Multiple Level Programming: An Introduction

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Brief Contents

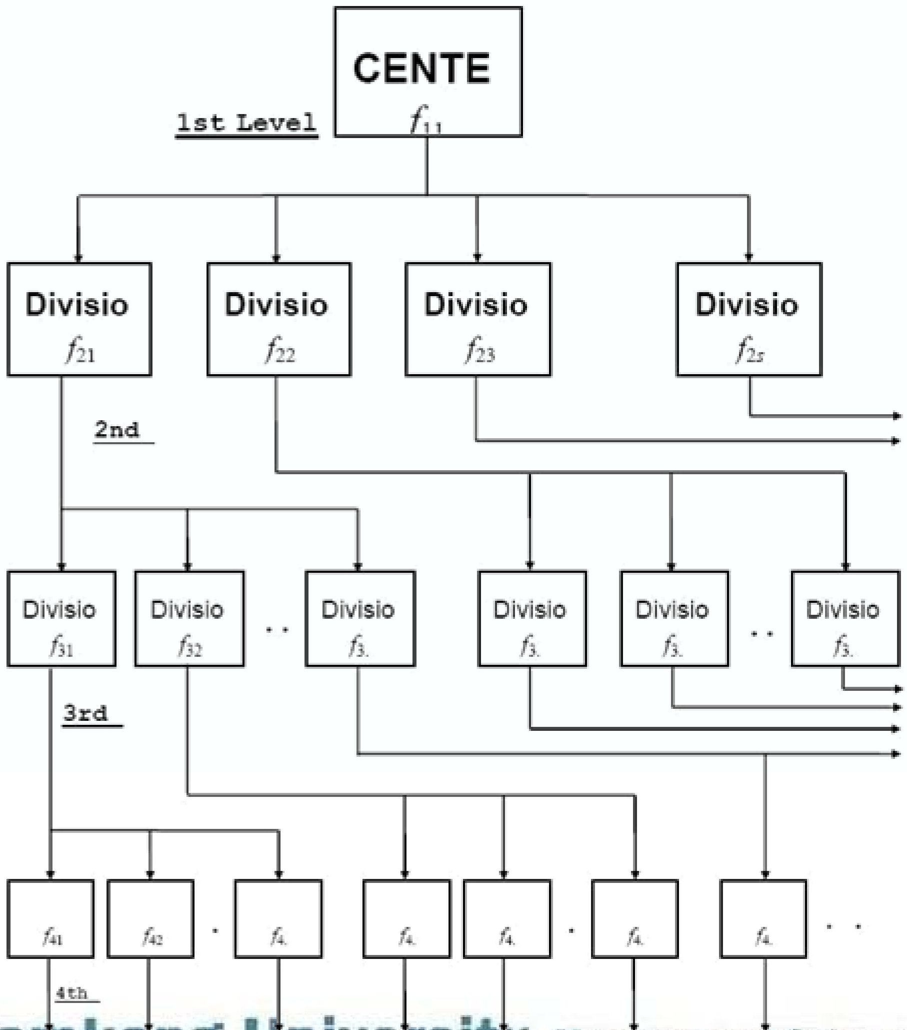
Introduction

- **What is multiple level programming?**
  - **Decentralized planning in organizations**
- **Where are its applications?**
  - **Many areas with conflict resolution**
- **What's techniques deal with the problems?**
  - **Traditional and non-traditional techniques**
- **Future Research**

Definition

- **Multiple Level Programming (MLP)**
  - **To solve decentralized planning problems with multiple executors in a hierarchical organization**
  - **Explicitly assigns each agent a unique objective and set of decision variables as well as a set of common constraints that affects all agents**

Hierarchical Structure



MLP Formulation

Multi-level (k levels decentralized) mathematical programming :

$\underset{x1}{\text{Max}} \quad f_{11}(\boldsymbol{x}) = \sum_{j=1}^{n} c_{11j}^{T} x_j ,$ (1st level)

where $x_{21}, x_{22}, \ldots, x_{2p}$ solve individually,

$$\left\{ \begin{array}{l} \underset{x21}{\text{Max}} \quad f_{21}(\boldsymbol{x}) = \sum_{j=1}^{n} c_{21j}^{T} x_j , \\ \ldots \\ \underset{x2p}{\text{Max}} \quad f_{2p}(\boldsymbol{x}) = \sum_{j=1}^{n} c_{2pj}^{T} x_j , \end{array} \right.$$ (2nd level)

...

