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Optimal Procurement Strategies for a Risk-Averse Buyer When Price Is Uncertain

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Abstract

A procurement model is built based on the expected utility theory when purchase price is uncertain. The optimal purchasing strategy for a risk-averse manufacturer is obtained. And the influence of different factors on the optimal purchasing strategy is analyzed. The results show that the relationship between the optimal purchasing quantity and the degree of risk aversion depends on the relative level of procurement cost at that time. As the price volatility or drift rate increases, the purchase amount in the early stages will increase, and the purchase amount in the later stages will decrease. Finally, it is demonstrated that the procurement strategy of a risk-neutral manufacturer is a bang-bang strategy, which is different from the procurement strategy of a risk-averse manufacturer.

Keywords: Price risk, procurement strategy, risk aversion, expected utility.

1. Introduction

At the age of economic globalization, manufacturing companies (buyers) often procure raw materials from global markets. There are two pricing mechanisms in raw material markets for iron ore, chemical products, and electronic products. One is a long-term agreement pricing mechanism between a buyer and a seller, and the other is a market price mechanism for spot trading. However, the long-term pricing of raw materials is gradually reduced in some markets, while spot trading is increasing. For example, China rejected the long-term pricing model of iron ore in 2010 that had lasted for nearly 40 years, and switched to a monthly pricing model. In addition, with the rapid development of IT, the spot market has been rapidly developed. Many online trading markets such as ChemConnect, E-Steel and Converge have emerged, which dramatically reduce the cost of spot trading.

Manufacturers can immediately obtain raw materials from the spot market to meet production needs. However, fluctuations in spot price have caused difficulties to procurement activities. Due to various factors such as climates, interest rates and exchange rates, supply and demand in raw material markets often change. Changes in supply and demand lead to fluctuations in spot price, as the spot price can quickly reflect the supply and demand. This has caused trouble for the spot market procurement, often causing great losses to the company and even leading to bankruptcy. Therefore manufacturing firms that procure materials from the spot market need to develop appropriate strategies to control procurement risk and cost.

2. Literature Review

The classic theory of purchasing and inventory management mainly focuses on the uncertainty of demand. In recent years, more and more research considers the uncertainty of purchasing cost (see Gaudenzi et al.[8], Hallikas and Lintukangas [10], Pellegrino, Costantino and Tauro [15]). Gaudenzi et al. provided strategies to mitigate the commodity price risk by implementing various sourcing, contracting, and financing strategies. Hallikas and Lintukangas investigated the effectiveness of two supply chain risk management strategies in mitigating price risk, namely, switching suppliers and substituting commodities. Pellegrino et al. investigated actions that influence a company's supply risk management performance. The current research in this area can be divided into two categories: one is the study of purchasing strategies using only a single market, and the other is the study of a combination of multiple contracts such as forward and option contracts (see Feng, Mu and Hu [7], Kleindorfer and Wu [12], Xanthopoulos, Vlachos and Iakovou [18]). This study belongs to the former category, so the following mainly summarizes the research of this category.

Fabian, Fisher and Sasieni [6] studied a procurement model considering the price uncertainty. Kalymon [11] established a model of price following the Markov stochastic process, and the result showed that the optimal purchasing strategy depended on the price level. When the price followed the Markov process and the demand followed a Poisson distribution, Yang and Xia [19] got that a basic stock strategy was the best strategy. Berling and Martinez-de-Albeniz [2] described the characteristics of a price-dependent basic stock strategy.

The above studies investigated the procurement strategy in discrete time, while other studies considered the procurement strategy in continuous time. Arnold et al. [1] used the optimal control method to find the optimal purchasing strategy in an uncertain environment. Guo et al. [12] also used control theory to analyze an inventory problem. However, these procurement models didn't consider the risk-averse characteristics of buyers.

