time.

	Age based on the inspection		**Calendar based on the inspection**	
	$\overline{S}_{\max}$	I^*	$\overline{S}_{\max}$	I^*
$\beta = 0.5$	0.8619	171h	0.8619	180h
$\beta = 1$	0.9557	356h	0.9557	364h
$\beta = 2$	0.9869	911h	0.9869	919h

4. Results and Discussions

By considering both scenarios A_0 and A_1, the considered model can be extended from the one discussed by Cui and Xie [3] if $T_\varepsilon = 0$. Their argument provides only instantaneous system availability. Therefore by a combination of such instant system availability with limiting average of the system availability have been discussed. It is noticed that the optimal failure finding interval is closely similar since the shape to the parameter β is bigger than 1. The below given Fig (5.1) presents the average of system availability on optimal failure finding interval if β is attributed a value of 0.8 As the function is not with a single modal, the optimal failure finding interval is changed compared to that one with small β value, said $\beta = 2$. as an example.

Optimal failure finding interval with maximum limiting average of the system availability is presented in table 2. From this table we make comparison between the optimal failure finding interval using scenarios A_0 and A_1 with all inspection procedures.

As $\beta < 1$, the optimal failure finding interval using scenarios A_0 is too short compared to the one using scenarios A_1. Maximum limiting average of the system availability $\overline{S}_{\max}$ is very high with renewed after each failure under scenarios A_1 than at every inspection time under scenarios A_0. By using scenarios A_1, if β is closed to or bigger than 1, Maximum limiting average of the system availability system $\overline{S}_{\max}$ with optimal failure finding interval (I^*) are not sensitive with β value as well as the procedure used. All procedures on maximum limiting of the system availability $\overline{S}_{\max}$, are similar into the range of 0.95.

Which proves that the optimal failure finding interval is always closed same, even when the shape to the parameter β is replaced by the other value, it will stay almost up to 10 with identical η value. For all inspection procedures, the optimal failure finding interval under calendar based on the inspection procedures approximately $334-348$ hours is slightly bigger than for underage based on procedures $326 - 340$ hours to the similar β value. It is interested to observe how the outcome results to the study procedures changing with constant value of β but only for scale parameter η. Since η changing from the value less than 8000 hours, the optimal failure finding interval result is 277 hours with maximum limiting average of the system availability 0.9460. Hence, in time $\beta > 1$, the optimal failure finding interval to the system availability is more sensitive on η accordingly compared to β.

Table 2: Optimal failure finding interval due to comparison to the maximum availability system.

	scenarios A_0				**scenarios A_1**			
$\eta = 8000h$	**Age based**		**Calendar based**		**Age based**		**Calender based**	
	$\overline{S}_{\max}$	I^*	$\overline{S}_{\max}$	I^*	$\overline{S}_{\max}$	I^*	$\overline{S}_{\max}$	I^*
$\beta = 0.5$	0.8619	17h	0.8619	180h	0.9673	458h	0.9672	467h
$\beta = 1$	0.9557	356h	0.9557	364h	0.9557	340h	0.9557	348h
$\beta = 2$	0.9869	911h	0.9869	919h	0.9531	322h	0.9531	330
$\beta = 4$					0.9536	326h	0.9536	334h

4.1. Numerical example for explanation

An industrial plant for smelting company in the United States of America steams and the process of the tube with the subject is to evaluate pressure loads. The protection of the pipes and materials equipment to the different unpredicted inconvenient of pressure variation, the safety valves have been established. The primary purpose of the safety valves is to liberate the pressure in case it arrives at the critical point. The failure of these tube affects downstream to the materials equipment, or motivate the failure of the tube. The results are unpredicted downtime and sudden replacement which is so expensive. The failure prevention to the safety valves, inspection time, is applied in normal time estimated period. At every inspection time which is supposed to be taken along in working time, valves are completed washed and proved to be useful at the critical pressure point. As the valves are tested to work due to inspection time, it must be revisited the maintenance operation office for the next found failure. In the same way, as the valves are tested having a failure, this will give another working time to break up the machine system which is becoming as good as new same as before. Therefore we assume that $T_\varepsilon = 8$ hours with $T_\epsilon = 8$ hours or $T_\Upsilon = 16$ hours.