where $x_{K1}, x_{K2}, \ldots, x_{Ks}$ solve individually,

$$\left\{ \begin{array}{l} \underset{xK1}{\text{Max}} \quad f_{K1}(\boldsymbol{x}) = \sum_{j=1}^{n} c_{K1j}^{T} x_j , \\ \ldots \\ \underset{xKs}{\text{Max}} \quad f_{Ks}(\boldsymbol{x}) = \sum_{j=1}^{n} c_{Ksj}^{T} x_j , \end{array} \right.$$ (Kth level)

$$\text{s.t.} \quad \sum_{\forall k,i}^{n} \boldsymbol{A}_{ki} x_i \leq \boldsymbol{b} ,$$

$$x_j \geq \boldsymbol{0} , \quad j = 1, 2, \ldots, n$$

and $n = 1+p+q+ \ldots +s$,

where $j = 1, 2, \ldots, n$ represents the jth decision variable, and $k = 1, 2, \ldots, K$ represents the kth level, respectively. In addition, the decision variable set $\cup_{k,i} \{x_{ki} \mid \forall\ i \text{ and } k\} = \{x_1, x_2, \ldots, x_n\} = \{x\}$.

Characteristics (I)

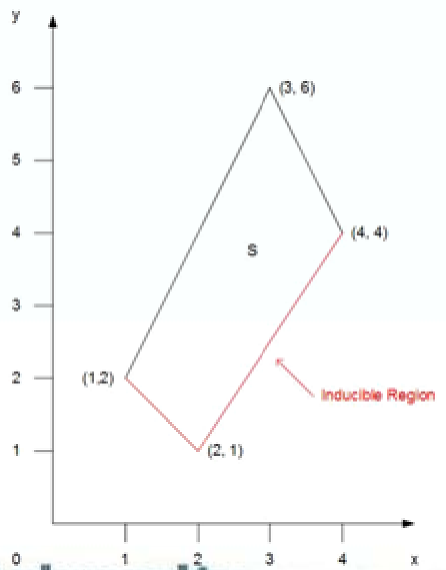
- **Common Characteristics of MLP**
  1) **Interactive decision-making units exit within a predominantly hierarchical structure**
  2) **Execution of decisions is sequential, from top level to bottom level**
  3) **Each unit independently maximizes its own net benefits, but is affected by actions of other units through externalities**
  4) **The external effect on a decision-maker’s problem can be reflected in both his objective function and his set of feasible decision space**

Characteristics (II)

- **Consider a constrain region of the bi-level programming problem**
  - **Follower's rational reaction set**
  - **Inducible region – non-convexity**

$F(x, y) = x - 4y$

$f(y) = y$



Non-convex in general

Bi-level Programming– a simple case

Problem formulation

$$\max_{x1} \quad f_1(x_1, x_2) = c_{11}^{T} x_1 + c_{12}^{T} x_2 \qquad \text{(upper level)}$$

where x_2 solves,

$$\max_{x2} \quad f_2(x_1, x_2) = c_{21}^{T} x_1 + c_{22}^{T} x_2 \qquad \text{(lower level)}$$

s.t.

$$(x_1, x_2) \in X = \{(x_1, x_2) \mid A_1 x_1 + A_2 x_2 \leq b, \text{ and } x_1, x_2 \geq 0\}$$

where c_{11}, c_{12}, c_{21}, c_{22}, and b are vectors, A_1 and A_2 are matrices, and X represents the constraint region.

Wen and Hsu, 1991

Bi-level Programming

- **A Special Case of Two-person, Non-zero Sum Non-cooperative Game**
  - **A general Stackelberg's (leader-follower) duopoly model**
- **Nested Optimization Problem**
  - **NP-hard complexity**

Applications (I)

- **Agricultural model**
  - Agricultural policy- Nile Valley case (Parraga, 1981)
  - Milk industry (Candler and Norton, 1977)
  - Mexican agriculture model (Candler and Norton, 1977)
  - Water supply model (Candler et al., 1981)
- **Government policy**
  - Distribution of government resources (Kyland, 1975)
  - Environmental regulation (Kolstad, 1982)
- **Finance model**
  - Bank asset portfolio (Parraga, 1981)
  - Commission rate setting (Wen and Jiang, 1988)

Applications (II)

- **Economic systems**
  - Distribution center problem (Fortuny and McCarl, 1981)
  - Principle-agent model (Arrow, 1986)
  - Price ceilings in the oil industry (DeSilva, 1978)
- **Welfare**
  - Allocation model of strategic weapons (Bracken et al., 1977)
- **Transportation**
  - Highway network system (LeBlance and Boyce, 1986)
- **Others**
  - Network flows (Shih and Lee, 1999; Shih, 2005)
  - Supply chain (Viswanarthan et al., 2001)

Techniques (I)