The above models assume that decision makers are risk-neutral, while in reality decision makers tend to be risk averse. Bouakiz and Sobel [3] used an exponential utility model to analyze a multi-period Newsboy problem. Shu L et al. [17] used an increasing and concave utility function to describe risk aversion. Oberlaender [14] studied the dual-source procurement problem of buyers with different risk preferences. Seifert et al. [16], Chiu and Choi [4] used the Mean-variance criterion for conducting risk analysis in stochastic supply chain operational models. And some studies applied Value at Risk (VaR) and Conditional Value at Risk (CVaR) models to the risk assessment in electricity trading (see Dahlgren, Liu and Lawarree [5], Li et al. [13]). These scholars used different models to introduce the risk attitude of buyers in a volatile environment.

This paper investigates optimal procurement strategies for a risk-averse manufacturer when purchase price is uncertain. Based on the existing research, we establish a procurement model that uses the expected utility to introduce the risk aversion of a buyer. When the buyer's utility function is in the form of a quadratic function and raw material price follows the geometric Brownian motion, an analytic solution of the buyer's optimal purchasing strategy is obtained. Then, we examine the impact of demand, risk aversion and price risk on the optimal purchasing strategy. Finally, we compare purchasing strategies of a risk-neutral buyer and a risk-averse buyer.

3. Problem and Model

A risk-averse manufacturer (Decision Maker, DM) needs raw materials for production at the end of a period (T). The manufacturer can buy raw materials from a spot market at any time t during the period ($[0, T]$), where the spot price is volatile. The demand for raw materials at T is D, which is uncertain. The mean and standard deviation of demand are μ_D and σ_D, respectively.

The spot price of raw materials is $p(t)$ and the quantity purchased is $q(t)$ at time t ($0 \leq t \leq T$). The price is not affected by the manufacturer's purchasing quantity. Assume that $q(t) \geq 0$, and the lead time is equal to 0.

The manufacturer has to develop appropriate procurement strategies for raw materials to control procurement risk and cost. From the initial time 0 to the end of the period T, the DM's total utility function $v(x(0))$ is the sum of purchasing utilities during the period (see Oberlaender [14], Shu L et al. [17]).

$$\max v(x(0)) = E_0\Big[\int_0^T \delta^t u(l(t))dt\Big] \tag{3.1}$$

$$\begin{aligned} \text{s.t.} \qquad & x(0) = x_0 \\ & x(T) = x(0) + \int_0^T q(t)dt = D \end{aligned} \tag{3.2}$$

$x(t)$ is the inventory level of raw materials at time t, and $x'(t) = q(t)$.
δ is the discount factor of the DM, and $\delta = e^{-r}$, where r is a discount rate.
$l(t)$ is the cash flow at time t, and $l(t) = -p(t)q(t)$.
$u(l)$ is the utility function of the DM. $u(l)$ is an increasing and concave function, characterizing the risk attitude of the DM.

The boundary constraint (3.2) indicates that the inventory of raw materials at time T must meet production demand. Assume that the demand at time T is larger than the inventory at the beginning ($D > x_0$). Otherwise the manufacturer does not need to procure any raw materials. Eq.(3.1) shows that the total expected utility of the DM is the sum of utilities at each time in the procurement cycle. The aim of the DM is to maximize the total expected utility level $v(x(0))$.

In the procurement practice, raw materials are generally purchased in batches. A manufacturer's optimal purchasing strategy can be obtained by the discretization of model (3.1). The time period $[0,T]$ is divided into N discrete intervals to obtain $N+1$ times, denoted by i, $i=0,\dots,N$. Then the total utility function of DM can be expressed as

$$\begin{aligned} \max v(x_0,q(i)) &= E_0\Big[\sum_{i=0}^{N}\delta^i u(l(i))\Big] \qquad (3.3)\\ \text{s.t.} \qquad x(N) &= x_0+\sum_{i=0}^{N} q(i)=D. \end{aligned}$$