We suppose that the failure time to the safety valves obeys the Weibull distribution and shape parameter β with scale parameter η. Hence we obtain,

$$\Phi(t) = \exp(-(\frac{t}{n})^{\beta}).$$

And shape with scale parameter estimated as $\beta = 0.855$ and $\eta = 4.399$ years respectively by failure data identification. In this case, the instant system availability is computed respectively using the above relations (5), and (6) as calendar based on the inspection procedure is assumed otherwise the relation (2) may be applied as age-based on the inspection procedure is also assumed.

Here it is not necessary to find an optimal finding failure interval from the instant system availability function. Then, the limiting average to the availability system preferably to be computed then the instant system availability to find out and appropriateness of the inspection interval time. By the relation (6), a limiting of average of the system availability to time-based the inspection procedure is given by,

$$\overline{S} = \frac{\mu}{I\sum_{\iota=1}^{\infty}\Phi(t_\iota)} = \frac{\eta\Gamma(1+\frac{1}{\beta})}{I + I\sum_{\iota=1}^{\infty}\exp(\frac{-((\iota(I-T_\varepsilon)+T_\varepsilon-T_F)}{\eta})^\beta}$$

With Γ is given as the gamma function.

By the relation (4), a limiting of average to the availability system for time-based on the inspection procedure is given by the following,

$$\overline{S} = \frac{\mu}{(I+T_\varepsilon)\sum_{\iota=0}^{\infty}\Phi(\iota I) + T_\Upsilon - T_\varepsilon} = \frac{\eta\Gamma(1+\frac{1}{\beta})}{(I+T_\varepsilon)\sum_{\iota=0}^{\infty}(\exp(\frac{-\iota I}{\eta})^\beta) + T_\Upsilon - T_\epsilon}$$

The given Figure 1 below illustrates the limiting average of the system availability as a function of the failure finding interval time I for the calendar based on the inspection procedure. The age-based on the inspection procedure plot has to be neglected at this level which approach of the one plotted on periodic based on the inspection procedure as a result of the estimated loss value of T_ε with T_Υ. Both procedures give the identical optimal finding of failure interval time of 34 days, which very approach the limiting average of the system availability evaluated as (0.9804) where it is different only from the 6^{th} digit decimal numbers. The present practical made in the company was the inspection of the safety valves for one time per year. Therefore the finding failure interval time is $I = 1$ year = 3605 days. It evaluates a very less limiting average of the system availability as 0.8985 for calendar based on the inspection procedure with a value of 0.8984 attributed to the age based on the inspection procedure. Therefore the suggestion of obtaining the maximization of average of the system available to the safety valves, the present inspection frequency must be varied from year to approximately month.

All procedures give the similar optimal finding failure interval time which is identical to the limiting average of the system availability. Therefore the application of calendar based on the inspection procedure is recommended. It is simply to make a schedule plan than for the age based on the inspection procedure. It is very important to get out the required solution since the solution of all inspection procedures is quite different from each other. As downtime to the inspection time with replacement or repair time are comparatively large. By theoretical research, the age-based on the inspection procedure is well functioning efficiently compared to the calendar based inspection procedure as it can schedule an unessential inspection time before being renewed. In this case , suppose that $T_\varepsilon = 24$ hours with $T_\Upsilon = 48$ hours. With all procedures, we get the maximum similarity to the system availability as 0.9661. The optimal failure finding interval time with a calendar based on the inspection procedure is given for 58 days, which is relatively great than that of age based on the inspection time. Also, since the inspection procedure with replacement or repair time varies up to the (3.5), period of limiting average of the system availability of the age-based procedure is delicately greater to the periodic based on inspection time is 0.9362 caused by previous one with 0.9360 for the next procedure.

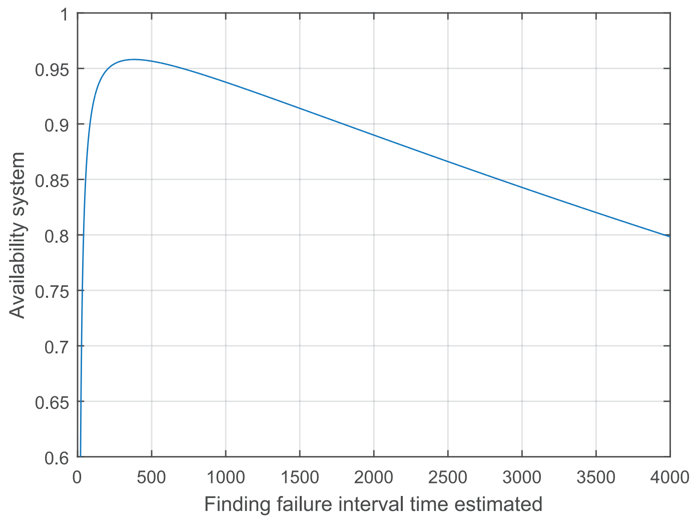

Figure 3: Limiting average availability system due to Calendar and beta=0.8.

