A Cloud QoS Risk Elements Transmission GERT Network Model

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Abstract

Due to the characteristics of dynamic, virtualization and widespread uncertainties or threat in cloud oriented computing environment, cloud Quality of Service violation is of frequent occurrence, which resulting in low cloud consumer satisfaction and economic loss. This paper raised the cloud Quality of Service Risk Elements transmission model based on cloud workflow inspired by Graphic Evaluation and Review Technique. We construct the model by choosing the QoS criteria of cloud services as risk elements, paths in workflow graph as risk element transmission route. We use moment generating function and Mason formula to calculate risk level of cloud workflow. Finally, we give a case study of the model.

Keywords: Risk transmission, QoS risk, cloud service, risk elements.

1. Introduction

With cloud computing in-depth study and promote the application of the technology, more and more services are transferred from local network to cloud environment. One of the main components of cloud services is the service level agreement (SLA), that refers to the contractual obligations between a Cloud Service Provider (CSP) and a service consumer (or client) (see Jin, et al. [12]). The SLA can document the promised Quality of Service (QoS) from a service provider and the para-functional requirements of service delivery to the client. However, the lack of adequate confidence in a cloud service in terms of the uncertainties associated with its level of quality may prevent a cloud service consumer from adopting cloud technologies (see Macias [16]). CSPs offer access to their resources through formal SLA, and need well-balanced infrastructures so that they can maximise the QoS they offer and minimise the number of SLA violations.

The uncertainties associated with QoS, which described as risk elements in this paper, is a collection of risk factors in term of risk management. Several researches have been done in the area of cloud QoS and risk management in cloud computing environments. For example, Hammadi, et al. [9] proposed a framework for the purpose of
Some studies have been conducted to investigate the definition, types of risks related to cloud computing and the assessment methods and frames to identify and mitigate these risks, but few have been devoted to express the dynamic characteristics of risks. This paper focuses on a specific aspect of risk assessment as QoS risks in cloud computing based on Graphic Evaluation and Review Technique (GERT) for the following reasons. Cloud services evaluation and selection is an important application realm of cloud computing for improving the quality of service and business value, but the major concern of the current evaluation and selection methods lies on certain types of quantifiable criteria (e.g. cost, performance, and availability). Non-quantifiable criteria (e.g. trust, reputation, risk) have been almost neglected (see Alabool, et al. [2]), so non-quantifiable criteria such as risk and its dynamic characteristics calls for more attention and research. A more comprehensive risk measures means that more adaptive and more reliable cloud services can be selected. On the other hand, risk management mechanisms need to be incorporated into cloud infrastructures, in order to move beyond the best-effort approach to service provision that current cloud infrastructures follow (see Ferrer, et al. [7]). If we can analyze the typical cloud model considered dynamic feature of the risk, we may get the analytical model or algorithm with high adaptability.

The remainder of the paper is organized as follows. The general cloud service model is described in Section 2. The cloud QoS risk elements transmission GERT network model is formulated in Section 3, which includes four components: model formulation, risk elements transfer function construction, equivalent transfer probability and moment generating function, and cloud workflow QoS risk level and solving algorithm. Simulation example and discussion are present in Section 4 and conclusions are given in Section 5.

2. General Cloud Service Model

In this paper we based the proposed model on the grounds of a general cloud service model regardless of whether service-based platforms are IaaS, PaaS, or SaaS.
Definition 1. Cloud Workflow Graph (CWG). A cloud workflow is abstracted as a DAG, which is a quadruple $G = (V, E, t_0, t_n)$ consisting of following elements:

- $V$ is a finite set of vertices. Each vertex is associated with an executable atomic task $t_i$ ($0 \leq i \leq n$). When the workflow starts, $t_i$ can choose several cloud services distributed in different regions, cloud services can include composite services, atomic services. Atomic services can be divided into abstract services and concrete services (see Hock-Koon, et al. [10]). Abstract services represent the functional descriptions of services. Concrete services also called service instances, are published by CSPs. When a concrete service is selected, $t_i$ is represented as $\langle t_i, s_i \rangle$.
- $E \subset V \times V$ is a set of edges connecting vertices. An edge is a triple $e = \langle t_i, p_i, t_j \rangle \in E$. Edges express dependencies between tasks and are associated with an enabling probability $p_i$. When a task has only one outgoing transition, the enabling probability is 1. In such a case, the probability can be omitted from the CWG.
- $t_0 \in V$ is the starting task of the CWG.
- $t_n \in V$ is the ending task of the CWG, signifying the completion of the cloud workflow.