- **Extreme-point Search**
  - Kth-best algorithm
  - Grid-search algorithm
  - Fuzzy approach (Shih 1995, 2002; Shih et al., 1996)
  - Interactive approach (Shih, 2002)
- **Transformation Approach**
  - Complement pivot
  - Branch-and-bound
  - Penalty function
- **Interior Point**
  - Primal-dual algorithm

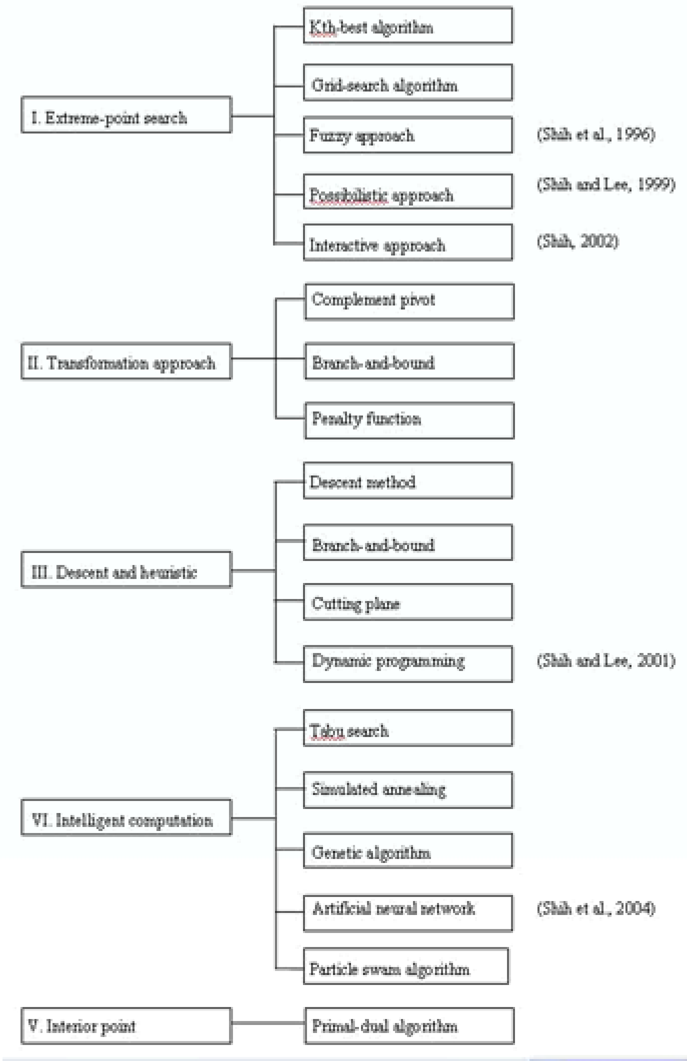
Lee and Shih, 2001; Shih et al., 2004

Techniques (II)

- **Decent and Heuristics**
  - Descent method
  - Branch-and-bound
  - Cutting plane
  - Dynamic programming (Shih and Lee, 2001; Shih, 2005)
- **Intelligent Computation**
  - Tabu search
  - Simulated annealing
  - Genetic algorithm
  - Artificial neural network (Shih et al., 2004)

Lee and Shih, 2001; Shih et al., 2004

Categories of Techniques



Example 1. A Trade-off Problem between Exports and Imports

Problem formulation

$$\max_{x_1} f_1 = 2x_1 - x_2 \quad \text{(effect on the export trade - 1st objective)}$$

where x_2 solves,

$$\max f_2 = x_1 + 2x_2 \quad \text{(profits on the product - 2nd objective)}$$

s.t.