The utility function u is concave, so $v(x_0,q(i))$ is concave and has a maximum value. Substitute the constraint into Eq.(3.3) to get

$$v(x_0,q(i)) = E_0\Big[\sum_{i=0}^{N-1}\delta^i u(-p_i q_i)+\delta^N u\Big(-p_N(D-x_0-\sum_{i=0}^{N-1} q_i)\Big)\Big]. \qquad (3.4)$$

It can be obtained from the FOC of Eq.(3.4) that

$$\delta^i E_0(u'(-p_i q_i^*)p_i)=\delta^T E_0\Big(u'\Big(-p_N(D-x_0-\sum_{i=0}^{N-1} q_i^*)\Big)p_N\Big), \quad i=0,1,\dots,N-1. \qquad (3.5)$$

The optimality condition (3.5) shows that the discounted value of the expected marginal utility of purchase at each time is equal. The optimal purchasing strategy $q^*(i)$ can be obtained from Eq.(3.5).

Suppose that the utility function $u(l)=l-\frac{1}{2}kl^2$, where $k>0$. The larger the k is, the more risk averse the buyer is. Geometric Brownian motion is a commonly used stochastic price model under the circumstance of fluctuating commodity price. The spot price $p(t)$ follows a stochastic process of geometric Brownian motion.

$$\frac{dp}{p}=\mu dt+\sigma dW \quad \text{or} \quad d\ln p=\Big(\mu-\frac{\sigma^2}{2}\Big)dt+\sigma dW,$$

where μ is the price drift rate, which represents the price trend. σ is the price volatility, which represents the degree of price fluctuations. dW is the Wiener process, which represents the random factors that affect the price changes. If the DM's utility function $u(l)$ is in the form of a quadratic function and the spot price follows the geometric Brownian motion, the analytic expression of the optimal strategy can be obtained. That is,

$$q^*(i)=\frac{k(\mu_D-x_0)+\sum_{j=0}^{N}\frac{\delta^j E(p_j)-\delta^i E(p_i)}{\delta^j E(p_j^2)}}{k\sum_{j=0}^{N}\frac{\delta^i E(p_i^2)}{\delta^j E(p_j^2)}}, \quad i=0,1,\dots,N. \qquad (3.6)$$

And the amount of purchase at each time $q(i) = \max(q^*(i), 0)$.

4. Sensitivity Analysis of Optimal Purchasing Strategy

After obtaining the optimal purchasing strategy, we further analyze the impact of production demand, risk aversion and price risk on the optimal strategy. Eq. (3.6) is used to analyze the change of the optimal strategy with various factors. And a numerical analysis is conducted by Monte Carlo method. The settings in the numerical analysis are based on a raw material (copper) market. In the numerical analysis, $x_0 = 0$, $p_0 = 580$, $\mu_D = 50$, $\delta = 0.9$, $k = 0.0001$, $\mu = 16.4\%$ and $\sigma = 34.9\%$. The purchasing period is 3 months long, which is divided into 3 intervals. And the purchasing times are denoted by 0,1,2,3.

4.1. The impact of production demand

Proposition 1. *The optimal purchasing quantity increases with the expected demand, but it is irrelevant to fluctuations in demand.*

Proof. From Eq.(3.6), if the expected demand μ_D increases, the optimal purchasing strategy $q^*(i)$ increases linearly with the demand. Howerver, σ_D does not affect the optimal purchasing strategy. Proposition 1 holds.

Figure 1 shows the comparison of purchasing quantities when the expected demand is 50, 60, and 70, respectively. The purchasing quantity at each time increases when the expected demand becomes larger.