The cloud workflow, which enables the business process to be executed automatically, also can make use of cloud computing to extend its functions (see Aalst, et al. [1]). With rescaling the server cluster, a workflow model can coordinate various cloud services to achieve a complicated business logic in a simple and cheap way (see Wang, et al. [20]).

Definition 2. Execution path and Execution plan. In a CWG, a set of tasks with the relation of $(t_i \rightarrow t_{i+1}) \land (t_{i+1} \rightarrow t_{i+2}) \land \cdots \land (t_{j-1} \rightarrow t_j)$ forms an execution path $\pi_{i\rightarrow j} = (t_i, t_{i+1}, t_{i+2}, \ldots, t_j)$. A set of services with the relation of $((t_i, s_i) \rightarrow (t_{i+1}, s_{i+1})) \land ((t_{i+1}, s_{i+1}) \rightarrow (t_{i+2}, s_{i+2})) \land \cdots \land ((t_{j-1}, s_{j-1}) \rightarrow (t_j, s_j))$ forms an execution plan $\rho_{i\rightarrow j} = ((t_i, s_i), (t_{i+1}, s_{i+1}), (t_{i+2}, s_{i+2}), \ldots, (t_j, s_j))$. The relationship between tasks and cloud services can be explained as service $s_i$ belongs to the concrete service associated with task $t_i$. In other words, service $s_i$ provides the operation required by task $t_i$.

Definition 3. QoS Criteria of Cloud Service. The quality of a cloud service is generally multidimensional, we define $Q(s) = \{q_1(s), q_2(s), \ldots, q_n(s)\}$ as $n$ dimensions QoS criteria of service $s$, where $q_i(s)$ represent the $i$th criterion agreed in advance between customer and CSP for service $s$. Then, the $Q(s)$ of task $t_k$ can be expressed by $Q_k(s) = \{q_1^k(s), q_2^k(s), \ldots, q_n^k(s)\}$.

Currently, no regulated cloud service quality standard exists. The literatures (see INSPIRE European Commission [11] and Liam, et al. [15]) described in detail various criteria of QoS, these criteria may include but not limited to: Usage cost (U), Fault rate (F), Response time (R), Operability (O), Availability (A).

Definition 4. Risk Element $r_k(s)$. Let $Q_k(s)$ of a task $t_k$ in a cloud workflow, which may be affected by uncertainties associated with Cloud SLA that can make a difference between the actual QoS quality and its agreed in advance.
3. Cloud QoS Risk Elements Transmission GERT Network Model

GERT (see Feng [6]) is a new type of generalized analysis method on stochastic network developed on the basis of Performance Evaluation Review Technique (PERT), and has been widely used in every field of society. The static and dynamic characteristics and the probability distribution of the system can be obtained by means of this method. Shi, et al. [19] modeled a risk analysis model of complex supply chain based on GERT stochastic network, which can provide emergency decision for manufacturers. Chong, et al. [4] proposed a reliability GERT stochastic network to analyze the reliability of multi component complex system. So GERT provides a good description for the relations and the transition among various risk elements, and reflect the random properties of risk elements transmission. We proposed a cloud QoS risk elements transmission GERT network model inspired by this analysis method.

3.1. Model formulation

Cloud services distributed in different regions, which are linked by workflow for implementing business processes in the context of cloud computing applications, form a complicated GERT network. Each task node of cloud workflow. Nodes in risk elements transmission GERT network consist of various task nodes in cloud workflow. The flow of risk elements forms the edge of the network. Volatility and uncertainty of various QoS criteria constitutes risk flow. GERT model analysis transitive relation between risk elements according to signal flow graph theory, and calculate the numerical characteristics of probability distribution by moment generating function, thus obtain the risk level of the whole system.

Based on above description and discussion, the proposed cloud QoS risk elements transmission GERT network model shows as follows.