$$3x_1 - 5x_2 \le 15 \quad \text{(capacity)}$$

$$3x_1 - x_2 \le 21 \quad \text{(management)}$$

$$3x_1 + x_2 \le 27 \quad \text{(space)}$$

$$3x_1 + 4x_2 \le 45 \quad \text{(material)}$$

$$x_1 + 3x_2 \le 30 \quad \text{(labor hours)}$$

$$x_1, x_2 \ge 0 \quad \text{(non-negative)}$$

Kth-best Algorithm– Extreme-point

- ## Solving procedure
  - Step 1. Solve the upper-level problem

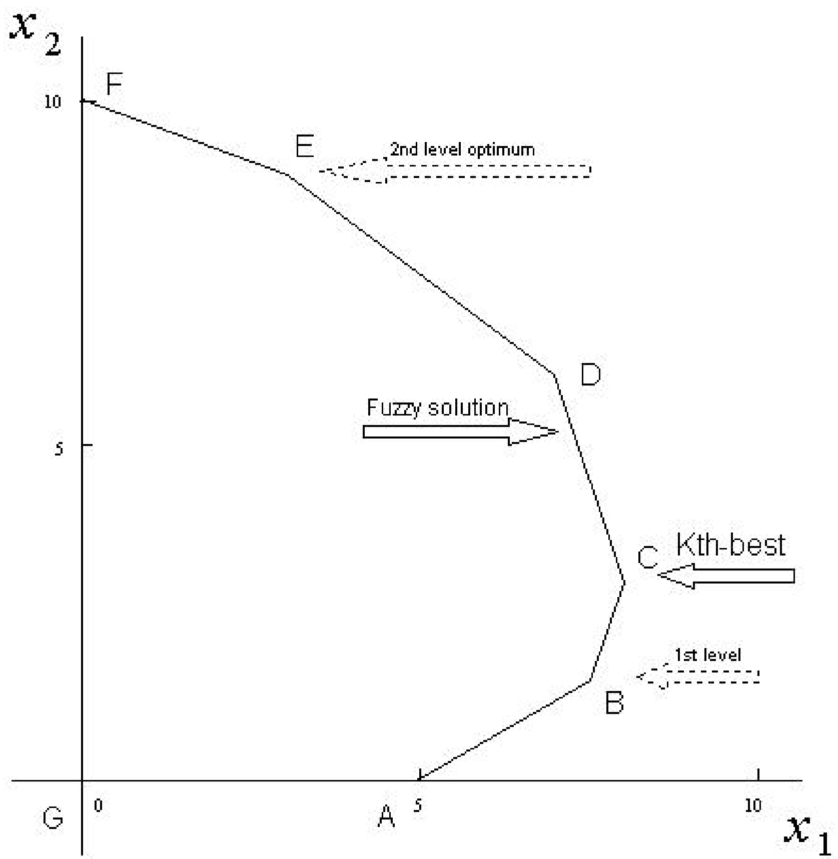
    i=1, $x_{[1]}$*= (7.5,1.5) at vertex B
  - Step 2. Solve the lower-level problem with x_1 = 7.5

    Solution x+= (7.5,4.5) between vertex D and vertex C

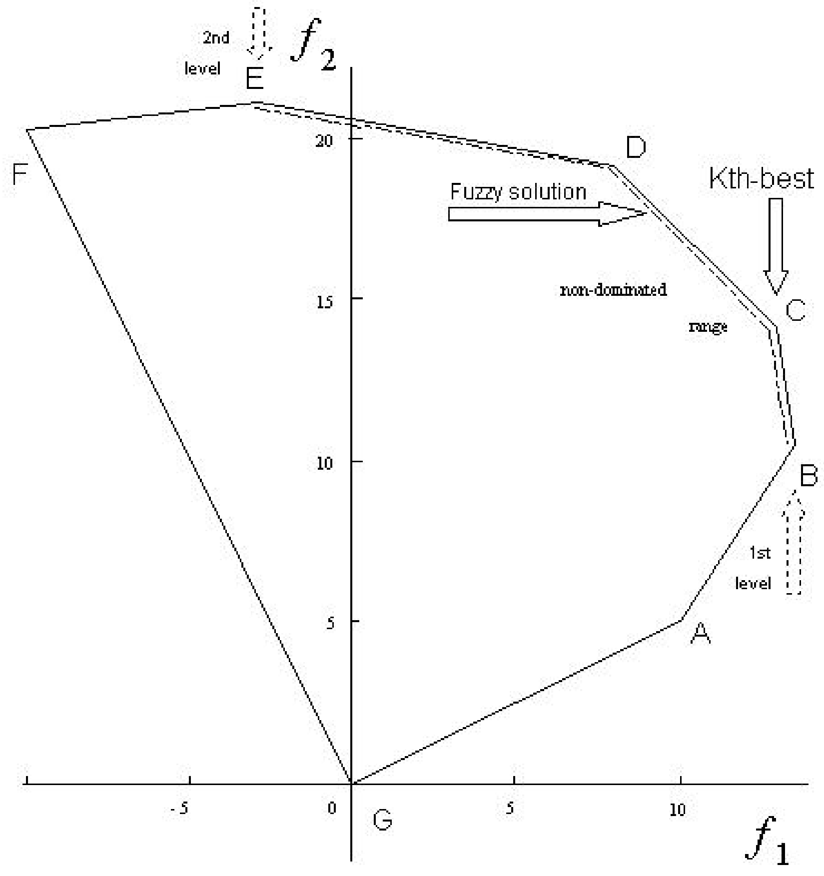
    $x+ \neq x_{[1]}*$, go to Step 3.
  - Step 3. Consider the neighboring set of $x_{[1]}$* (vertex A and vertex C)
  - Step 4. Update label i=i+1=2, and choose $x_{[2]}$* = (8,3) (vertex C). Go to Step 2.
  - Step 2. Let x_1 = 8 to the lower level problem