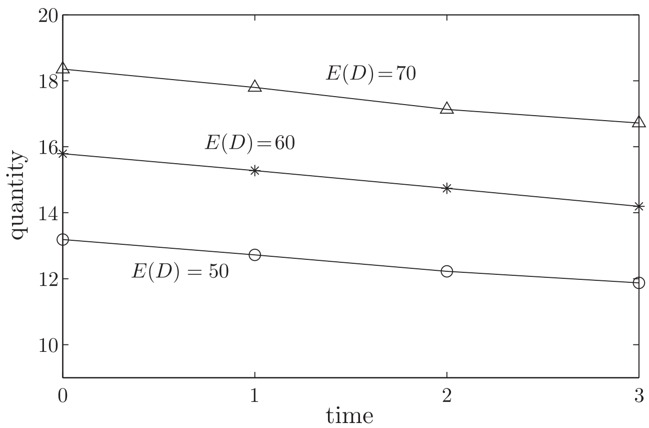

Figure 1: Impact of production demand on purchasing quantities.

4.2. The impact of risk aversion

Let $DC_i = \sum_{j=0}^{N} \dfrac{\delta^j E(p_j) - \delta^i E(p_i)}{\delta^j E(p_j^2)}$, and $BQ_i = (\mu_D - x_0) \Big/ \Big[\sum_{j=0}^{N} \dfrac{\delta^i E(p_i^2)}{\delta^j E(p_j^2)} \Big]$.

BQ_i represents the base purchase quantity at time $t(i)$. DC_i represents the relative level of purchase cost at time $t(i)$ during the procurement period.

Proposition 2. *When $DC_i > 0$, $q^*(i)$ increases as risk aversion increases.*

When $DC_i = 0$, $q^*(i)$ is not affected by risk aversion.

When $DC_i < 0$, $q^*(i)$ decreases as risk aversion increases.

Proof. Eq.(3.6) becomes

$$q^*(i) = BQ_i + \frac{DC_i}{k \sum_{j=0}^{N} \frac{\delta^i E(p_i^2)}{\delta^j E(p_j^2)}}. \tag{4.1}$$

From Eq.(4.1), the relationship between $q^*(i)$ and k is determined by the sign of DC_i. Proposition 2 holds.

DC_i=0 indicates that the purchase cost is just at an average level, and the optimal purchase quantity is equal to the base purchase quantity ($q^*(i) = BQ_i$) at $t(i)$. $DC_i < 0$ indicates that the purchase cost is relatively higher than the average level, and the optimal purchase quantity is lower than the base purchase quantity at $t(i)$. $DC_i > 0$ indicates that the purchase cost is relatively lower than the average level, and the optimal purchase quantity is higher than the base purchase quantity at $t(i)$.

Figure 2 shows the impact of risk aversion on the optimal number of purchases. The procurement curve tends to be flat as the risk aversion of the DM increases. When $k = 0.00001$, $q^*(0) = 14.27$ and $q^*(3) = 10.91$. When $k = 0.0001$, $q^*(0) = 13.17$ and $q^*(3) = 11.84$. As k increases, the amount of purchase at the early period will decrease, while the amount of purchase at the later period will increase.

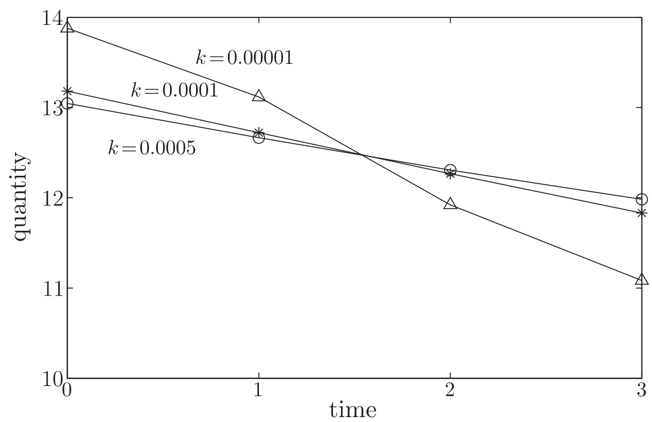

Figure 2: Impact of risk aversion on purchasing quantities.