![Figure 1: Cloud QoS risk elements transmission GERT network model.](image)

**Definition 5.** Cloud QoS Risk Elements Transmission GERT Network Model (CQRETM). CQRETM is a triple \((N, E, R)\) consisting of following elements:

- \(N = \{n_1, n_2, \ldots, n_m\}\) represents a collection containing only “or” type of risk transfer node;
- \(E = \{e_i \mid \langle v_i, v_j \rangle\}\) represents a collection of branches connecting nodes.
- \(R = N \times E \cup E \times N = \{r_{ij}(k), k \in \{1, 2, \ldots, n\}\}\) represents risk flow between branches.

Define \(n\) risk elements \(r_{ij}(k)\) associated with node \(i\) and \(j\) for \(n\) dimensions QoS attribute in \(Q_k(s)\), respectively.
In the CQRETM as shown in Figure 1, $R_{ij}$ indicate the risk flow from node $i$ to node $j$, $p_{ij}$ denote the transition probability between two nodes. In general, $p_{ij}$ is equal to the transition probability between two tasks in cloud workflow. We consider paths between any two tasks in CWG as risk element transmission route.

For a risk element $r_{ij}$, the moment generating function of $r_{ij}$ are discussed in two area. The moment generating function of $r_{ij}$ can be obtained directly, if its probability density function is known. For instance, the moment generating function of normal distribution $N(\mu, \sigma^2)$ is $M_x(s) = \exp(\mu s + \frac{1}{2} \sigma^2)$. If the probability density function is unknown, the probability distribution of the risk element can be estimated by using the statistical theory with some experts predict, historical data of past executions, service consumers' feedbacks, and service providers' profiles, etc. If the known data samples of $r_{ij}$ is $N = \{x_1, x_2, \ldots, x_m\}$, the probability density function can be calculate by Maximum Entropy Model as fellows.

$$f(r_{ij}(k)) = \exp\left(\lambda_0 + \sum_{j=1}^{M} \lambda_k h'_k\right). \quad (3.1)$$

Where, $M$ is the number of known moment of density function $f(r_{ij}(k))$, $m_j$ is the $j$th moment, and $\lambda_0, \lambda_1 (j = 1, 2, \ldots, M)$ is Lagrange multiplier introduced by variational method.

### 3.2. Risk elements transfer function construction and solution

Risk elements transmission function play important foundation role for dynamic characteristic of cloud workflow QoS risk. From the equivalent risk transmission function, the equivalent moment generating function can be determined and others critical parameters can be computed further. For simplify and to make the expression clear, this paper only consider three risk elements. Without loss of generality, the proposed model can be extended to other QoS criteria. Total risk of cloud service QoS related to tasks is consist of three components: $C$ characterize the cost risk elements, $R$ for the response time risk elements, $A$ for the available risk elements, then the risk increment of whole GERT network is $R_{ij} = (C,R,A) = C + R + A$.

**Theorem 1.** Let $r_{ij}(1), r_{ij}(2), \ldots, r_{ij}(n)$ be $n$ independent risk elements from node $i$ to node $j$ applies to cloud workflow application, the probability of the process implementation is $p_{ij}$. Suppose each moment generating function of risk elements is exists, and the equivalent parameter between two nodes is linear combination of $n$ risk elements, namely, $r_{ij} = \lambda_1 r_{ij}(1) + \lambda_2 r_{ij}(2) + \cdots + \lambda_n r_{ij}(n)$, thus the equivalent transfer function of the branch from node $i$ to node $j$ can be calculated as fellows.

$$W_{ij}(s_1, s_2, \ldots, s_n) = \frac{\prod_{k=1}^{n} W_{x_{ij}(k)}(s_k \lambda_k)}{p_{ij}^{n-1}}. \quad (3.2)$$
Proof. If \( r_{ij} = \lambda_1 r_{ij}(1) + \lambda_2 r_{ij}(2) + \cdots + \lambda_n r_{ij}(n) \), and \( r_{ij}(1), r_{ij}(2), \ldots, r_{ij}(n) \) are mutually independent. Then,

\[
W_{ij}(s_1, s_2, \ldots, s_n) = p_{ij} M_p(s_1, s_2, \ldots, s_n) = E[e^{s_1 r_{ij}(1) + s_2 r_{ij}(2) + \cdots + s_n r_{ij}(n)}] \\
= p_{ij} \prod_{k=1}^{n} e^{s_k \lambda_k r_{ij}(k)} \\
= p_{ij} \prod_{k=1}^{n} M_{r_{ij}(k)}(s_k \lambda_k) \\
= \frac{1 \prod_{k=1}^{n} M_{r_{ij}(k)}(s_k \lambda_k)}{p_{ij}^{n-1}}.
\]