    Solution x+= (8,3). Since x+= $x_{[2]}$*, the procedure is terminated. $x_{[2]}$* is the optimum

Decision (Variable) Space



Objective (Function) Space



Karush-Kuhn-Tucker Conditions

- **Four sets of conditions**
  - **Stationarity**
  - **Complete slackness**
  - **Primal feasibility**
  - **Dual feasibility**

Karush-Kuhn-Tucker Conditions– Transformation approach

Problem formulation

$$\max_{x_1, x_2} f_1 = 2x_1 - x_2$$

s.t.

$$(x_1, x_2) \in X$$
$$w_1(-3x_1 + 5x_2 + 15) = 0$$
$$w_2(-3x_1 + x_2 + 21) = 0$$
$$w_3(-3x_1 - x_2 + 27) = 0$$
$$w_4(-3x_1 - 4x_2 + 45) = 0$$
$$w_5(-x_1 - 3x_2 + 30) = 0$$
$$-5w_1 - w_2 + w_3 + 4w_4 + 3w_5 = 2$$
$$w_1, w_2, w_3, w_4, w_5, x_1, x_2 \geq 0$$

Lee and Shih, 2001

Separation Procedure

Problem formulation

The constraint set

$w^T(A_1 x_1 + A_2 x_2 - b) = 0$**, where** w **is a dual vector.**

The transformed two terms

$w \le (1 - \eta) M$**, and**

$A_1 x_1 + A_2 x_2 - b \le M\eta$

where $\eta \in \{0, 1\}$ **and** M **is a large positive constant**

Lee and Shih, 2001

Concept of Fuzzy Approach

- **Fuzzy Membership Functions (Zadeh, 1965)**
  - **Tolerance of decisions**
  - **Achievement of goal**
- **Fuzzy Multi-objective Decision Making (Zimmermann, 1985)**
  - **Information aggregation**
- **Possibility theory (Zadeh, 1978)**
  - **Imprecise range**

→ Supervised search procedure

Shih, 1995; Shih et al., 1996

Fuzzy Approach

Problem formulation

Max $f_2 = 2x_1 - x_2$

s.t.

$(x_1, x_2) \in X$

$\mu_{f1}(f_1(x)) \geq \alpha$

$\mu_{x1}(x_1) \geq \beta$

$\alpha \in [0, 1]$ and $\beta \in [0, 1]$

$x_1, x_2 \geq 0$

Shih, 1995; Shih et al., 1996

Fuzzy Decision

Problem formulation

Max $\{\alpha, \beta, \delta\}$

s.t.