The relative level of procurement cost affects the purchasing quantity. A manufacturer with a low degree of risk aversion is aggressive, and will procure more raw materials at lower expected cost. A manufacturer with a high degree of risk aversion is relatively conservative, and tends to make average purchases at different times.

4.3. The impact of spot price risk

Proposition 3. *As the price volatility or drift rate increases, the amount of purchase in the early stages will increase, while the amount of purchase in the later stages will decrease.*

Proof. From Eq.(3.6), we can get

$$q^*(i) = \frac{k(\mu_D - x_0) + \dfrac{1}{p_0}\sum\limits_{j=0}^{N}[\exp(-\mu j - \sigma^2 j) - \delta^{i-j}\exp(\mu i - 2\mu j - \sigma^2 j)]}{k\sum\limits_{j=0}^{N}\exp(2\mu(i-j) + \sigma^2(i-j))}. \tag{4.2}$$

Based on Eq.(4.2), the numerator is greater than 0. When i is small (in the early stages), the denominator becomes smaller as the price volatility (σ) or drift rate (μ) increases. And the amount of purchase in the early stages increases. When i is large (in the later stages), the denominator becomes larger as the price volatility (σ) or drift rate (μ) increases. And the amount of purchase in the later stages decreases. Proposition 3 holds.

Figure 3 shows the impact of price volatility on the optimal purchasing quantity $q^*(i)$. When $\sigma = 0.3487$, $q^*(0) = 13.17$ and $q^*(3) = 11.84$. When $\sigma = 1.0000$, $q^*(0) = 14.57$ and $q^*(3) = 10.56$. That is, as σ increases, the amount of purchase in the early stages will increase, while the amount of purchase in the later stages will decrease. The manufacturer is risk averse and unwilling to take too much risk of price fluctuations. Adopting this strategy can reduce the risk of price fluctuations.

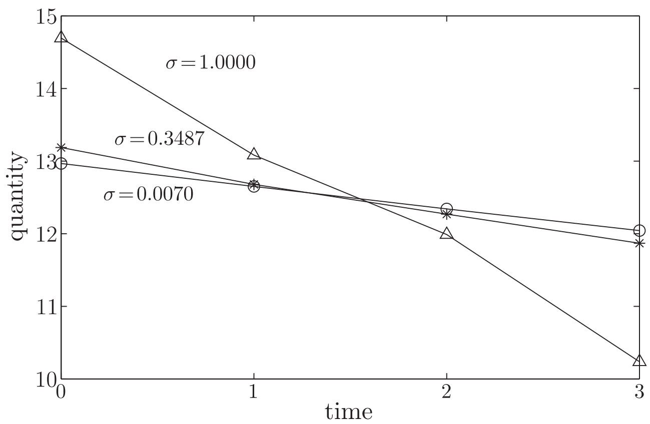

Figure 3: Impact of price volatility on purchasing quantities.

Figure 4. shows the impact of price drift on the optimal purchasing quantity $q^*(i)$. When the drift rate becomes larger, the amount of purchase in the early stages will increase and the amount of purchase in the later stages will decrease. The expected price of raw materials will be higher and the purchase cost will increase if the drift rate increases. Adopting this strategy can reduce procurement cost.

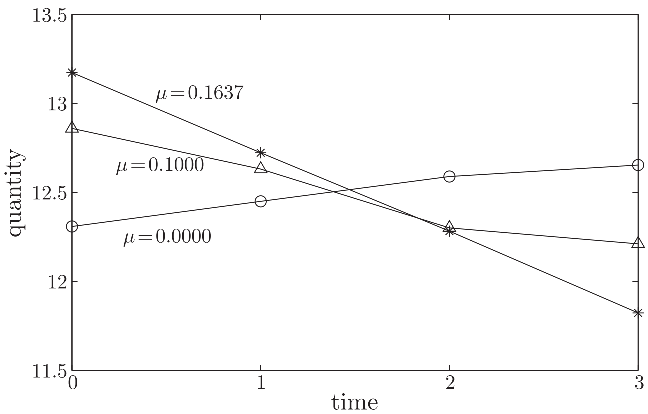

Figure 4: Impact of price drift on purchasing quantities.