Theorem 2. Let \( W_r(s_1, s_2, \ldots, s_n) \) be the equivalent transfer function of \( n \)th direct path from node \( i \) to node \( j \), \( W_i(L_m) \) be equivalent transfer coefficient of \( i \)th order ring of \( n \) order ring. The equivalent transfer function from node \( i \) to node \( j \) \( W_{uv}(s_1, s_2, \ldots, s_n) \) can be obtained as fellows.

\[
W_{uv}(s_1, s_2, \ldots, s_n) = \frac{\sum_{k=1}^{n} W_r(s_1, s_2, \ldots, s_n) \left[ 1 - \sum_{m \neq r} (-1)^m W_i(L_m) \right]}{1 - \sum_{m} (-1)^m W_i(L_m)}. \tag{3.3}
\]

Proof. Owing to \( W_i(L_m) \) is equivalent transfer coefficient of \( i \)th order ring of \( n \) order ring, the Eigen function of the CQRETM is \( \Delta = 1 - \sum_{m \neq r} (-1)^m W_i(L_m) \). The Eigen function of residual subgraph after eliminating all nodes and branches associated with \( r \)th path is \( \Delta = 1 - \sum_{m \neq r} (-1)^m W_i(L_m) \). In the light of Mason formula of signal flow graph, the equivalent transfer function from node \( i \) to node \( j \) is

\[
W_{uv}(s_1, s_2, \ldots, s_n) = \frac{\sum_{k=1}^{n} W_r(s_1, s_2, \ldots, s_n) \left[ 1 - \sum_{m \neq r} (-1)^m W_i(L_m) \right]}{1 - \sum_{m} (-1)^m W_i(L_m)}.
\]

3.3. Equivalent transfer probability and moment generating function

In CQRETM, equivalent transfer probability characterize the degree of possibility of risk occurrence between arbitrary nodes in cloud workflow. Each order origin moment can be calculated by means of moment generating function, and then each order center moment.

Let \( W_{uv}(s_1, s_2, \ldots, s_k, \ldots, s_n) \) be the equivalent transfer function from node \( u \) to node \( v \). In the light of features of multi parameter moment generating function. When \( s_k = 0 \), the equivalent transfer probability from node \( u \) to node \( v \) can be obtained by setting all the \( s_k \) in \( W_{uv}(s_1, s_2, \ldots, s_k, \ldots, s_n) \) equal 0 as shown in formula (3.4).

\[
p_{uv} = W_{uv}(s_1, s_2, \ldots, s_k, \ldots, s_n). \tag{3.4}
\]
The formula (3.5) can be further available.

\[ p_{uv} = W_{uv}(0, 0, \ldots, 0) = p_{uv}M_{uv}(0, 0, \ldots, 0) = p_{uv} \prod_{k=1}^{n} M_{r_{ij}(k)}(\lambda_{k}s_{k}) = p_{uv} \prod_{k=1}^{n} M_{r_{ij}(k)}(0). \] (3.5)

According to Mason formula, the equivalent moment function from node \( u \) to node \( v \) can be obtained as shown in formula (3.6)

\[ M_{uv}(s) = \frac{W_{uv}(s_{1}, s_{2}, \ldots, s_{k}, \ldots, s_{n})}{p_{uv}} = \frac{W_{uv}(0, 0, \ldots, 0)}{W_{uv}(0, 0, \ldots, 0)}. \] (3.6)

### 3.4. Cloud workflow QoS risk level and solving algorithm

One of the main objectives is assessment of cloud workflow QoS risk associated to an execution plan. The proposed risk level in this paper can be utilized to measure occurrence probability of risk caused by uncertainty of Cloud workflow QoS criteria.

The risk level are categorized in two distinct ways: single risk element and multiple risk element.

In case where single risk element is the only consideration, risk level of it can be defined as fellows.