$(x_1, x_2) \in X$

$\mu_{f1}(f_1(x)) = (f_1 - 0) / (13.5 - 0) \geq \alpha$

$\mu_{x1}(x_1) = (x_1 - 4.5) / (7.5 - 4.5) \geq \beta$

$\mu_{x1}(x_1) = (8 - x_1) / (8 - 7.5) \geq \beta$

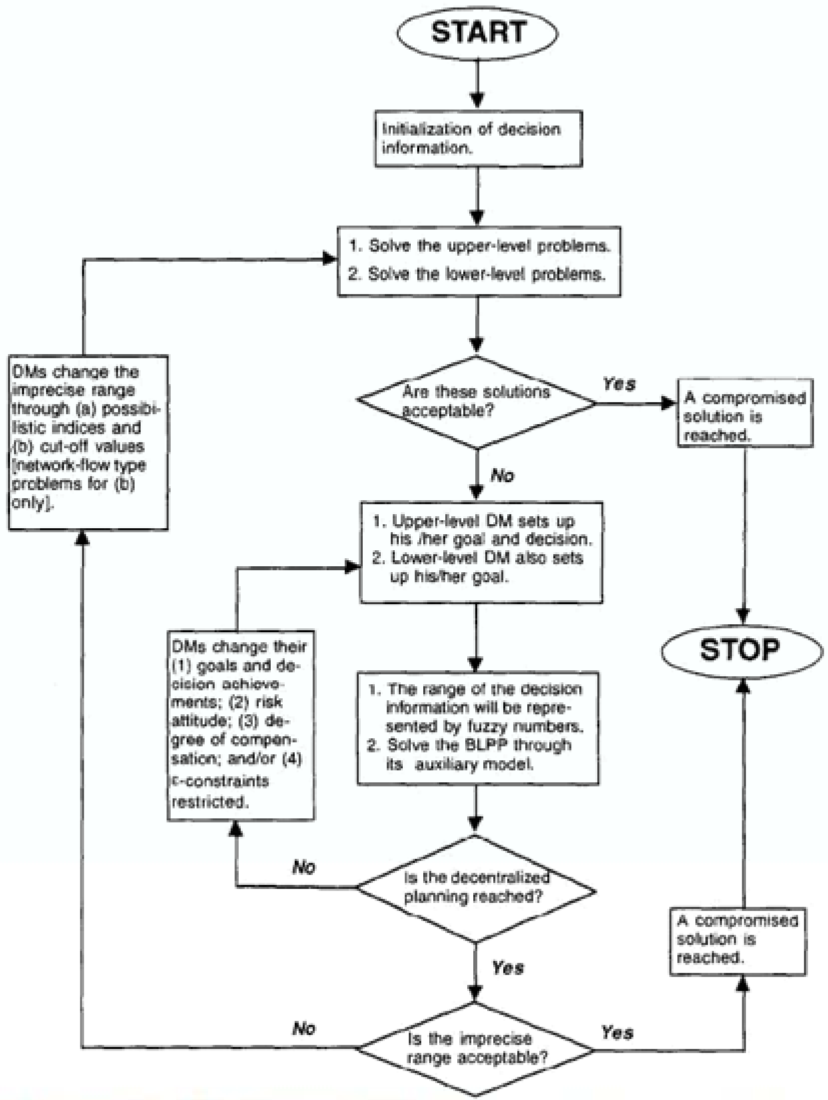
$\mu_{f2}(f_2(x)) = (f_2 - 10.5) / (21 - 10.5) \geq \delta$

$x_1, x_2 \geq 0$

$\alpha, \beta, \delta \in [0, 1]$

Shih, 1995; Shih et al., 1996

Interactive Approach



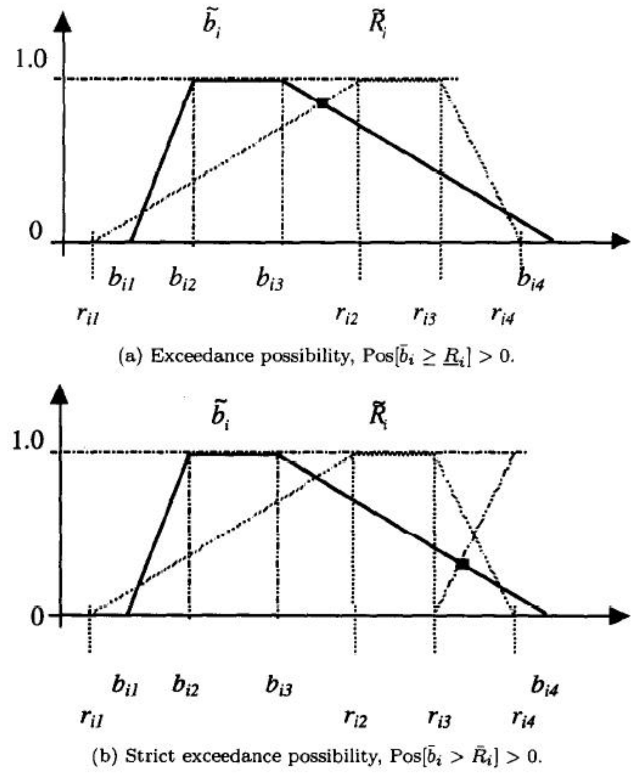
Shih, 2002

Advantages of Fuzzy Approach

- **Advantages**
  - Approximation of the natural of Large MLPPs
  - Not increase the computational complexity
  - Ease to extend to multiple levels
  - DMs involve the process
  - Efficient (Pareto) solution