5. Comparative Analysis of a Risk-Averse Strategy and a Risk-Neutral Strategy

If a DM is risk neutral, the DM's aim is to minimize the total procurement cost. That is,

$$\min v(x_0, q(i)) = E_0\Big[\sum_{i=0}^{N} e^{-ri} p(i) q(i)\Big] = \sum_{i=0}^{N} q(i) E_0[e^{-ri} p(i)] \tag{5.1}$$

$$\text{s.t.} \quad x(N) = x_0 + \sum_{i=0}^{N} q(i) = D,$$

$$q(i) \geq 0.$$

Eq.(5.1) is a linear programming problem, where $E_0[e^{-ri}p(i)]$ represents the expected discounted purchase cost of raw materials. The optimal solution can be obtained by analyzing the dual model of this problem. That is, $q^*(i) = \mu_D$, where the purchase time i takes $E_0[e^{-ri}p(i)]$ to the minimum value. $q^*(i) = 0$ at the rest of purchase times.

Therefore, a risk-neutral DM's optimal strategy is to procure raw materials when the expected purchase cost is at the lowest, and purchase 0 at the rest of purchase times. The purchasing quantity of a risk-neutral DM at time i is 0 or μ_D. This optimal strategy is called a bang-bang strategy.

When spot price follows the geometric Brownian motion, we can get $E_0[e^{-ri}p(i)] = e^{(\mu-r)i}$. If the price drift rate is greater than the discount rate ($\mu > r$), the DM purchases raw materials at the beginning of the period. If the price drift rate is less than the discount rate ($\mu \leq r$), the DM purchases raw materials at the end of the period.

In summary, the optimal procurement strategies of a risk-averse DM and a risk-neutral DM are different. A risk-averse DM makes multiple purchases of raw materials during the procurement period, while a risk-neutral DM only makes one purchase.

The procurement costs of a risk-averse buyer and a risk-neutral buyer are denoted as TC and TC0, and the expected utilities are denoted as EU and EU0 (see Table 1), respectively. Table 1 shows that the expected utility level of a risk-averse DM is higher than the utility of a risk-neutral DM, but the expected procurement cost of a risk-averse DM increases. Based on Table 1, a risk-averse manufacturer trades off the procurement cost and the risk. The cost of improving the expected utility is to increase the expected procurement cost.

Table 1: Expected cost and expected utility of two purchasing strategies.

k (10e-5)	TC0	TC	EU0	EU
2	29000.0	29195.5	-37410.0	-31360.4
4	29000.0	29198.2	-45820.0	-33522.2
6	29000.0	29199.1	-54230.0	-35683.2
8	29000.0	29199.6	-62640.0	-37842.4
10	29000.0	29199.9	-71050.0	-40003.6

6. Conclusion

Fluctuations of spot price bring risks to purchases. This paper studies the optimal purchasing strategy of a risk-averse manufacturer under price fluctuations. Based on the expected utility theory, a DM's risk aversion is introduced and a procurement model is established. Under the assumption that the DM has a quadratic utility function and the spot price follows the geometric Brownian motion, the optimal strategy of the risk-averse DM is obtained.

Then, the influence of production demand, risk aversion and price risk on the optimal purchasing strategy is analyzed. The results show that the optimal purchase quantity becomes larger with the increase of production demand, but is not affected by fluctuations in demand. The relationship between the optimal purchase quantity and risk aversion is determined by the sign of DC_i. As price volatility or drift rate increases, the manufacturer will increase purchase volume of the early stages, and reduce purchase volume of the later stages. Finally, the comparison shows that the procurement strategy of a risk-averse DM is different from the procurement strategy of a risk-neutral DM. A risk-averse DM makes multiple purchases of raw materials to meet demand, while a risk-neutral DM only makes one purchase.

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