**Definition 6.** Risk level \( \psi_{ij}(k) \) of risk element \( r_{ij}(k) \). Given that \( E(r_{ij}(k)) \) and \( V(r_{ij}(k)) \) respectively denote the expectation and variance of \( k \)th risk element \( r_{ij}(k) \), and \( \sqrt{V(r_{ij}(k))} \) indicate absolute measurement of \( r_{ij}(k) \). Accordingly, Risk levels \( \psi_{ij}(k) \) can be defined as \( \psi_{ij} = \frac{\sqrt{V(r_{ij}(k))}}{E(r_{ij}(k))} \), \( k \in \{1, 2, \ldots, n\} \).

Given that an execution plan \( \rho_{i \ldots j} = (\langle t_{i}, s_{i} \rangle, \langle t_{i+1}, s_{i+1} \rangle, \ldots, \langle t_{j}, s_{j} \rangle) \), \( T = \{ t_{p} \mid t_{p} \) is a task in \( \rho_{i \ldots j} \} \), \( S = \{ s_{p} \mid s_{p} \) is a concrete cloud service to fulfill the task \( t_{p} \)\}, \( p \in \{1, 2, \ldots, m\} \) and \( Q(s) = \{ q^{k}(S), q^{k}(S), \ldots, q^{k}(S) \} \), \( k \in \{1, 2, \ldots, m\} \).

The risk level \( \psi_{ij}(k) \) can be calculated by Algorithm 1.
Algorithm 1. Calculating risk level of $\pi_{i\cdots j}$ (Single risk element)

Input: $T, S$, two tasks $t_i, t_j$ in CWG.
Output: $\psi_{ij}(k), k \in \{1, 2, \ldots, n\}$ of risk element of $\rho_{i\cdots j}$.

Begin

1 Input transition probability of $t_k$ in $T$, and probability density function $f(x_i)$ of $Q_k(S)$.

2 for $h = 1 \rightarrow n$ //n risk element corresponding to $Q_k(s)$.

3 Calculate characteristic transfer function $w_{ij}^h(s)$ of $\pi_{i\cdots j}$.

4 Calculate moment generating function $M_{ij}^h(s)$ of $\pi_{i\cdots j}$.

5 Calculate the expectation $E(r_{ij}(s))$ and variance $\sqrt{V(r_{ij}(s))}$ of risk element $r_{ij}(h)$.

6 $\psi(h) = \frac{\sqrt{V(r_{ij}(s))}}{E(r_{ij}(s))}$

7 end //for

8 return $\psi_{ij}(k), k \in \{1, 2, \ldots, n\}$.

End

For step 3, characteristic transfer function $w_{ij}^h(s)$ is divided into three cases to study.

(1) *Sequence structure*. Because of the linear characteristics, multiple serial structures can be transformed into equivalent network of single vector, and then characteristic transfer function $w_{ij}^h(s)$ is given as follows:

$$w_{ij}(s) = w_{i,i+1}(s) \cdot w_{i+1,i+2}(s) \cdots w_{i+m-2,j}(s).$$  (3.7)

(2) *Parallel structure*. Parallel structure is analogous to that in circuit theory. Suppose there are $n$ paths from node $i$ to node $j$, $w_{ij}^h(s)$ is the characteristic transfer function related to $h$th path, and then characteristic transfer function $w_{ij}^h(s)$ of this structure is given as follows:

$$w_{ij}(s) = \sum_{i=1}^{n} w_{ij}^i(s).$$  (3.8)

Where, $w_{ij}^i(s)$ can be calculated using the formula (3.7).

(3) *Mixed structure*. In this paper, Network structure containing loops described as mixed structure as shown in Figure 2.
The characteristic transfer function $w_{ij}^h(s)$ of this structure is given as follows:

$$\frac{P_{ij}^1 M_{ij}^2(s)}{1 - P_{ij}^2 M_{ij}^2(s)}$$

(3.9)

For step 4, let $w_{ij}^h(s) = P_{ij} M_{ij}^h(s)$, where $P_{ij}$ and $M_{ij}^h(s)$ denote respectively the equivalent transition probability and moment generating function of $\pi_{i\cdots j}$. When $s = 0$, we can calculate $M_{ij}(0) = E(0) = 1$, which means must certainly exist according to properties of moment generating function. The $P_{ij}$ can be obtain as fellows.