Therefore, the real good application, for downtime caused by the inspected time with a replacement or repaired time that comparatively closed too small. Hence, the calendar based on the inspection time is commonly taken into consideration as the most real good and easy for application. When the finding failure interval is founded, we have to consider the instant system availability to be a function of time for knowing the real change to the system availability over the given time. The instant system availability over time to the system of calendar based on the inspection procedure is discussed if the finding failure interval time of 34 days has been assumed. In addition to this, the plot of the instant system availability function due to age based on the inspection procedure has been neglected since it closed similar to that one illustrated in Figure 3 considering the less value given to T_ε with T_Υ. respectively.

The situation seems that the downtime caused by inspection time and downtime for replacement or repair time are omitted if $T_\varepsilon = 8$ hours together with $T_\Upsilon = 16$ hours are too small relative to $\textit{Finding failure interval time} = 34 \times 24 = 866$ hours. Therefore, if we consider the downtime for inspection time to be neglected, means that $T = 0$, the limiting average of the system availability as illustrated in Figure 3 is having significant value, specifically, if the finding failure interval time is too small.

It is noticed that since $T_\varepsilon = 0$, periodic based on the inspection procedure with age-based on inspection procedure, the availability function is almost same. The maximum for average of the system availability is gained by finding failure interval time, providing availability of 0.9997. In different real application procedure, the inspection time is always taken into consideration in the study process. In exemplary, when state system device is revealed failed, the unique solution is to apply replacement or repair time in which may save time consumer. Theoretical and practical experiences have shown that the inspected time is relatively close to and even important to downtime needed as replacement time which can not be neglected anymore.

We are very interested to noticed how the outcome results varied since the shape parameter β is taken as 1 having the similar mean with slightly modifying the scale parameter such that $\eta = 4.735$. The finding failure optimal is almost seen as identical within 34 days of time. The maximum limiting average of the system availability is also considered great, which is closed related to 0.9804 as have been seen before. The results prove that the optimal finding failure interval time is always same even since the shape of parameter β is still changing and continue to be similar up to 10 with the identical mean.

5. Conclusion

In this model, the study of instant system availability and limiting average of system availability have been discussed. We have observed that downtime caused by inspection and downtime caused by replacement or repair time cannot be neglected. In this model, the inspection downtime is supposed to be maintained constant as the downtime to replacement or repair time is taken to be in the similar case. In this article, calendar based on the inspection policy and age based on the inspection policy have been introduced to make the novelty of the model formulated. We have used the calendar and age-based on the inspection interval together with scenarios A_0 and A_1 to detect failure finding interval of the system which was our principally objective. Moreover, the results showed that the optimum failure finding interval changes due to parameter β and maximum limiting availability of the system is obtained and closed similar into the range of 0.95 as illustrated in Table 1. The comparison of maximum average availability and optimum failure finding interval have been achieved using scenarios A_0 and A_1 combined with age and calendar based procedures. Results shown that the optimal failure finding interval is 277 hours in case maximum limiting average of availability is approximately 0.946 for $\beta > 1$. A numerical example has been actively discussed to demonstrate the practical application of this model. Therefore the calendar based on the inspection is recommended to be used for detecting the failure finding interval of the system than age-based on inspection. Our model may be extended to the expected cost by considering the minimum repair or minimum replacement procedure to discover the hidden failure of the system.

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School of Mathematical Sciences, Dalian University of Technology, Dalian 116024, PR China, Liaoning, China.

E-mail: hagenimanae@yahoo.fr; hagenimanaem@gmail.com

Major area(s): Probability and statistic in reliability theory.

School of Mathematical Sciences, Dalian University of Technology, Dalian 116024, PR China, Liaoning, China. (Corresponding author)

E-mail: lxsongdl@163.com

Major area(s): Probability and statistics.

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