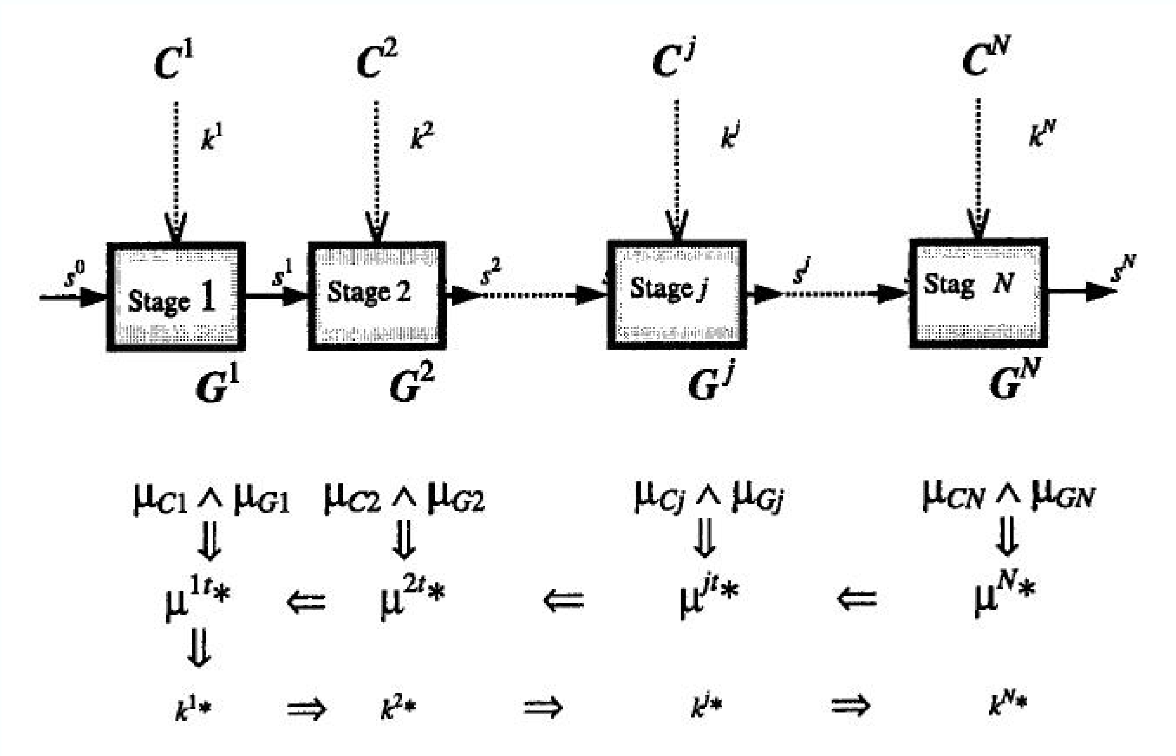
Nested Optimization $\Rightarrow$ Sequential Optimization

Extension to Vague Information



Vague/Imprecise data $\Rightarrow$ Possibilistic Distribution

Dynamic Aspect of MLP (I)



Dynamic environment $\Rightarrow$ Multi-stage MLP
(discrete space)

Shih, 2005; Shih and Lee, 2001

Dynamic Aspect of MLP (II)

- **Applications:**
  - **Shortest path problems**
  - **Knapsack problems**
  - **Other networks**

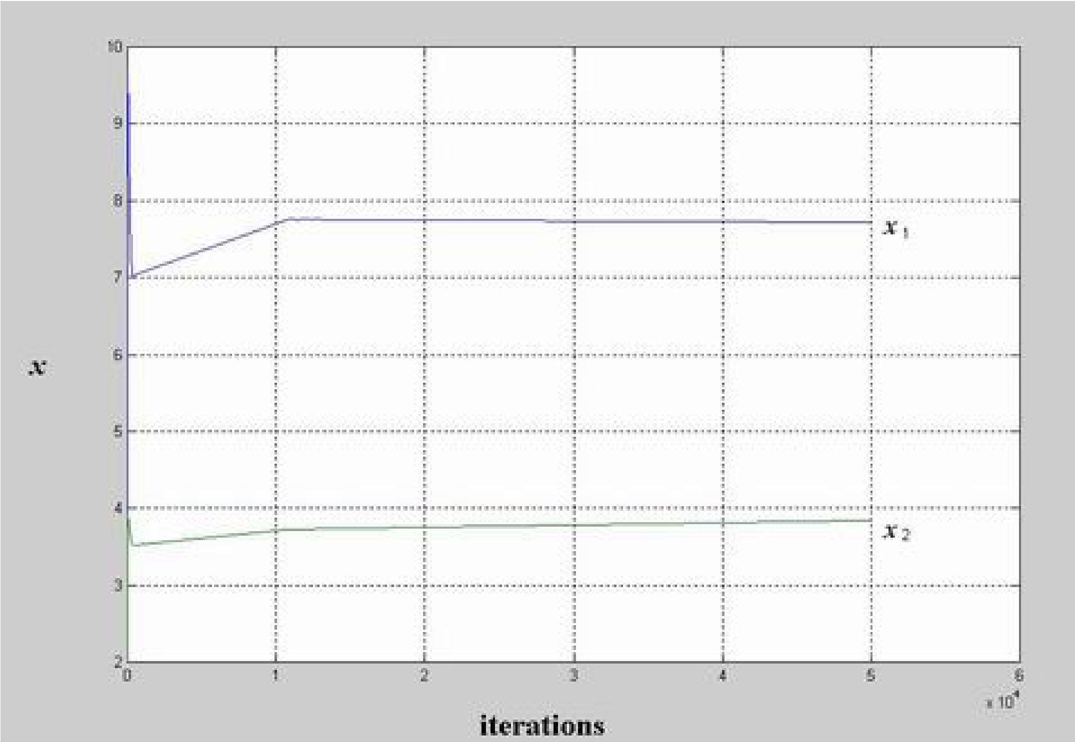
Shih, 2005; Shih and Lee, 2001

Neural Network Approach

- Use of dynamic behavior of artificial neural networks with parallel processing
- Based on Hopfield and Tank (1985)- recurrent network
- Transforming to the energy function without constraints
- Optimum solution with a steady state