$$P_E = \frac{w_{ij}^h(s)}{M_{ij}^h(s)} \bigg|_{s=0} = w_{ij}^h(0).$$

(3.10)

For step 5, here the expectation and variance expressed as the average value of an attribute of QoS and the expectation degree of dispersion under the influence of various risk factors, respectively. The formula of expectation and variance can be described as fellows.

$$E(r_{ij}(h)) = \frac{dM_{ij}^h(s)}{ds} \bigg|_{s=0}$$

(3.11)

$$V(r_{ij}(h)) = \frac{d^2M_{ij}^h(s)}{ds^2} \bigg|_{s=0} - \left[ \frac{dM_{ij}^h(s)}{ds} \right]_{s=0}^2.$$

(3.12)

When the number of risk elements exceeds 2, the concept of risk level need to redefine as follows.

**Definition 7.** Risk level $\psi_{ij}^k$ of risk element $r_{ij}(k)$. $\psi_{ij}^k$ can manifest the QoS risk degree with respect to $k$th risk element of an execution path $\pi_{i\cdots j}$. Let $E(R(k))$ and $V(R(k))$ be the expectation and variance of $k$th risk element $r_{ij}(k), k \in \{1, 2, \ldots, n\}$, and $\sqrt{V(R(k))}$ indicate absolute measurement of a risk. Accordingly, Risk levels $\psi_{ij}^k$ can be defined as $\psi_{ij}^k = \sqrt{V(R(k))}/E(R(k)), k \in \{1, 2, \ldots, n\}$.

**Definition 8.** Total risk level $\Psi_{ij}$ of an execution path $\pi_{i\cdots j}$. $\Psi_{ij}$ can demonstrate the QoS risk degree with respect to an execution path $\pi_{i\cdots j}$ in multiple risk elements environment. Let be $E(Y)$ and $V(Y)$, which respectively denote the expectation and variance of risk flow $R$, and $\sqrt{V(Y)}$ indicate absolute measurement of a risk. Accordingly, Risk levels $\Psi_{ij}$ can be defined as $\Psi_{ij} = \sqrt{V(Y)}/E(Y)$.

The risk level $\psi_{ij}^k$ and $\Psi_{ij}$ can be calculated by Algorithm 2.
Algorithm 2. Calculating risk level $\psi_{ij}^k$ and $\Psi_{ij}$ of $\pi_{i\cdots j}$ (Multiple risk elements)

Input: $T, S$, two tasks $t_i, t_j$ in CWG.

Output: $\psi_{ij}(k), k \in \{1, 2, \ldots, n\}$ and $\Psi_{ij}$.

Begin

1. Input transition probability of $t_k$ in $T$, and probability density function $f(x_i)$ of $Q_k(S)$.

2. Calculate equivalent characteristic transfer function $W_{ij}(s_1, s_2, \ldots, s_n)$ of $\pi_{i\cdots j}$ using formula (3.2) and (3.3).

3. Calculate equivalent moment generating function $M_{uv}(S)M_{ij}^h(s)$ of $\pi_{i\cdots j}$ using formula (3.6).

4. Calculate the expectation $E(R(k))$ and variance $\sqrt{V(R(k))}$ of risk element $r_{ij}(h)$.

5. $\psi_{ij}^k = \frac{\sqrt{V(R(k))}}{E(R(k))}$

6. Calculate the expectation $E(Y)$ and variance $V(Y)$.

7. $\Psi_{ij} = \frac{\sqrt{V(Y)}}{E(Y)}$

8. return $\psi_{ij}(k), k \in \{1, 2, \ldots, n\}$ and $\Psi_{ij}$

End

For step 4, $E(R(k))$ indicates the first moment. If the equivalent transfer function from node $i$ to node $j$ is $W_{ij}(s_1, s_2, \ldots, s_n)$, the first moment $E(R(k))$ can be calculated by formula (3.13).

$$E[R(k)] = \frac{\partial}{\partial S_k} \left[ \frac{W_{ij}(s_1, s_2, \ldots, s_k, \ldots, s_n)}{W_{ij}(0, 0, \ldots, 0)} \right]_{s_1 = s_2 = \cdots = s_k = \cdots = s_n = 0}$$  \hspace{1cm} (3.13)

Specific calculation formula is as follows.

$$\frac{\partial}{\partial S_k} \left[ \frac{W_{ij}(s_1, s_2, \ldots, s_k, \ldots, s_n)}{W_{ij}(0, 0, \ldots, 0)} \right]_{s_1 = s_2 = \cdots = s_k = \cdots = s_n = 0} = \int_{-\infty}^{\infty} f(r_{ij}(1))dr_{ij}(1) + \int_{-\infty}^{\infty} f(r_{ij}(2))dr_{ij}(2) + \cdots$$

$$+ \frac{\partial}{\partial S_k} \left[ \int_{-\infty}^{\infty} e_{ij}^{r_{ij}(k)} f(r_{ij}(k))dr_{ij}(k) \right]_{s_k = 0} + \cdots + \int_{-\infty}^{\infty} f(r_{ij}(k))dr_{ij}(k)$$
of a linear combination of \( n \) variables:

\[
E \left[ \sqrt{V(R(k))} \right] = \sum_{i=1}^{n} \lambda_i R(i)
\]

This indicates the second moment. If the equivalent transfer function from node \( i \) to node \( j \) is \( W_{ij}(s_1, s_2, \ldots, s_k, \ldots, s_n) \), the incremental variance \( \sqrt{V(R(k))} \) equals the value that setting all the \( s_k \) in \( W_{uv}(s_1, s_2, \ldots, s_k, \ldots, s_n) \) equal 0 after the partial derivatives of second order to \( s_k \) for risk element \( r_{ij}(k) \) minus \( E[R(k)] \) squared.

\[
V_{ij}[R(k)] = E[R(k) | 2] - [E[R(k)] | 2]
\]

\[
= \frac{\partial^2}{\partial S_k^2} \left[ \frac{W_{ij}(s_1, s_2, \ldots, s_k, \ldots, s_n)}{W_{ij}(0, 0, \ldots, 0)} \right] \bigg|_{s_1 = s_2 = \cdots = s_k = \cdots = s_n = 0}
- \left[ \frac{\partial}{\partial S_k} \left( \frac{W_{ij}(s_1, s_2, \ldots, s_k, \ldots, s_n)}{W_{ij}(0, 0, \ldots, 0)} \right) \right] \bigg|_{s_1 = s_2 = \cdots = s_k = \cdots = s_n = 0}^2
\]

(3.14)

Where, \( E[R(k)] = \frac{\partial}{\partial S_k} \left[ \frac{W_{ij}(s_1, s_2, \ldots, s_k, \ldots, s_n)}{W_{ij}(0, 0, \ldots, 0)} \right] \bigg|_{s_1 = s_2 = \cdots = s_k = \cdots = s_n = 0} \).

For step 6, if first moments of \( n \) risk elements with respect to the execute path from node \( i \) to node \( j \) are respectively \( E[R(0)], E[R(2)], \ldots, E[R(k)], \ldots, E[R(n)] \), the expectation of a linear combination of \( n \) risk elements equals a linear combination of expectation of order to \( s_k \) for risk element \( r_{ij}(k) \) minus \( E[R(k)] \) squared.

\[
E[Y] = E[Y] = \lambda_1 R(1) + \lambda_2 R(2) + \cdots + \lambda_n R(n)
\]

\[
= \lambda_1 E[R(1)] + \lambda_2 E[R(2)] + \cdots + \lambda_n E[R(n)] = \sum_{i=1}^{n} \lambda_i E[R(i)]
\]

(3.15)

Similarity, if variance of \( n \) independent risk elements with respect to the execute path from node \( i \) to node \( j \) are respectively \( V[R(1)], V[R(2)], \ldots, V[R(n)] \), the variance of a linear combination of \( n \) risk elements can be obtained as follows.

\[
V[Y] = V[Y] = \lambda_1 R(1) + \lambda_2 R(2) + \cdots + \lambda_n R(n)
\]

\[
= \lambda_1^2 V[R(1)] + \lambda_2^2 V[R(2)] + \cdots + \lambda_n^2 V[R(n)] = \sum_{i=1}^{n} \lambda_i^2 V[R(i)].
\]

(3.16)
4. Application Example

According to the Algorithm 1, the risk level between any two tasks in workflow $T$ can be calculated. In order to verify the effectiveness of the proposed algorithm, the risk levels of the task $t_1$ to task $t_9$ in Figure 3 is calculated as an example.