Shih et al., 2004

Neural Network Approach



Future Research

- **Conditions of existing Pareto-optimal**
- **Use of hybrid algorithms for uncertainty**
- **Solutions of multi-subunits**
- **Extension to *n*-level problems**
- **Applications of real-world problems (nonlinear or stochastic coefficients, chance constraints, multi-level multi-objectives)**

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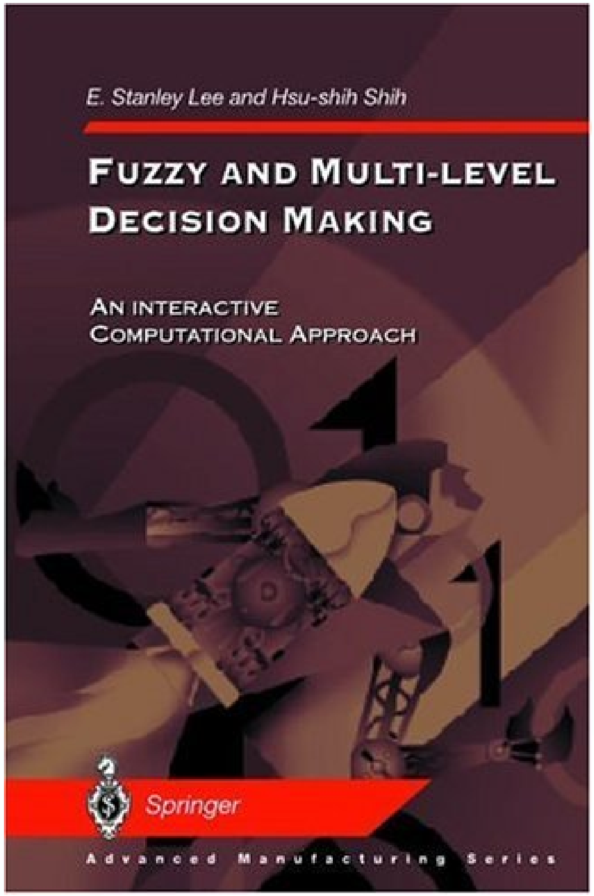

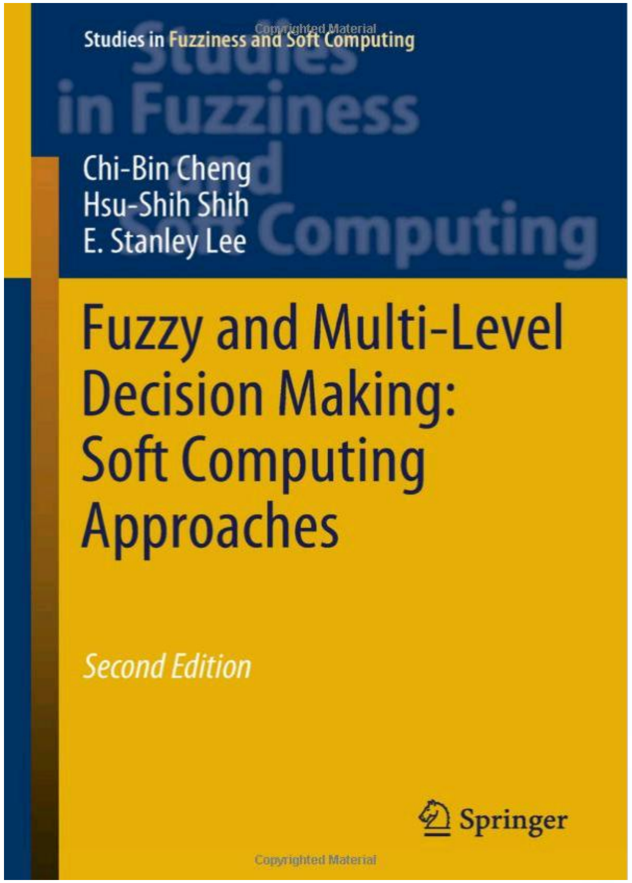

Questions & Comments

Thank you!