The first case, we considered only single risk element, i.e., response time attribute $(R)$ is considered, and it may be fluctuated due to risk factor such as some network problems, etc. According to the Definition 3, response time of QoS is a risk element and write for $r_R(s)$. Assuming the risk element $r_R(s)$ are all in line with normal distribution. Let $f(x) = x^2$, Table 1 show pertinent parameters of $C, R$ and $A$.

The equivalent transfer function of $\pi_{1...9}$ is calculated as follows:

$$w_{19}^R(s) = w_{12}w_{24}w_{57}w_{79} + w_{12}w_{25}w_{57}w_{79} + w_{13}w_{36}w_{68}w_{89}$$

$$= 0.18e^{13s+6.5s^2} + 0.12e^{19s+6s^2} + \frac{0.54e^{25s+16s^2}}{1 - 0.2e^{6s+4s^2}}.$$ 

$$P_{19} = w_{19}^R(0) = 0.975.$$
Consequently, the equivalent moment generating function $M_{19}(s) = w^R_{19}(s)$ form $t_i$ and $t_j$ can be derived from above.

Then, the expectation, variance and the risk level of risk element $r_R(s)$ can be obtained according to the following operation procedure.

\[
E(r_R(s)) = \left. \frac{dM_{19}(s)}{ds} \right|_{s=0} = 22.5075;
\]
\[
V(r_R(s)) = \left. \frac{d^2M_{19}(s)}{ds^2} \right|_{s=0} - \left[ \left. \frac{dM_{19}(s)}{ds} \right|_{s=0} \right]^2 = 582.0825 - 506.5876 = 75.5;
\]
\[
\psi(r_{19}^R) = \sqrt{\frac{V(x_R(s))}{E(x_R(s))}} = 38.6%\]

The result of $P_{19} = 97.5$ is reasonable because a workflow denotes an execution sequence of a business process in the elastic cloud environment. The risk level of response time from task $t_1$ to $t_9$ is 38.6%, which is relatively greater. Risk managements and control measures should be strengthened for the purpose of risk mitigation.

In the second case, we considered multiple risk elements, i.e., $C$, $R$, and $A$ mentioned in Section 3.2.

According to Algorithm 2, the total risk level can be calculated as follows:

\[
E(Y)_{19} = \sum_{k=1}^{3} \frac{\partial}{\partial s_k} \left[ \frac{W_{ij}(s_k)}{W_{ij}(0,0,0)} \right] \bigg|_{s_1=s_2=s_3=0} = 345 + 22.5075 + 3.99178
\]
\[
V(Y)_{19} = \left( \frac{\partial^2}{\partial s_k^2} \left[ \frac{W_{ij}(s_1,s_2,\ldots,s_k,\ldots,s_n)}{W_{ij}(0,0,\ldots,0,\ldots,0)} \right] \bigg|_{s_1=s_2=\ldots=s_k=\ldots=s_n=0} \right)^2
\]
\[
= 75.5 + 10546.125 + 0.6787
\]
\[
\Psi_{19} = 0.277422
\]

It should be noted that the risk level can also be specified similarly under the condition of other probability distribution.

5. Conclusion and Future Work

This paper gives the typical analytical model of QoS risk in the general cloud model. Because the transmission characteristic of GERT network is especially suitable for the solution of risk element transmission, so a cloud QoS risk element transfer model based on workflow is established. The proposed model is of certain generality and typicality. Different risk level definition is defined according to the number of risk elements, and two algorithm for the risk level is given based on GERT model. At the end of paper,
a numerical example is given to solve the model. The results show that, when the expectation and variance of QoS criteria of a task are obtained, the risk level of its completion probability is further analyzed.

In the future, we plan to integrate proposed model to scheduling strategy for cloud workflow applications.

Acknowledgements

This study is funded by the National Nature Science Foundation of China (Grant number: 71271084, 71671065, 71662022) and the technology project of State Grid Corporation of China (Project number: 5204BB1600CP).

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(Received February 2017; accepted April 2018)